The Real Predictive Ability of New Keynesian Models

Christian Matthes *
Universitat Pompeu Fabra and Barcelona GSE

Mu-Chun Wang †
Goethe University Frankfurt

April 22, 2011

Abstract

This paper examines the out of sample forecasting performance of the New Keynesian dynamic general equilibrium model of Smets & Wouters (2007). In particular, we assess the forecasting performance of the Smets & Wouters (2007) model vis-a-vis two variants of Real Business Cycle models that lack nominal frictions. Thus we are able to quantify the contribution of *nominal* frictions to the forecasting performance with regard to *real* variables. We find that some nominal frictions are indeed helpful to forecast real variables while indexation to lagged inflation in the price and wage setting process seems to add little to the forecasting performance of these models.

*Address: Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. E-mail: christian.matthes@upf.edu

†Corresponding author. Address: RuW-49, Grueneburgplatz 1, 60629 Frankfurt, Germany. E-mail: muwang@wiwi.uni-frankfurt.de. We want to thank Uwe Hassler and seminar participants at the University of Hamburg and at the 14th International Conference on Computing in Economics and Finance in Paris for useful comments.

1 Introduction

New Keynesian dynamic general equilibrium models feature prominently in macroeconomic research at central banks and in academic circles. Recent vintages of these models are often found to have similar success in forecasting macroeconomic variables as do leading statistical models such as vector autoregressions (see for example Smets & Wouters (2003)). The model in Smets & Wouters (2007) builds on earlier work such as Christiano, Eichenbaum & Evans (2005) who build dynamic equilibrium models that incorporate numerous frictions, both nominal and real, helping the model fit the data and giving monetary policy an important role in these models by breaking the Pareto optimality of equilibrium allocations. In addition to those frictions this class of models usually features a relatively large number of shocks, which besides improving fit also allows researchers to use more observable variables when estimating the model using likelihood-based estimation. A number of papers have explored the forecasting performance of DSGE models. Besides Smets & Wouters (2004), other papers that focus (at least in part) on forecasting issues are Del Negro & Schorfheide (2004) and Wang (2009), among others.

This paper examines how *nominal* frictions introduced in the New Keynesian framework contribute to the out of sample forecasting performance of *real* variables, namely real investment, real GDP and real consumption. The results are then compared to two stripped down versions of the Smets-Wouters model, one without the nominal frictions and one without nominal and most real frictions. All models are estimated using Bayesian methods. As a side product, we are also able to draw some conclusions on how important real frictions are when forecasting real variables, namely by comparing the forecasting performances of the two benchmark models.

Thus, the the main difference in focus between this paper and others in the literature is that here we focus on the relative merits of different structural models when it comes to forecasting, in contrast to exploring the forecasting performance of those models relative to purely statistical time series models. This paper is also related to research that investigates the role of different frictions in dynamic equilibrium models and how those frictions help the model to replicate VAR impulse responses, such as Christiano et al. (2005). Given that there is a close relation between impulse responses and forecasting [1] our results could be useful in that context as well. Another paper focusing on the relative importance of different frictions is Kano & Nason

[1] An impulse response analysis forecasts the response of the model after a certain shock

(2008), which compares model-implied spectra to spectra estimated using a statistical model.
An advantage of focusing on forecasting performance is that different metrics to measure that performance are readily available and easily interpretable. Furthermore, forecasting performance is interesting in its own right, in particular since models of the class we consider here are now regularly used at central banks for both policy evaluation and forecasting. Throughout this paper we focus on the out of sample forecasting performance of models. While there is considerable discussion among economists about whether or not in-sample or out-of-sample forecast measures should be used (Inoue & Kilian 2002), we follow most of the applied forecasting literature in focusing on the latter.
Section 2 introduces the models used in this paper, section 4 explains the measure of forecasting accuracy, section 3 gives a short summary of the Bayesian estimation procedure and section 5 gives the results. The last section concludes.

