The Influence of Fiscal and Monetary Policies on the Shape of the Yield Curve

Yoosoon Chang* Fabio Gómez-Rodríguez† Christian Matthes‡

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Abstract

We investigate the influence of the U.S. government's spending and taxation decisions, along with the monetary policy choices made by the Federal Reserve, on the dynamics of the nominal yield curve. Aggregate government spending moves the long end of the yield curve, whereas monetary policy and changes in taxation move the short end of the yield curve on impact. Disentangling different types of government spending, we find that only government consumption exerts a discernible influence on the short end of the yield curve. The effects are generally transient and disappear after one year.

JEL Classification: E50, E62, G10

Key Words: Yield Curve, Fiscal Policy, Monetary Policy, Functional Time Series

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*Indiana University, yoosoon@indiana.edu.

†Lehigh University & Central Bank of Costa Rica, fag221@lehigh.edu

‡Indiana University, matthesc@iu.edu.

1 Introduction

How do policy actions influence the yield curve? In this paper, we study this question using a flexible statistical framework paired with standard identification assumptions on fiscal and monetary policy shocks. Why is this an interesting question? First, the yield curve is important to the economy because many interest rates (such as mortgage rates) move closely with the yield curve.

Second, macroeconomists have recently uncovered conditions under which fiscal and monetary policies can have the same effect on economic outcomes (Correia *et al.*, 2008; Wolf, 2021). Our paper complements those theoretical studies and highlights that, at least historically, fiscal policy (in particular, government spending) has moved the yield curve in a different fashion than monetary policy. Finally, in a more narrow sense, our results have implications for how government and central bank decisions change the borrowing costs of the government. Governments finance a substantial fraction of their outlays through debt issuance. Although there is a large literature on how government decisions affect aggregate macroeconomic outcomes (for example, Romer and Romer (2010) and Mertens and Ravn (2013) study the effects of various tax changes, while Blanchard and Perotti (2002), Auerbach and Gorodnichenko (2012), Ramey (2011), and Ramey and Zubairy (2018) study the effects of government spending), there is surprisingly no work on the effects of government decisions on its borrowing costs encoded in the yield curve of government liabilities.[1] Our paper tackles this question.

The question of how much a government's decisions change its borrowing costs is crucial for determining fiscal policies. This is most clearly evident from the literature on optimal fiscal policies in equilibrium models, where a government has to take into account how its actions (and central bank actions, if a central bank is present in the model) will shift the yield curve (see, for example, Lucas and Stokey (1983), Barro (1979), and in particular models of optimal fiscal policy that explicitly incorporate the yield curve such as Buera and Nicolini (2004) and Angeletos (2002))[2]. We want to contrast the effects of fiscal policy on the yield curve with those of monetary policy in the same framework instead of relying on the large

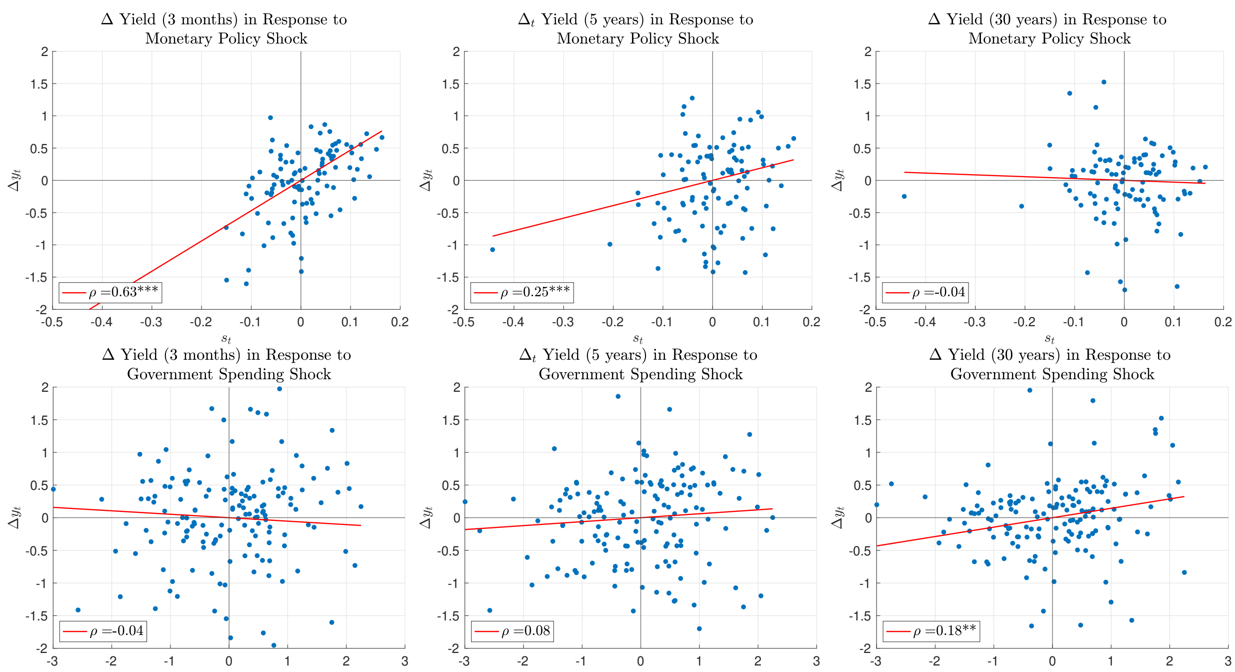
[1] In the current paper we focus on the yields on nominal U.S. government debt.The reasons for this choice are threefold: First, the inflation-indexed bond market (TIPS) is substantially smaller and less liquid than the corresponding market for nominal debt, the time series on TIPS yields is much shorter, and finally, the nominal yield curve itself is a prominent object of study in economics and finance.

[2] While this point is clearly evident in models of *optimal* fiscal policy under *rational expectations*, knowledge about the effect of fiscal policies on prices are key ingredients in any model of fiscal policy - for a model of fiscal policy where policymakers do not have rational expectations see for example Karantounias (2020).

literature that already exists that studies the effects of monetary policy on the yield curve (e.g. Piazzesi, 2005; Ireland, 2015), because it removes the risk that different specification choices across studies influence the results.

Not only is there a theoretical motivation for studying the effects of fiscal policy on the yield curve, there is also indirect evidence that fiscal policy could have substantial effects on the yield curve. In particular, we are motivated by two sets of empirical findings: Ang and Piazzesi (2003) and Evans and Marshall (2007) highlight that macroeconomic factors are important drivers of the nominal yield curve. Furthermore, the literature on the macroeconomic impact of fiscal policy changes cited above generally finds substantial macroeconomic effects of fiscal policies.

Figure 1: Correlation Between Policy Shocks and Changes in the Yield Curve



Note: Scatter-plots showing changes in yields Δy_t and instruments for policy shocks: monetary policy (Aruoba and Drechsel, 2022) and government spending shock (Auerbach and Gorodnichenko, 2012). We report the estimated correlation in the legend of each panel, where the significance levels are denoted by $^*: 10\%, ^{**}: 5\%, ^{***}: 1\%$. Information on data sources can be found in Appendix B.

To give examples of direct empirical evidence on the effect that government decisions have on the yield curve, Figure 1 shows scatterplots of yield changes at various maturities versus instruments for monetary policy shocks (Aruoba and Drechsel, 2022) and government spending shocks (Auerbach and Gorodnichenko, 2012) in the same quarter. We can see that there are significant correlations between these shocks and the yield curve, but the policy action affects different maturities of the yield curve - monetary policy affects the short end of the yield curve, whereas government spending affects the long end. This plot is only suggestive because (i) it does not take into account that the measures of policy shocks we use are only instruments, and (ii) it does not jointly study the entire yield curve. Our empirical approach addresses both these shortcomings (and also studies tax changes as well as more disaggregated government spending categories). However, this difference between policies is a key and robust takeaway of our paper.