2 Models

This section provides the log-linearized equilibrium conditions of the models used in the paper. For the Smets & Wouters (2007) model a more detailed exposition can be found in the original paper.
All variables are log-deviations from the steady state balanced growth path. Starred variables are steady state (balanced growth path) values.

2.1 A medium-scale New Keynesian model - Smets & Wouters

Let c_y, i_y,g_y and k_y denote the steady state ratios of consumption, investment, exogenous government spending and capital to output. Steady state variables that are not ratios are denoted by a $*$. We define $z_y = R^k_* k_y$ where R^k_* is the steady state rental rate of capital.
The aggregate resource constraint in the economy is given by:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon^g_t \tag{1}$$

ε^g_t denotes an exogenous spending shock. c_y is given by $1 - g_y - i_y$. The representative household chooses the path of real consumption to satisfy the following Euler Equation:

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2(l_t - E_t l_{t+1}) - c_3(r_t - E_t \pi_{t+1} + \varepsilon^b_t) \tag{2}$$

where

$$c_1 = (\lambda/\gamma)/(1 + \lambda/\gamma) \tag{3}$$
$$c_2 = [(\sigma_c - 1)W_*^h L_*/C_*]/[\sigma_c(1 + \lambda/\gamma)] \tag{4}$$
$$c_3 = (1 - \lambda/\gamma)/[(1 + \lambda/\gamma)\sigma_c] \tag{5}$$

γ is the steady state growth rate of the economy which enters the model as a deterministic growth rate to labor productivity. λ governs the strength of the first-order external habit formation in the representative agent's utility function. σ_c is the inverse of the intertemporal elasticity of substitution.

Investment is determined by another Euler Equation:

$$i_t = i_1 i_{t-1} + (1 - i_1)E_t i_{t+1} + i_2 q_t + \varepsilon_t^i \tag{6}$$

The no arbitrage condition for the value of capital q_t is given by:

$$q_t = q_1 E_t q_{t+1} + (1 - q_1)E_t r_{t+1}^k - (r_t - E_t \pi_{t+1}) + \varepsilon_t^b \tag{7}$$

where

$$i_1 = 1/(1 + \beta\gamma^{1-\sigma_c}) \tag{8}$$
$$i_2 = i_1 \gamma^2 \varphi \tag{9}$$
$$q_1 = \beta\gamma^{-\sigma_c}(1 - \delta) \tag{10}$$

β is the representative agent's discount factor, δ is the appreciation rate for capital in the model economy and φ is the steady-state elasticity of the capital adjustment cost function. orientation The aggregate production function follows a standard Cobb-Douglas specification except for ϕ_p, which is one plus the share of fixed costs in production. α is the share of capital in production.

$$y_t = \phi_p(\alpha k_t^s + (1 - \alpha)l_t + \varepsilon_t^a) \tag{11}$$

Determination of capital services is given by the equation below:

$$k_t^s = k_{t-1} + z_t \tag{12}$$

where

$$z_t = [(1 - \psi)/\psi]r_t^k \tag{13}$$

and ψ is a parameter that takes values between 0 and 1 and measures the elasticity of the capital utilization adjustment cost.

$$r_t^k = -(k_t - l_t) + w_t \tag{14}$$

Installed capital follows the law of motion:

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i \tag{15}$$

where

$$k_1 = (1 - \delta)/\gamma \tag{16}$$

$$k_2 = (1 - (1 - \delta)/\gamma)(1 + \beta\gamma^{1-\sigma_c})\gamma^2 \varphi \tag{17}$$

Prices are given as a mark up μ_t^p over marginal cost:

$$\mu_t^p = mpl_t - w_t = \alpha(k_t^s - l_t) + \varepsilon_t^a - w_t \tag{18}$$

Monopolistic competition and exogenous sluggish price adjustment a la Calvo leads to the following New Keynesian Phillips Curve:

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p \tag{19}$$

$$\pi_1 = \iota_p / (1 + \beta\gamma^{1-\sigma_c}\iota_p) \tag{20}$$

$$\pi_2 = \beta\gamma^{1-\sigma_c} / (1 + \beta\gamma^{1-\sigma_c}\iota_p) \tag{21}$$

$$\pi_3 = 1/(1 + \beta\gamma^{1-\sigma_c}\iota_p)[(1 - \beta\gamma^{1-\sigma_c}\xi_p)(1 - \xi_p)/\xi_p((\phi_p - 1)\varepsilon_p + 1)] \tag{22}$$

Prices are indexed to inflation according to the following scheme: prices of firms that can not reoptimize their price in a given period are changed by the factor $\pi_{t-1}^{\iota_p} \pi_*^{\iota_p}$.

The degree of price stickiness is given by ξ_p.

Wages are set as a mark-up μ_t^w over marginal cost, similarly to prices:

$$\mu_t^w = w_t - mrs_t = w_t - \left(\sigma_l l_t + \frac{1}{1 - \lambda/\gamma}(c_t - \lambda/\gamma c_{t-1}) \right) \tag{23}$$

σ_l is the elasticity of labour supply with respect to the real wage. Real wages are determined by

$$w_t = w_1 w_{t-1} + (1 - w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w \tag{24}$$

where the parameters in the equation above are given by

$$w_1 = 1/(1 + \beta\gamma^{1-\sigma_c}) \tag{25}$$

$$w_2 = (1 + \beta\gamma^{1-\sigma_c}\iota_w)/(1 + \beta\gamma^{1-\sigma_c}) \tag{26}$$

$$w_3 = \iota_w / 1 + \beta\gamma^{1-\sigma_c} \tag{27}$$

$$w_4 = 1/(1 + \beta\gamma^{1-\sigma_c})[(1 - \beta\gamma^{1-\sigma_c}\xi_w)(1 - \xi_w)/(\xi_w((\phi_w - 1)\varepsilon_w + 1))] \tag{28}$$

ι_w and ξ_w are defined analogously to their counterparts in the price setting equations. Monetary policy is determined by a Taylor-type rule where the relevant output variables are deviations from potential output. Potential output is the level of output in the case of flexible prices and wages and without the two mark-up shocks.

$$r_t = \rho r_{t-1} + (1-\rho)(r_\pi \pi_t + r_y(y_t - y_t^p)) + r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^r \tag{29}$$

The exogenous variables are determined by the following processes:

$$\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b \tag{30}$$
$$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a \tag{31}$$
$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i \tag{32}$$
$$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r \tag{33}$$
$$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a \tag{34}$$
$$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p \tag{35}$$
$$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w \tag{36}$$

All η's are independent of each other and across time. We assume they are normally distributed with standard deviations $\sigma_b, \sigma_a, \sigma_i, \sigma_r, \sigma_g, \sigma_p$ and σ_w.

2.2 A RBC model with real frictions

Next we strip the Smets-Wouters model of its nominal aspects to arrive at a RBC-type model with real frictions. Variables are defined as in the Smets

& Wouters (2007) model above.

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g \tag{37}$$

$$c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t + \varepsilon_t^b) \tag{38}$$

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i \tag{39}$$

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - r_t + \varepsilon_t^b \tag{40}$$

$$y_t = \phi_p (\alpha k_t^s + (1 - \alpha) l_t + \varepsilon_t^a) \tag{41}$$

$$k_t^s = k_{t-1} + z_t \tag{42}$$

$$z_t = [(1 - \psi)/\psi] r_t^k \tag{43}$$

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i \tag{44}$$

$$r_t^k = -(k_t - l_t) + w_t \tag{45}$$

$$0 = w_t - \left(\sigma_l l_t + \frac{1}{1 - \lambda/\gamma} (c_t - \lambda/\gamma c_{t-1}) \right) \tag{46}$$

$$0 = \alpha (k_t^s - l_t) + \varepsilon_t^a - w_t \tag{47}$$

To summarize differences between the RBC model and the New Keynesian model in their log-linearized representation: in the RBC model we are removing the Calvo price and wage stickiness and shutting down the market power in both the labor and intermediate goods markets. Hence, there is no New Keynesian Phillips Curve, and the monetary authority plays no role. On the real side of the model, there are only minor changes compared to the New Keynesian models. The equations (37)-(45) are almost the same as (1)-(18), except that expected inflation is dropped from the consumption Euler equation (38) and the no arbitrage condition of the real value of capital (40). A big difference between the models is the wage setting process. Since both price and wage mark ups are zero, the real wage is simply equal to the marginal rate of substitution *and* the marginal product of labor given by equations (46) and (47). [2] Regarding the exogenous shocks, the price mark up, the wage mark ups, and the monetary policy shock disappear in the RBC model. Thus, the number of exogenous shocks reduces to four. The stochastic specification for the remaining shocks are otherwise the same as in the New Keynesian model.

In the figures below this model is called the *flex RBC* model.

[2] An alternative would have been to allow non-zero real wage and price mark-ups, i.e. to retain the assumption of monopolistic competition. We chose not to do so to bring this model closer in line with other RBC models used in the literature.

2.3 A simple RBC model

We then go on to remove all real frictions from the RBC model to arrive at a textbook version of that model. This model will help us assess how important those real frictions are for forecasting.
The following equations characterize the equilibrium for this model:

$$c_t = c_{t-1} + \frac{(\sigma_c - 1)\frac{W_* L_*}{C_*}}{\sigma_c}\left(l_t - l_{t+1}\right) - \frac{1}{\sigma_c} r_t \tag{48}$$

$$r_t = \left(1 - \beta\gamma^{-\sigma_c}\left(1-\delta\right)\right) r^k_{t+1} \tag{49}$$

$$y_t = \alpha k^s_t + \left(1-\alpha\right) l_t + \varepsilon^a_t \tag{50}$$

$$k_t = \frac{1-\delta}{\gamma} k_{t-1} + \left(1 - \frac{1-\delta}{\gamma}\right) i_t \tag{51}$$

$$k^s_t = k_{t-1} \tag{52}$$

$$r^k_t = -\left(k^s_t - l_t\right) + w_t \tag{53}$$

$$\varepsilon^a_t = \alpha r^k_t + \left(1-\alpha\right) w_t \tag{54}$$

$$w_t = \sigma_l l_t + c_t \tag{55}$$

$$y_t = c_y c_t + i_y i_t + \varepsilon^g_t \tag{56}$$

A New Keynesian model without indexation to lagged inflation

This model is essentially the same as the New Keynesian model with indexation except for the following modifications (with the reduced form parameters formed using indexation parameters that are set to 0):

$$\pi_t = \pi_2 E_t \pi_{t+1} - \pi_3 \mu^p_t + \varepsilon^p_t \tag{57}$$

$$w_t = w_1 w_{t-1} + (1-w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_1 \pi_t - w_4 \mu^w_t + \varepsilon^w_t \tag{58}$$

3 Estimation Procedure and Data

3.1 Estimation

The estimation procedure we follow has been used quite extensively in the macroeconomics literature during the last decade. We only sketch the algorithm and provide the additional information necessary to be able to reproduce our results. For a detailed exposition of these methods we refer to An & Schorfheide (2007).

The log-linearized equilibrium conditions given in the preceding section form a system of expectational difference equations. These can be jointly solved for a state space system like (59)-(60) via a number of algorithms, e.g. Sims (2002).