We want to analyze the yield curve without imposing too much structure. Thus, we take advantage of recent advances in the theory of functional time series (Chang *et al.*, 2016). We view the yield curve at each point in time as the realization of a random function. With minimal structure imposed on the yield curve, we can write this random function as a combination of countably many basis functions with time-varying weights. Furthermore, one can approximate this functional process well (in a sense we make precise later) using only a finite number of basis functions. This leaves us with only the task of tracking the finite-dimensional weights on these basis functions to characterize movements in the yield curve. We show how this approach can be cast as a state-space model to aid interpretation.

Our approach uses all available data on the yield curve and directly models the entire yield curve, allowing us to represent changes in yields over time more effectively compared to standard approaches such as principal component analysis, which achieves dimension reduction simply by focusing on specific combinations of interest rates with different maturities and does not explore how the yield curve itself as a curve changes over time. For more discussions and demonstrations of the relative advantage of the functional approach over the conventional approach, see Chang, Durlauf, Lee and Park (2023a) and Bjørnland, Chang and Cross (2023).

While this approach allows us to track movements in the yield curve, we want to go further and identify the *causal* link between a government's actions and changes in the yield curve. Our identification is completely standard in that we use well-established instruments for policy shocks: We borrow measures of *exogenous* variation (or shocks) to total government spending, defense spending, government consumption, and government investment from Auerbach and Gorodnichenko (2012), exogenous tax changes identified by Romer and Romer (2010), as well as the monetary policy shock measure of Aruoba and Drechsel (2022). We then estimate how these measures of policy changes are related to changes in the yield curve (the aforementioned weights in the basis functions, to be exact), which allows us to compute impulse responses of the entire yield curve to these policy changes.

In terms of related literature, one relatively close paper in terms of topic to ours is Berndt *et al.* (2012), who study the effects of defense spending shocks on the government's financing decision, i.e., whether the return on the government's portfolio changes after a defense shock or net surpluses change. Instead, our paper focuses on how different fiscal decisions affect nominal borrowing costs at different maturities. Plosser (1987) studies how *forecast errors* in fiscal variables propagate to the yield curve. Instead, we focus on *the causal effects* of changes in fiscal and monetary policy on the yield curve. Dai and Philippon (2005) study the effects of fiscal policy on the yield curve using a no-arbitrage framework and Blanchard and Perotti (2002) type identification assumptions. We study a broader set of fiscal (and monetary) policies, use a flexible statistical framework to model the dynamics of the yield curve, and exploit instruments to identify causal effects. The closest applied paper that uses ideas about estimating responses of entire functions to economic shocks is Inoue and Rossi (2021), who incorporate level, slope, and curvature yield curve factors from a Nelson-Siegel type approach in a VAR to assess the effects of unconventional monetary shocks. Chang *et al.* (2021) use functional methods to study how distributions at the micro-level are related to aggregate variables and how these distributions react to aggregate shocks.

In the next section, we use insights from the government budget constraint and the consumption Euler equation to both further motivate our study and provide possible explanations for how government policies can influence the yield curve. In Section 3 we give an overview of our econometric methodology aimed at economists interested in empirical applications. After that, we turn to our main results.

2 Two Concepts from Economic Theory

In this section, we highlight two concepts that are helpful in motivating our analysis and interpreting the link between changes in fiscal or monetary policies and any associated changes in the yield curve for nominal government securities.

First, following Berndt *et al.* (2012), we analyze the government budget constraint. In contrast to Berndt *et al.* (2012), we will analyze the *nominal* budget constraint due to our focus on the nominal yield curve. In nominal terms, the government's budget constraint is given by

$$B_{t+1} = R^b_{t+1}\left(B_t - S_t\right) \tag{1}$$

where B_t is the nominal value of outstanding government debt at the beginning of period t,

S_t is the nominal primary surplus, and R^b_{t+1} is the nominal gross return on the government's portfolio between t and $t+1$ [3]. Directly borrowing from Berndt *et al.* (2012), the government budget constraint can be approximated via log-linearization as follows:[4]

$$ns_t - b_t = E_t \sum_{j=1}^{\infty} \rho^j \left(r^b_{t+j} - \Delta ns_{t+j} \right) \tag{2}$$

where ns_t is the weighted log nominal primary surplus ratio (for our purposes it will suffice to think of it as a measure of nominal surpluses), $b_t = \log B_t$, $r^b_t = \log R^b_t$, and ρ is a parameter between 0 and 1.

The key insight for our analysis is that changes in the surplus-to-debt ratio $ns_t - b_t$ will have to manifest themselves in changes in expectations of (i) returns on the government portfolio or (ii) net surpluses. Berndt *et al.* (2012) focus on tracing out how changes in defense spending affect this decomposition. Our focus is different: We ask how changes in *different* fiscal policies such as changes in different components of government spending and changes in different tax rates as well as monetary policy affect the government's borrowing costs. While these costs are encoded in r^b_t (see Hall and Sargent (2011) for a clear exposition), we want to disentangle how borrowing costs change across maturities, i.e., we directly study the effects of fiscal policies on the *entire* yield curve.

More information on the impact of policies on the yield curve can be obtained using the insight that government securities must be priced in such a manner as to entice market participants to purchase these securities.

To analyze this angle further, we turn to standard consumption-based asset pricing (see, for example, Cochrane (2001) and Campbell (2017)). In particular, we assume the existence of a positive real stochastic discount factor $\mathcal{M}_t$ (which might not be unique). We will now study the yield of a (zero-coupon) nominal government bond that matures next period. Such a bond pays a nominal return $R^n_{t,t+1}$ which is known at time t. We can use the stochastic discount factor to determine the yield via

$$1 = E_t \left(\mathcal{M}_{t+1} \frac{R^n_{t,t+1}}{\pi_{t,t+1}} \right) \tag{3}$$

[3] Hall and Sargent (2011) have made substantial progress in computing theory-consistent measures of R^b_{t+1}. Other papers that have used the government budget constraint to analyze fiscal policy include Hilscher *et al.* (2014), Giannitsarou and Scott (2008), and Chung and Leeper (2007).

[4] One can verify that the analogous conditions derived by Berndt *et al.* (2012) for their log-linearization of the *real* budget constraint also hold for our log-linearization of the nominal counterpart.

where π denotes (gross) inflation.

Given that the yield is known at time t, we get that

$$\frac{1}{R^n_{t,t+1}} = E_t\left(\mathcal{M}_{t+1}\frac{1}{\pi_{t,t+1}}\right) \tag{4}$$

Next, we turn to multi-period risk-free nominal bonds (which deliver a known nominal return $R_{t,t+j}$ in j periods) in order to study the entire yield curve. Note that for zero-coupon bonds, the yield is just $R^{1/j}_{t,t+j}$.

The stochastic discount factor $\mathcal{M}_t$ implies an associated discount factor $\mathbb{M}^*_t$ such that the following equation holds:

$$1 = R^n_{t,t+j}E_t\left(\mathbb{M}^*_{t+j}\frac{1}{\pi_{t,t+j}}\right)$$

Re-arranging this equation yields

$$\frac{1}{R^n_{t,t+j}} = cov_t\left(\mathbb{M}^*_{t+j}, \frac{1}{\pi_{t,t+j}}\right) + E_t(\mathbb{M}^*_{t+j})E_t\left(\frac{1}{\pi_{t,t+j}}\right) = cov_t\left(\mathbb{M}^*_{t+j}, \frac{1}{\pi_{t,t+j}}\right) + \frac{1}{R_{t,t+j}}E_t\left(\frac{1}{\pi_{t,t+j}}\right)$$

where $R_{t,t+j}$ is the j-period return on a risk-free real asset.[5] We can use this equation to identify important drivers of the nominal yield curve. Note that the terms on the right-hand side of the previous equation are not independent, so shocks could move all objects on the right-hand side. Both the levels of the real interest rate and expected inflation as well as the covariance between the inverse of inflation and the stochastic discount factor can be important. In particular, we now know that if a shock moves the nominal yield curve, and in particular $R^n_{t,t+j}$, such a shock has to move either expectations of the (inverse of) inflation and real returns or the comovement between inflation and the stochastic discount factor (or a combination of these terms).

For illustrative purposes, we find it useful to make a strong assumption on $\mathcal{M}$: We use the stochastic discount factor based on the consumption Euler equation for log utility.[6] In that case we get

$$\mathcal{M}_{t+1} = \beta\frac{C_t}{C_{t+1}}$$

This tells us that an investor with log-utility really cares about states of the world where

[5] The inverse of this return is equal to the expected j period stochastic discount factor, adjusted for inflation.

[6] Unfortunately log utility does not fit assets prices well generally, but it is useful to gain intuition.

consumption growth is low.[7] In term of the earlier decomposition, the key covariance term on the right-hand side now becomes

$$cov_t\left(\beta^j \frac{C_t}{C_{t+j}}, \frac{1}{\pi_{t,t+j}}\right)$$

What we can take away from this analysis is that fiscal or monetary policy induced changes in nominal yields must make investors either update their views on average real returns (which are directly linked to real consumption growth with this specific stochastic discount factor) and average inflation or the comovement between inflation and real consumption growth growth.[8] In particular, changes in fiscal and monetary policies could change investors' views of the government and thus lead them to update their perceptions of future economic growth and/or future inflation.

3 A Hitchhiker's Guide to Functional Time Series Methods

In this section, we give a high-level overview of the functional time series methodology that we use throughout our paper.[9] When large amounts of data are available on economic variables that are theoretically linked via a functional relationship (such as various nominal yields linked via the yield curve), such a functional approach can efficiently exploit this functional relationship.

We assume that observations of the nominal yield curve in a period t can be described by a function $y_t(\tau)$ defined over an interval I of possible maturities (between three months and 30 years in our case) taking real values (*that is,* $y_t : I \to \mathbb{R}$). The yield at time t for a security that matures in $t + \tau$ is thus given by $y_t(\tau)$ where τ is a value taken from the set I. We treat the function $y_t(\tau)$ as a random variable in a functional space, as it varies non-deterministically from one period to the next. To be concise, we will drop the argument τ from the y_t function unless needed.

The functional form of the yield curve we use here (Gürkaynak, Sack and Wright, 2007)

[7] The risk free real return on a j-period security with log utility is given by $\left[E_t\left(\beta^j \frac{C_t}{C_{t+j}}\right)\right]^{-1}$.

[8] Even with richer stochastic discount factors such as those derived using Epstein-Zin utility, consumption growth is still a key determinant - see for example Campbell (2017).

[9] More details are provided in the appendix or in Chang, Hu and Park (2022) and Chang, Park and Pyun (2023b).

allows us to obtain a yield for all values of τ between the aforementioned bounds of three months to thirty years. We describe in Appendix D how we, in practice, use a grid of values to represent the interval I and their corresponding images (yields) for each quarter t.

So far we have not restricted the yield curve in any way - the function $y_t(\tau)$ can take on arbitrary value for each maturity τ at any point in time t. We next describe the mild restriction we impose on the function $y_t(\tau)$ before turning to a description of a finite-dimensional approximation of this function, which we can then exploit in our empirical analysis.

3.1 Restrictions on the Yield Curve

In order to econometrically exploit the fact that all yields are linked via the yield curve, we will put one mild restriction on the yield curve. We only study yield curves that are in the space $H = \mathcal{L}^2(I)$, the space of square integrable functions.[10] While this space of functions is very general (it includes functions that are not continuous, for example), it still imposes a surprising amount of regularity. In particular, we can now define a scalar product and a norm in H: For f and g in the space H we obtain

$$\langle f,g\rangle = \int_I f(x)g(x)dx \quad \text{and} \quad \|f\| = \sqrt{\langle f,f\rangle}. \tag{5}$$

In addition to the inner product and the norm, we also can define a tensor[11].

$$(f \otimes g)v = \langle v,g\rangle f \tag{6}$$

for all v in H. In Appendix E we show how to use these constructs (scalar and tensor products) to define the expectation function and the covariance operator of random functions in H.

Using results from functional analysis[12] we find that the space H is a *separable Hilbert space*. These are spaces that admit a scalar product, such as the one defined above, and have *countable* bases. This means that every yield curve in H can be expressed as the linear

[10] The space of all (real) functions f_t defined over I such that $\int_I |f(x)|^2 dx < \infty$.

[11] If $H \equiv \mathbb{R}^n$, we have $f \otimes g = fg'$, *i.e.*, $f \otimes g$ reduces to the outer product, in contrast to the inner product $\langle f,g\rangle = f'g$, where f' and g' are the transposes of f and g. Note that $(f \otimes g)v = (fg')v = (v'g)f$ for all $v \in \mathbb{R}^n$ in this case

[12] See for example Folland (1999).

combination of countable many functions $\{v_i\}_{i=1,2,3,\ldots}$:[13]

$$y_t = \sum_{i=1}^{\infty} \alpha_{it} v_i. \tag{7}$$

Since the functions $\{v_i\}$ are independent of t, once they are determined, the yield curve y_t is fully characterized by the sequence of real numbers $(\alpha_{1t}, \alpha_{2t}, \ldots)$. In other words, the yield curve can be analyzed through a sequence of real numbers, and every sequence of real numbers can be traced back to a yield curve by combining the basis functions $\{v_1, v_2, \ldots\}$ with the sequence $(\alpha_{1t}, \alpha_{2t}, \ldots)$.

This approach is different from models of the yield curve that start with focusing on the level, slope, and curvature of the yield curve (Diebold and Rudebusch, 2012): We are not imposing a particular set of functions to describe the yield curve - instead, we choose basis functions that jointly describe most of the fluctuations in the yield curve.

3.1.1 A Finite Dimensional Representation of the Yield Curve

The dimension of a space is given by the number of elements in its basis. By this logic, the space H is infinite dimensional as the basis $\{v_i\}_{i=1,2,3,\ldots}$ that we used in (7) has infinitely many elements.

The next step in our approach is to define a finite-dimensional subspace of H. We do this by considering only functions resulting from a linear combination of the first m elements of the basis $\{v_1, v_2, \ldots, v_m\}$, these functions define the finite-dimensional space H_m (a subspace of H).

The function y_t is not an element of H_m given that we need more than just the first m elements of the basis to represent it as we can see in (7). However, we can consider the projection of y_t on H_m given by

$$\tilde{y}_t = \sum_{i=1}^{m} \alpha_{it} v_i. \tag{8}$$

This gives us an equation akin to an observation equation in a state space model

$$y_t = \sum_{i=1}^{m} \alpha_{it} v_i + w_t, \tag{9}$$

[13]Note that we have omitted the argument τ of the function, but v_i (and y_t) still refers to a *function*.

where $w_t = y_t - \tilde{y}_t$ is the approximation error we make by restricting ourselves to H_m. Under suitable conditions, this approximation error becomes asymptotically negligible. In what follows, we assume that $\{v_i\}$ is an orthonormal basis, i.e. $\|v_i\| = 1$ for all i and $\langle v_i, v_j \rangle = 0$ for all $i \neq j$. Under this assumption, we have

$$\alpha_{it} = \langle v_i, y_t \rangle$$

for all i and t.