$$y_t = Ax_t + Bu_t \tag{59}$$

$$x_t = Cx_{t-1} + Dv_t \tag{60}$$

This state space system (consisting of the vector of observables y_t and the vector of generally unobserved states x_t) is then used in conjunction with the Kalman Filter to evaluate the density of observables given a vector of parameters. The time series used in our estimations are quarterly. We use independent priors for each structural parameter and combine those with the likelihood function computed with the Kalman Filter to arrive at a posterior distribution, the object of interest in Bayesian statistical inference. The priors we choose are identical to those of Smets & Wouters (2007). As we move from the New Keynesian model to the variants of the RBC model, we keep the priors for those parameters that also appear in the respective RBC Models. The prior distributions are given in the appendix. Since the mapping from structural parameters to the entries of the parameter vectors in (59)-(60) is in general non-linear we use the Metropolis-Hastings algorithm to generate 1000000 draws that approximate draws from the desired posterior. We use a random walk proposal density with mean zero normal innovations and a scaled version of the inverse Hessian at the posterior peak as covariance matrix. We estimate our models and generate forecasts using the Dynare package of Juillard et al.

3.2 Data

We use the following observables in our estimation exercises [3]:

1. $\Delta \log GDP$, the log difference of real GDP per capita
2. $\Delta \log C$, the log difference of real consumption per capita
3. $\Delta \log INV$, the log difference of real fixed private domestic investment per capita
4. $\Delta \log W$, the log difference of real wages per hour in the non-farm sector

[3] This is the same data set used in Smets & Wouters (2007). All series are seasonally adjusted.

5. $\Delta \log P$, the log difference of the GDP implicit price deflator
6. $\log H$, the log of average non-farm hours times civilian employment per capita
7. $\log R$, the log of the Federal Funds Rate divided by 4

For the Smets & Wouters (2007) model and the New Keynesian model without indexation we use all of the observables above, while for the RBC model with real frictions we use observables 1 through 4 and for the textbook RBC model we use observables 1 and 2 only to avoid stochastic singularity when calculating the likelihood function. This choice might be criticized since we give the New Keynesian models more data to work with, but we see that as a way of disciplining the estimation. If we let all models use the same observables all nominal variables in the New Keynesian model are unobserved (some of them will be unobserved state variables). Those state variables can then take on highly improbable values to improve in sample fit, while out-of sample fit of the real variables could suffer substantially (remember that the unobserved states are crucial in forecasting via the state space system and also that we are not interested in forecasting nominal variables). Since we are interested in realistic forecasting exercises such as those undertaken by central banks we view our approach as reasonable enough. All real variables are obtained by using the GDP deflator. Per capita variables are calculated by dividing by the size of the population 16 years and older. Hours and wages are obtained from the Bureau of Labor Statistics and all other variables besides the Federal Funds Rate are from the Bureau of Economic Analysis. The sample starts in the first quarter of 1965 and ends in the fourth quarter of 2001. We actually have data until the last quarter of 2004, but we set those values aside for our forecasting exercises.

4 Formation of Forecasts and Measure of Forecast Performance

For the out-of-sample forecasting experiments, we use rolling estimations with sample size $R = 100$. While this is a small number for models of the size we use we had to strike a balance between individual sample size and the number of samples we could use to measure forecast performance. Forecast horizons h up to 12 quarters are considered. The first estimation sample starts in 1965:1 and ends in 1989:4 so that the first forecasting date is 1990:1. After all models have been estimated, the first set of out-of-sample forecasts is computed. Then, sample range shifts one-step forward to 1965:2-1990:1 in order to compute the second set of forecasts. All models are fully re-estimated for each rolling sample with the estimation procedures described earlier. The estimation is performed $S = 49$ times to obtain a series of forecasts for each forecast horizon and each model. The last sample is 1977:1-2001:4 and the last forecasting date is 2004:4.