Let us now introduce a mapping from H_m to $\mathbb{R}^m$

$$H_m \ni \tilde{y}_t \mapsto \alpha_t = \begin{pmatrix} \alpha_{1t} \\ \alpha_{2t} \\ \vdots \\ \alpha_{mt} \end{pmatrix} \in \mathbb{R}^m.$$

This mapping is one-to-one correspondence between H_m and $\mathbb{R}^m$.Therefore, with the basis $\{v_1, v_2, \ldots, v_m\}$ and $\alpha_t = (\alpha_{1t}, \alpha_{2t}, \ldots, \alpha_{mt})'$ we can recover $\tilde{y}_t$ through (8). The mapping is an isometry between H_m and $\mathbb{R}^m$, which preserves the norm.[14]As a result, we can study a vector autoregression (VAR) for α_t by least squares, rather than having to work directly in a functional space.

Using functional principal components, whose properties we discuss in Appendix C, we determine a basis of functions $\{v_i\}_{i=1,2,3,\ldots}$ such that its first m elements generate $y_t^{(m)} = \alpha_t \in \mathbb{R}^m$ a "best" approximation of y_t. Note that we can thus effectively choose a very efficient set of basis functions for our purposes rather than restrict ourselves to an a priori chosen basis function such as monomials $\{1, \tau, \tau^2, \ldots\}$.

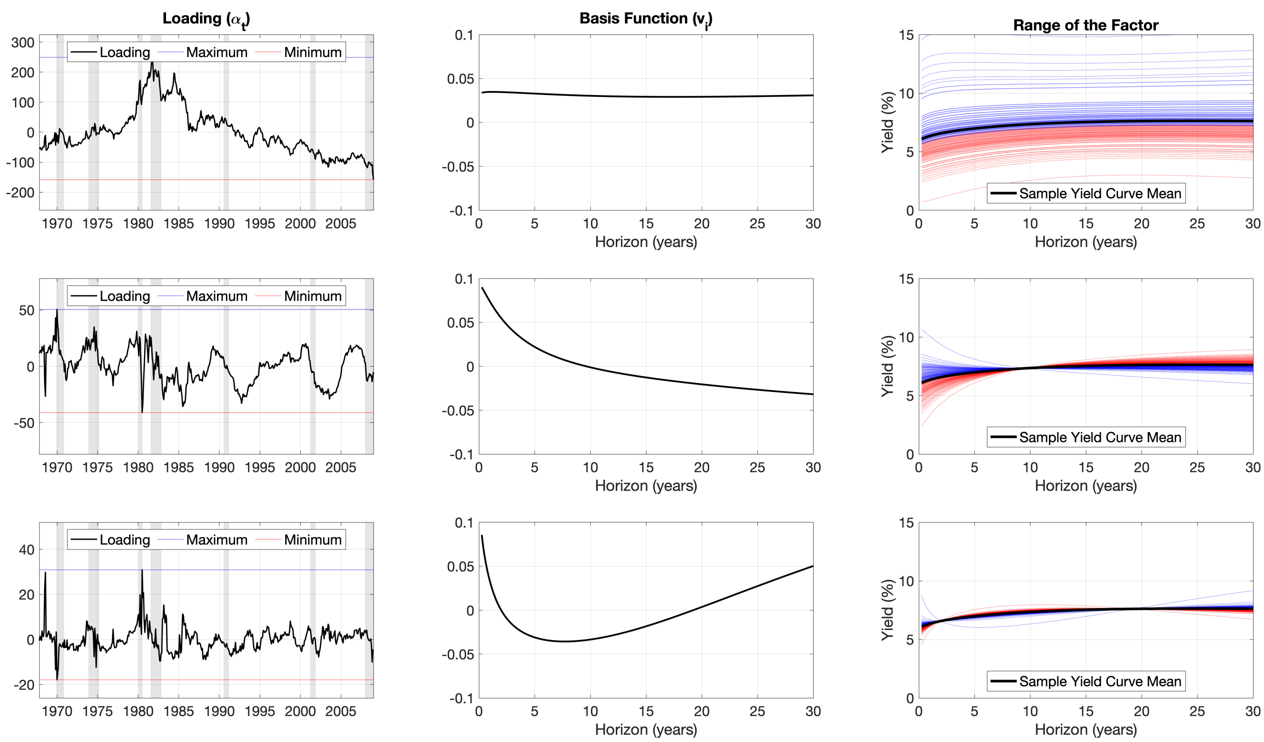
Since functional principal components algorithm depend on the data, and since the sample of the yield curve we use vary with the external shock being analyzed, so does the portion of the variability explained by these approximations. Our choice of $m = 3$ explains more than 90% of the variability of y_t in every case. This principal components analysis (detailed in the appendix) also delivers a time series for the vector of weights $\alpha_t = \begin{pmatrix} \alpha_{1t} & \alpha_{2t} & \ldots & \alpha_{mt} \end{pmatrix}'$.

[14]We can show that

$$\|\tilde{y}_t\|^2 = \sum_{i=1}^{m} \langle v_i, y_t \rangle^2 = \sum_{i=1}^{m} \alpha_{it}^2 = \|\alpha_t\|^2,$$

where we use the same notation $\|\cdot\|$ to denote the norm of a function in H_m and the norm of a vector in $\mathbb{R}^m$.

Figure 2: Description of the Functional Principal Components



Note: The first column describes the time series of weights (α_{it}) for each component (one in each row). The shape of the component is described in the second column. The last column shows the range of effects that each component has on the yield curve using the sample mean yield curve (black line) as a benchmark. The blue (red) lines in the top/middle/bottom panel signify the yield curves obtained with positive (negative) realizations of the first/second/third weight and the associated basis function.

Figure 2 shows an example of the α_{it} values (left column), the v_i's (center column), and the range of yield curves generated by time series fluctuations in the α vector, using our yield curve data as described in Section 4, and in particular the sample mean as the benchmark value that is perturbed by movements in α.

This vector α_t cannot be directly interpreted as yields, as the measurement equation highlights that only together with the basis functions $\{v_1, v_2, \ldots, v_m\}$ can we recover the yield curve. It does, however, serve as the state in our state-space model for the yield curve.[15] An important feature of this approach is that for a fixed value of τ, the yield $y_t(\tau)$ is a *linear* combination of the elements of α_t, which makes construction of impulse responses straightforward since we assume a linear law of motion for that vector, as we discuss next.

3.2 The Dynamics of α_t and the Identification of Impulse Responses

We posit a VAR law of motion for α_t and the instrument for the policy shock of interest m_t.[16] In particular, we focus on a VAR(1) for the sake of parsimony:

$$\gamma_t = A\gamma_{t-1} + u_t, \tag{10}$$

where $\gamma_t \equiv [m_t \ \alpha_t']'$. From an applied perspective, our approach can be thought of as modeling observations on the yield curve (and the instrument) at each point in time t through a state-space framework with a set of observation equations (equation 9 and the identity $m_t = m_t$) that link the yield of an asset with a specific maturity to a set of basis functions that depend on the maturity and weights on each basis function, which vary over time, but do not depend on maturity. These weights represent (a subset of) the states in our state-space model, which we model as a Vector Autoregression (VAR) as in equation 10.

We identify the shock of interest by assuming a linear relationship between the forecast error u_t and the vector of structural shocks of interest e_t as

$$u_t = \Omega e_t, \tag{11}$$

and assume that Ω is computed via the lower triangular Cholesky decomposition of the covariance matrix of u_t so that $E(e_t e_t') = I$,[17] as proposed by Plagborg-Møller and Wolf (2021). The policy shock of interest is related to the first element of e_t, as we discuss below. This approach has a number of advantages, even beyond its simplicity. First, it automatically cor-

[15] The analogy to state space models might be slightly misleading because we first compute the states via principal components and then go on to model the law of motion for the estimated states, whereas standard applications of state space models often employ a filtering algorithm (think about the Kalman Filter, for example) that exploits a posited law of motion for the states when estimating the states. Our approach is instead very much reminiscent of the standard two-step approach to linear factor models in standard time series analyses (see, for example, Stock and Watson (2016)). The resulting model of the yield curve is still in state-space form.

[16] We estimate a separate VAR for each instrument because the sample sizes for the different instruments are not the same.

[17] I denotes the identity matrix.

rects for possible autocorrelation of the instrument and dependence of the instrument on past yield curve movements (which are generally thought to encode macroeconomic outcomes). To see this, it is useful to write the first equation of the set of Equations (10), using Equation (11):

$$m_t = A_{1,1}m_{t-1} + \sum_{j=1}^{m} A_{1,j+1}\alpha_{jt} + \Omega_{1,1}e_t^1, \tag{12}$$

where $A_{i,j}$ is the element of the matrix A in row i and column j. Following Plagborg-Møller and Wolf (2021), it is worthwhile to point out that this identification approach will correctly identify normalized impulse responses even if the yield curve itself does not contain enough information to identify the shock of interest ε_t (i.e., non-invertibility) and if there is measurement error w_t present in e_t^1, so that $e_t^1 = \theta\varepsilon_t + w_t$, where $\theta \neq 0$ is a parameter that influences the strength of identification and w_t is an i.i.d. measurement error. This comes at a cost, as we can only identify normalized impulse responses if there is non-invertibility. Throughout this paper, we plot impulse responses that increase the first element of e_t by one unit (which is equal to a one standard deviation change in the first element of the one-step ahead forecast error u_t). This has the advantage of giving us some sense of magnitude of the effects of the shock is indeed invertible.

In terms of inference, we use a bootstrap procedure that is detailed in Appendix F. [18] α_t and its associated basis functions have a clear interpretation in our application, as we highlight in Figure (2), which plots the basis functions (center column) associated with the first three elements of α_t - the basis functions resemble the level, slope, and curvature of the yield curve (Diebold and Rudebusch, 2012). Note, however, that we did not impose these shapes ex ante.

4 Yield Curve Data and Instruments

For the nominal yield curve, we use the data constructed by Gürkaynak, Sack and Wright (2007) that can be downloaded from the Board of Governors' website[19]. Our sample for the yield curve begins in June 1961. We use quarterly data; in particular, we use the curve observed the last day of each quarter as our quarterly yield curve. This ensures that shocks occurring at any point in a quarter can influence the yield curve in that same quarter. The exact sample to determine the response of the yield curve to each policy shock is the largest

[18] This bootstrap procedure is valid, as shown by Chang, Park and Pyun (2023b).

[19] https://www.federalreserve.gov/data/nominal-yield-curve.htm

intersection of the yield curve sample and the corresponding shock sample.

As instruments for various government spending shocks we use shocks identified using a linear VAR as described by Auerbach and Gorodnichenko (2012).[20] Using shocks identified by a VAR as an instrument is common in applied work (see Känzig, 2021, for example). For tax shocks, we use the series of exogenous shocks extracted and described by Romer and Romer (2010).

Our instrument for monetary policy shocks is taken from Aruoba and Drechsel (2022), who use machine learning techniques and natural language processing to supplement Federal Open Market Committee (FOMC) staff numerical forecasts with information from FOMC staff documents to predict FOMC decisions. The difference between actual decisions and the forecasts is then our instrument for a monetary policy shock. One key advantage of using Aruoba and Drechsel (2022) is that it is available for much longer periods than the popular instruments based on the high frequency variation of interest rate futures around FOMC meetings (Kuttner, 2001; Gertler and Karadi, 2015). In Appendix A.1 we show that our findings are robust to using the instrument created by Miranda-Agrippino and Ricco (2021) instead.

5 Response of the Yield Curve to Policy Shocks

Throughout, we will present impulse responses for the yield curve by plotting how the entire yield curve changes h periods after a shock. We refer to h as the *horizon* of the response, not to be confused with the *maturity* of the yield curve, which is represented on the x-axis in our response plots (measured in years).

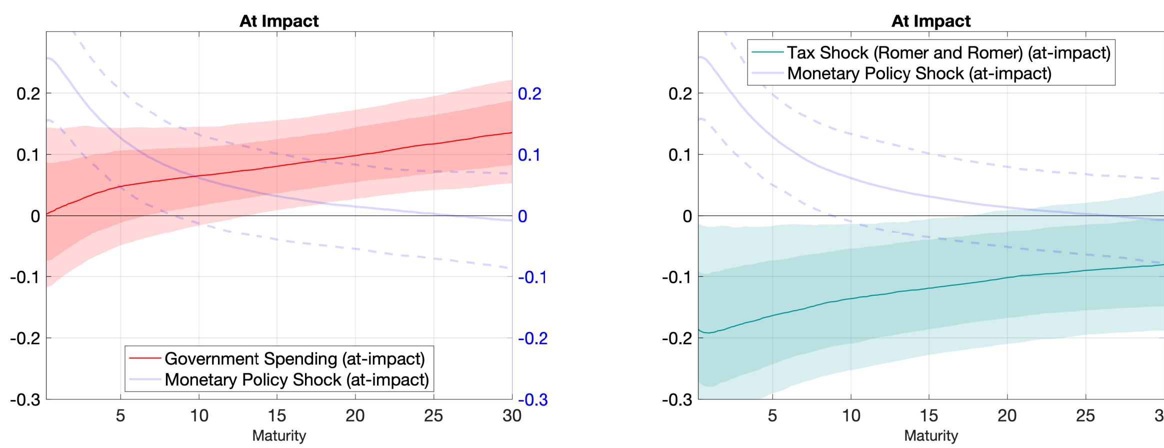
First, we focus on the impact response ($h = 0$) - remember that our yield curve data are the value of the yields on the last day of the quarter. The responses to fiscal shocks are plotted using red/orange for various measures of government spending shocks, and green for tax shocks. To aid comparison with monetary policy, we always plot the responses of the yield curve to a monetary policy shock in light blue in each figure. The left panel of Figure 5 shows the response to overall government spending, estimated using as an instrument the government spending shock constructed as in Auerbach and Gorodnichenko (2012). While

[20] One slight deviation from that paper is that we control for forecasts of overall government spending in *all* our VAR specifications. We do so to ensure that our identified shocks are truly unforecastable (Auerbach and Gorodnichenko (2012) do not do this for all specifications). However, it turns out that the impact of this change is minimal; the results are very similar if we take the exact specifications from Auerbach and Gorodnichenko (2012).

monetary policy moves the short end of the yield curve (the response is significant at the 90 percent level up to a maturity of approximately 10 years), the response to government spending shocks shows the exact opposite response: there is no significant response up to a maturity of 12 years, after which the response turns significant.

The right panel shows the response to a tax shock, where the instrument for a tax shock is those tax changes labeled exogenous by Romer and Romer (2010). Interestingly, the response to tax shocks is approximately the mirror image of the monetary policy response, both changing the yield curve on impact (though in opposite directions). Taxes have a more persistent effect than monetary policy, affecting the yield curve significantly up to a maturity of 15 years.

Figure 3: Response at Impact of the Yield Curve to a Government Spending Shock (Auerbach and Gorodnichenko, 2012) and Exogenouse Tax Change (Romer and Romer, 2010)

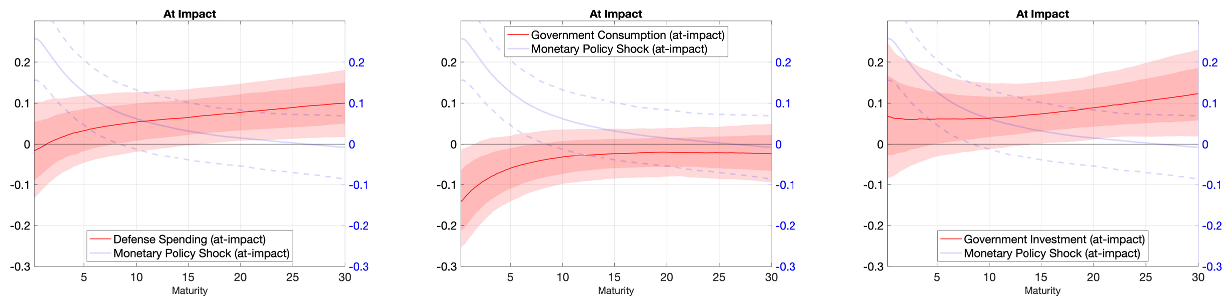


Note: For reference the response to monetary policy (Aruoba and Drechsel, 2022) and its 90% confidence band is represented in blue. The lighter (darker) shade signifies 90 % (68%) confidence bands. 90% and 68% confidence bands estimated using bootstrap methods.

Next, we ask whether different components of government spending have the same influence on the yield curve. This question is motivated by Boehm (2020), who shows that the spending multiplier can differ substantially between government investment and consumption. We construct the instruments for defense spending, government consumption, and government investment following Auerbach and Gorodnichenko (2012). The left panel of Figure 4 shows that defense spending shocks qualitatively induce the same pattern as overall government spending, only the long end of the yield curve moves significantly.[21] The same is true for government investment, as the right panel shows. Government consumption, on the

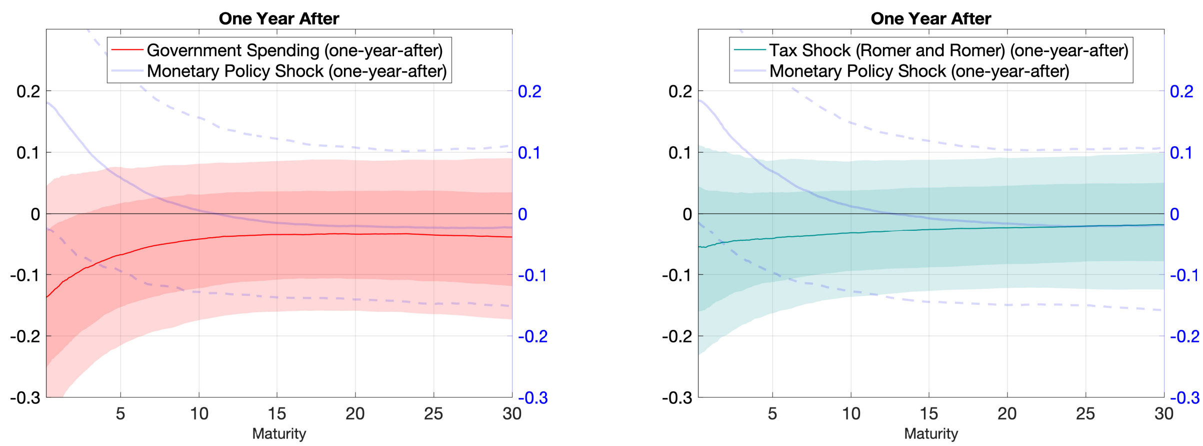
[21]Readers might wonder why we don't use the government defense spending news shock of Ramey (2011) - our sample only starts after the Korean war, and it is well known (Ramey, 2016) that this instrument has low instrument relevance in any sample starting after the Korean war.

Figure 4: Response at Impact of the Yield Curve to a Government Defense Spending Shock (left), Government Consumption Spending Shock (center), and, Government Investment Spending Shock (right) (Auerbach and Gorodnichenko, 2012)



Note: For reference the response to monetary policy (Aruoba and Drechsel, 2022) and its 90% confidence band is represented in blue. The lighter (darker) shade signifies 90 % (68%) confidence bands. 90% and 68% confidence bands estimated using bootstrap methods.

Figure 5: Response One Year After of the Yield Curve to a Government Spending Shock (Auerbach and Gorodnichenko, 2012) and Exogenous Tax Change (Romer and Romer, 2010)



Note: For reference the response to monetary policy (Aruoba and Drechsel, 2022) and its 90% confidence band is represented in blue. The lighter (darker) shade signifies 90 % (68%) confidence bands. 90% and 68% confidence bands are estimated using bootstrap methods.

other hand, only moves the very short end of the yield curve, with an even shorter impact than monetary policy.

Finally, we now look at responses not on impact but one year after the shock happened. Figure 5 focuses on overall government spending and taxes - the response to components of government spending one year after the shock can be found in Appendix A.2. We see that at the 90 percent significance level, all responses are insignificantly different from zero after one year. The initial effects of any policy shock on the yield curve thus wear off relatively quickly. Any prolonged effects on the real economy of policy shocks, as long as they are transmitted through yields, must therefore come from persistent effects of short-term changes in yields.

6 Conclusion

We study the effects of monetary and fiscal policies on the yield curve and find that they have qualitatively very different consequences for the yield curve. These findings are useful for both policymakers, who often view the yield curve as a major aspect of policy transmission, in addition to directly encoding a government's borrowing costs. Furthermore, our results can be useful as calibration targets for macroeconomists who want to develop quantitative equilibrium models that take the yield curve seriously.

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Online Appendix for "The Influence of Fiscal and Monetary Policies on the Shape of the Yield Curve"

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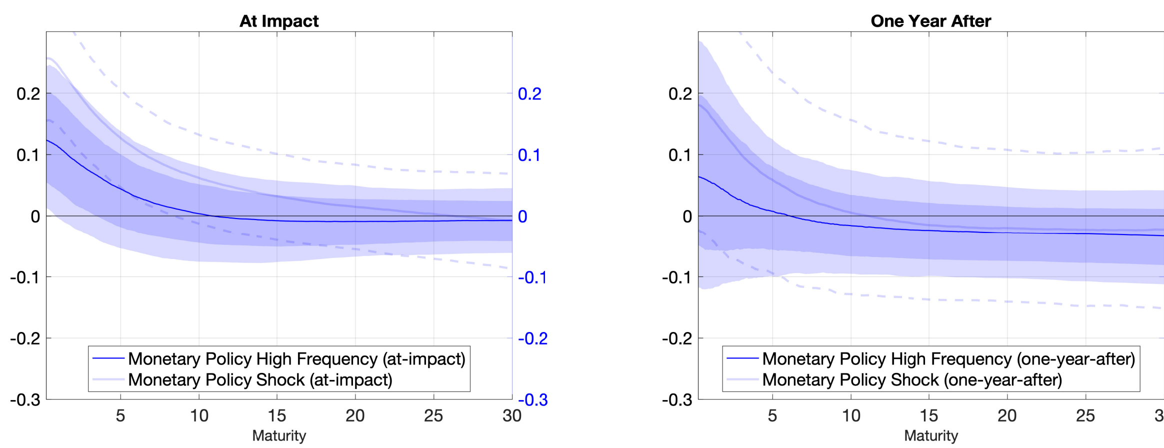
A Additional Figures

A.1 Response of the Yield Curve to Monetary Policy Identified Using High Frequency Variation

When comparing the responses of the yield curve to monetary policy shocks, two different studies provide valuable insights. Aruoba and Drechsel (2022) and Miranda-Agrippino and Ricco (2021) utilize different shocks to analyze the transmission of monetary policy to the yield curve. The shock identified by Aruoba and Drechsel (2022) focuses on a natural language analysis, while the shock from Miranda-Agrippino and Ricco (2021) represents a different perspective, based on high frequency estimation.

Figure A-1: Response at Impact of the Yield Curve to a Monetary Policy Shock (Miranda-Agrippino and Ricco, 2021)

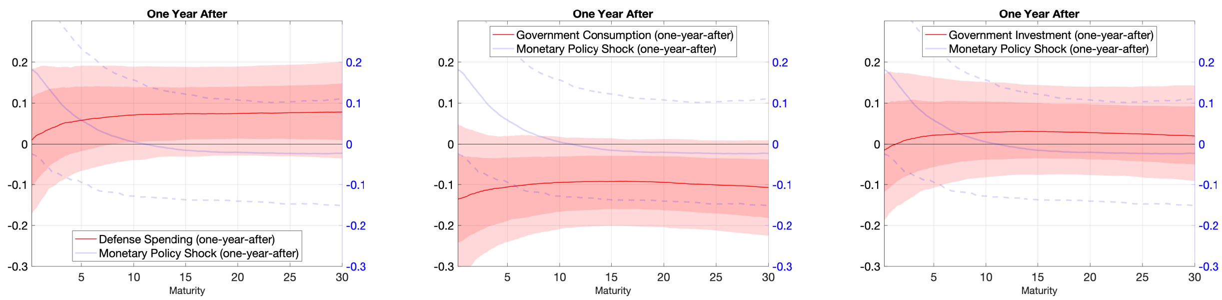


Note: For reference the response to monetary policy (Aruoba and Drechsel, 2022) and its 90% confidence band is represented in light blue. The lighter (darker) blue shade signifies the 90% (68%) confidence band for the response to high frequency monetary policy shock (Miranda-Agrippino and Ricco, 2021). The confidence bands are estimated using bootstrap methods.

A.2 Additional Results

The following figure shows the response of the yield curve to components of government spending one year after the shock. These plots confirm that the effects of fiscal policy become very weak or even negligible after just one year.

Figure A-2: Response after one year of the yield curve to a Government Defense Spending Shock (left), Government Consumption Spending Shock (center), and, Government Investment Spending Shock (right) (Auerbach and Gorodnichenko, 2012)



Note: For reference the response to monetary policy (Aruoba and Drechsel, 2022) and its 90% confidence band is represented in light blue. The lighter (darker) blue shade signifies the 90% (68%) confidence band for the response to high frequency monetary policy shock (Miranda-Agrippino and Ricco, 2021). The confidence bands are estimated using bootstrap methods.

B Data

In this section of the paper, we provide a comprehensive overview of the various data sources utilized in this paper.

B.1 Yield Curve Data

The yield curve data we use as starting point are taken from the Federal Reserve Board[1] based on the model by Gürkaynak, Sack and Wright (2007). There we obtain a daily estimation of six parameters $\beta_0, \beta_1, \beta_2, \beta_3, \phi_1$ and ϕ_2. These parameters are used to obtain the following function:

$$y_t(\tau) = \beta_0 + \beta_1 w_1^{\phi_1}(\tau) + \beta_2 w_2^{\phi_1}(\tau) + \beta_3 w_3^{\phi_2}(\tau) \tag{A-1}$$

where the functions $w_1^{\phi_1}(\tau)$,$w_2^{\phi_1}(\tau)$, and $w_3^{\phi_2}(\tau)$ are defined as follows:

$$w_1^{\phi_1}(\tau) = \frac{1 - e^{-\frac{\tau}{\phi_1}}}{\frac{\tau}{\phi_1}}$$

$$w_2^{\phi_1}(\tau) = \frac{1 - e^{-\frac{\tau}{\phi_1}}}{\frac{\tau}{\phi_1}} - e^{\frac{-\tau}{\phi_1}}$$

$$w_3^{\phi_2}(\tau) = \frac{1 - e^{-\frac{\tau}{\phi_2}}}{\frac{\tau}{\phi_2}} - e^{\frac{-\tau}{\phi_2}}$$

[1] https://www.federalreserve.gov/data/nominal-yield-curve.htm

Conditional on ϕ_1 and ϕ_2 these specification is a linear combination of three basis functions but since these parameters vary over time, contrary to our approach in the main text, the basis functions are not time-invariant. The daily sample of the yield curve starts from June 14^{th} 1961 and is updated weekly.

B.2 External Shocks

Throughout the paper, we make use of several external shocks borrowed from the literature. We consider three main groups of shocks: Monetary Policy, Government Spending, and Tax Changes. In the following section, we describe the data that provides the information for generating each of these external shocks.

B.2.1 Monetary Policy.

The main references to obtain measures of monetary policy changes are: Aruoba and Drechsel (2022) and Miranda-Agrippino and Ricco (2021). Additionally, we also consider Romer and Romer (2004), but in order to obtain the largest sample possible (intersection with the yield curve), we consider the extension of Romer and Romer (2004) by Silvia Miranda-Aggripino reported in her personal webpage: http://silviamirandaagrippino.com/s/Narrative-MP.zip

Aruoba and Drechsel (2022) In this paper, the authors apply natural language processing techniques to analyze documents prepared by economists at the FOMC meetings. The goal is to capture the information available to the committee at the time of policy decisions. Using machine learning techniques, they then predict changes in the target interest rate based on this information and obtain a measure of monetary policy shocks as the residual. This shock is available from 1982Q3 to 2008Q4. This shock is available at: http://econweb.umd.edu/ drechsel/files/Aruoba_Drechsel_Data.xlsx

Miranda-Agrippino and Ricco (2021) This shock uses a high-frequency instrument for monetary policy shocks that accounts for informational rigidities. The series of shocks are available in the following link: http://silviamirandaagrippino.com/s/Instruments_web-x8wr.xlsx. This shock's sample is from 1991Q1 to 2009Q4.

Romer and Romer (2004) In this paper the authors develop a measure of U.S. monetary policy shocks for the period 1969–1996. Quantitative and narrative records are used to infer the Federal Reserve's intentions for the federal funds rate around FOMC meetings. This series is regressed on the Federal Reserve's internal forecasts to derive a measure free of systematic responses to information about future developments. As mentioned earlier, we use an extension of this shock provided in the webpage of Silvia Miranda-Aggripino. The sample period for the extended shock is 1969Q1 to 2007Q4.

B.2.2 Government Spending

We considered two main references to obtain government spending shocks: Auerbach and Gorodnichenko (2012) and Ramey (2011).

Auerbach and Gorodnichenko (2012) We use the linear (without regime switching) version of the VAR model described in Auerbach and Gorodnichenko (2012) to obtain government spending shocks. The identification (ordering of the variables), decomposition of the government spending variable, and the way they control for predictable components of fiscal shocks are maintained in our analysis.

The variables included in the model are as follows: government spending, which represents the log real government (federal, state, and local) purchases (consumption and investment); government revenue, which represents the log real government receipts of direct and indirect taxes net of transfers to businesses and individuals; and output, which represents the log real gross domestic product (GDP) in chained 2000 dollars.

To remove the effects of anticipated shocks, we also consider quarterly forecasts of fiscal and aggregate variables (government purchases, output, taxes) from the University of Michigan's Research Seminar in Quantitative Economics (RSQE) macroeconometric model, the Survey of Professional Forecasters (SPF) and the forecasts prepared by the staff of the Federal Reserve Board (FRB) for the meetings of the FOMC. These forecasts are included in the SVAR to eliminate the effects of "innovations" in fiscal variables that were predicted by professional forecasters.

The resulting shocks are obtained using the inverse of the Cholesky decomposition of the estimated covariance matrix and the estimated residuals. The sample period for the shocks is from 1967Q3 to 2008Q4. Replication files are obtained from http://doi.org/10.3886/E114783V1

Ramey (2011) A narrative method is used to construct richer government spending news variables from 1939 to 2008. The author uses *Business Week*, as well as several newspaper sources, to construct an estimate of changes in the expected present value of government spending. The series are extended to 2013 by the author after the publication. Although the shock series goes back much further, its intersection with the yield curve sample covers only from 1961Q2 to 2013Q4. https://econweb.ucsd.edu/ vramey/research/govdat3908.csv.

B.2.3 Tax Changes.

For tax changes we replicate the estimation by Mertens and Ravn (2013) to obtain measures of personal and corporte income tax changes. We also estimate the response of the yield curve to changes in tax legislation described in Romer and Romer (2010).

Personal and Corporate Income Tax Changes The authors distinguish between changes in personal and corporate income taxes and develop a new narrative account of federal tax liability changes in these two tax components. They develop an estimator which uses narratively identified tax changes as proxies for structural tax shocks and apply it to quarterly post-WWII data. The sample period for this shock is 1961Q2 to 2006Q4. The replication files can be obtained from: http://doi.org/10.3886/E112644V1

Tax Changes The authors use the narrative record, such as presidential speeches and congressional reports, to identify the size, timing, and principal motivation for all major postwar tax policy actions. With their analysis they separate legislated changes into those taken for reasons related to prospective economic conditions and those taken for more exogenous reasons. The sample period for this shock is 1961Q3 to 2007Q4, and the shock series can be obtained from https://www.aeaweb.org/aer/data/june2010/20080421_app.zip.

C Functional Principal Components

The eigenvalues of a compact operator are defined as the non-zero scalars λ for which there exists a non-zero function v in the underlying vector space such that the operator S applied to v is a scalar multiple of v. More formally, for a compact operator S defined on a Banach space or a Hilbert space, the eigenvalue-eigenvector equation is given by:

$$Sv = \lambda v$$

Here, v is the eigenfunction associated with the eigenvalue λ, and λ is a scalar. Compact operators are typically defined on infinite-dimensional spaces, and their eigenvalues may include accumulation points or have a discrete or continuous spectrum depending on the properties of the operator and the underlying space. The eigenvalues of a compact operator provide important information about its spectral properties and behavior. In the case of a bounded compact operator on a Hilbert spaces, the eigenvalues are a sequence of numbers with 0 as only acummulation point.

For a time series of functions $y_1, y_2, y_3, \ldots, y_T$ with sample mean $\bar{y}$ we define the operator

$$S = T^{-1} \sum_{i=1}^{T} (y_i - \bar{y}) \otimes (y_i - \bar{y})$$

the operator S is the sample covariance operator of y_t.

Let $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots$ the eigenvalues of S we call the eigenfunctions $v_1, v_2, v_3, \ldots$ the **functional principal components of** y_t.

There are multiple norms we can use for S. We use the trace norm $\|S\|$ to measure the variability of y_t. The trace norm equals the summation of all eigenvalues of the compact operator

$$\|S\| = \lambda_1 + \lambda_2 + \lambda_3 + \cdots$$

A screeplot measures the amount of the total variability explained by a subset of principal components:

$$s(m) = \frac{\lambda_1 + \lambda_2 + \cdots + \lambda_m}{\lambda_1 + \lambda_2 + \lambda_3 + \cdots}$$

In the Appendix D we show how to computationally obtain the functional principal components of a functional time series.

D How to Model the Yield Curve Computationally?

In the main text, we use the interval $I = [0.25, 30]$ representing maturities from three months to 30 years. We use a grid of 1024 equidistant points, from $x_1 = 0.25$ to $x_{1024} = 30$.

Computationally, the sample of the yield curve is a matrix Y of dimensions $T \times 1024$, where each row contains the values y_{it} for $i = 1, \ldots, 1024$ and $t = 1, 2, 3, \ldots, T$, representing the yield in period t and maturity x_t.

The main operation using the data are described as follows:

1. Sample mean yield curve: $\bar{y} = (\bar{y}_{\cdot 1}, \bar{y}_{\cdot 2}, \ldots, \bar{y}_{\cdot 1024})$. This represents the mean yield curve in the sample.

2. The scalar product between two functions, is the inner product of their vector representation.

3. The tensor product is the outter product of their vector representation.

4. The sample variance matrix S with dimensions 1024×1024, calculated as:

$$S = (Y - \bar{y})'(Y - \bar{y})$$

5. The estimated functional principal components are obtained from the eigenvalue decomposition of the matrix S. The principal components are given by the eigenvectors of S, and the portion of the variance explained by each component is given by the eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots$

E Random Functions

In this section, we describe some of the main concepts of random functions. Similar to random variables and random vectors, we define a random function on a probability space $(\Omega, \mathcal{A}, P)$, which consists of a sample space, an event space, and a probability measure, respectively.

Expected function. Given a random function f and an arbitrary element v of H, the scalar product $\langle f, v \rangle$ becomes a random variable. This random variable has an expected value denoted as $\mathbb{E}\langle f, v \rangle$. The mapping

$$v \mapsto \mathbb{E}\langle f, v \rangle$$

is proven to be a linear functional from H to $\mathbb{R}$. By Riesz' representation theorem, there exists a non-random element in H referred to as "$\mathbb{E}f$", such that

$$v \mapsto \mathbb{E}\langle f, v \rangle = \langle \mathbb{E}f, v \rangle$$

In other words, we use the representation of a linear functional as the scalar product with a fixed element of H to characterize the expected function of the random function f.

Covariance operator. If f and g are random functions taking values in H, then their covariance operator $\mathbb{E}(f \otimes g)$ is generally defined as a linear operator satisfying

$$\Big\langle u, [\mathbb{E}(f \otimes g)]v \Big\rangle = \mathbb{E}\langle u, f\rangle\langle v, g\rangle$$

for all u and v in H.

The combination of these two concepts allows us to define, for example, a functional white noise (ε_t) as follows: We set $\mathbb{E}\varepsilon_t = 0$ for all $t \geq 1$, and (ε_t) to be serially uncorrelated with $\mathbb{E}(\varepsilon_t \otimes \varepsilon_t) = \Sigma$ for all $t \geq 1$.

F Bootstrapping

We use the bootstrap to determine confidence intervals for the statistics we estimate regarding the yield curve and its reactions to external shocks.

In our investigation of the yield curve's response to an external shock, we consider a sample of the yield curve that corresponds to the sample of the external shock. The majority of the external shocks examined in this study occur at a quarterly frequency, so we utilize the most recent daily observation of the yield curve from the corresponding quarter. This ensures that we have two identical samples, in terms of size and frequency, for both the yield curve and the external shock.

In the following, we outline the procedure for generating copies of functional time series and the external shock, each of size n:

1. Obtain the residuals from estimating model (10): $\hat{u}_t = \gamma_t - \hat{A}\gamma_{t-1}$.
2. Randomly select, with replacement, a sample of n residuals from the set $\{\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_n\}$, and center (demean) it to obtain a new set of residuals (u_t^*).
3. Generate a new time series γ_t^* using the equation $\gamma_t^* = \hat{A}\gamma_{t-1}^* + u_t^*$ with the (u_t^*) generated in the previous step and the initial value $\gamma_0^* = \gamma_0$. Note that $\hat{A}$ is the same as in the first step.

With the new copy of the time series $\{\gamma_t^*\}_{t=0,1,2,\ldots,n}$, estimate model (10) and obtain $\hat{u}_t^* = \gamma_t^* - \hat{A}^*\gamma_{t-1}^*$. With the new estimation, obtain impulse response functions of γ^* as discussed in the main text. Then, use the basis functions $v_1, v_2, v_3, \ldots, v_m$ to recover a yield curve from the components of the response that belong to the α vector. Repeat the steps 2-3 a large number B of times (e.g., $B = 1000$). Calculate the desired confidence bands as the quantiles of the B saved estimates.