We measure the forecast performance of the different models by comparing them to naive forecasts using the entire sample mean of each variable as the predictor. For each rolling sample we thus calculate the average of the observables we are interested in and denote that average $\overline{y}_s$ where $s = 1, ..., S$ For our DSGE models the forecasts are formed using the state space system (59)-(60), by iterating on the last estimate of the unobserved state using the state equation (60) and then backing out the corresponding value for the observable using the measurement equation (59). We do this for a subset of the parameter draws obtained using the Metropolis-Hastings algorithm [4] Let $T = R + s - 1$ be the end of each rolling sample s. We denote the h-step ahead point forecasts of model i and sample s at time T as $\widetilde{y}^{i,s}_{T+h|T}$ [5]. Thus for the mean forecast we have $\widetilde{y}^{mean,s}_{T+h|T} = \overline{y}_s$. The root mean squared forecast error of model i at forecast horizon h is given by:

$$RMSFE(h, i) = \sqrt{\frac{1}{S}\sum_{s=1}^{S}(y_{T+h} - \widetilde{y}^{i,s}_{T+h|T})^2} \tag{61}$$

We then look at the percentage gains or losses in the RMSFE's when we use a DSGE model relative to the naive mean forecast:

$$rRMSFE(h, i) = 1 - \frac{RMSFE(h, i)}{RMSFE(h, mean)} \tag{62}$$

$rRMSFE(h, i)$ is often referred to as the forecast content function. If the model i forecast is considered informative, its $rRMSFE(h, i)$ should be larger than zero. In the figures below $rRMSFE(h, i)$ is reported as a percentage.

[4] Since the draws generated by the Metropolis-Hastings algorithm are highly correlated using uniformly drawn parameter values from the original sample increases efficiency without losing too much information to obtain the posterior forecast distribution. The mean of the posterior forecast distributions is taken as the point forecast of the relevant variable. This distribution only takes into account parameter uncertainty.

[5] We assume that only the most recent finished subsample is used to form forecasts.

5 Results

5.1 Estimation Results of DSGE Models

Figure 1 reports posterior means of selected parameters with associated 90% highest posterior density intervals (HPDI) over the rolling subsamples of the New Keynesian model and the RBC model with real frictions.[6] First, our estimated parameters of the New Keynesian model are very similar to those of Smets & Wouters (2007). Second, the estimated parameters of the RBC model are, with minor exceptions, mostly in line with the New Keynesian model. There are moderate differences in estimates of the production fix cost and the standard deviation of the productivity shock between these two models. However, the HPDIs of nearly all parameters of the RBC model with real frictions overlap in large part with those from the New Keynesian model. This indicates that estimates of parameters governing real behavior are robust against different specifications of the nominal side of the economy. Regarding the evolution of the estimates, the estimated technology growth rate is increasing over time for both models, indicating an acceleration of technology growth over the past 20 years. Otherwise, the estimated real frictions and other behaviour parameters remain fairly stable over subsamples. Finally, an important observation is the decline of the estimated volatilities of the structural shocks. Both productivity and government spending shock volatilities fall substantially over time.

Figure 2 reports the estimation results of some selected parameters which only appear in the New Keynesian model and represent the nominal side of the model. In terms of monetary policy, it is interesting to note that both the estimated central bank response to inflation and the interest rate smoothing parameter have hardly changed over time, whereas the response to the output gap has declined somewhat. At first glance, these results seem to contradict those of Clarida, Gali & Gertler (2000) where substantial differences in the Taylor rule have been estimated between the Pre-Volcker (1960:1-1979:2) and Volcker-Greenspan (1979:3-1996:4) periods. However, our sample selection is quite different from theirs. Our first subsample starts in 1965:1 and ends in 1989:4 which encompasses the Volcker disinflation period and covers the beginning of Greenspan's tenure. Hence, our results are only loosely connected with the notion of different monetary policy regimes.

Regarding the nominal frictions on prices and wages, we can observe a large increase in the Calvo price parameter over time, while no significant movement of the Calvo wage parameter can be seen. Initially, wages are

[6] Figures for all estimated parameters can be provided by the authors upon request.

much stickier than prices with degree of stickiness around 0.7 against 0.55, respectively. By the end of the sample, price and wage stickiness are almost equal. The degrees of price and wage indexation to lagged inflation have fallen only slightly. These results are in line with the estimates from Smets & Wouters (2007). Most importantly, the standard deviations of aggregate nominal shocks - like in case of real shocks - have dropped considerably over time.

Overall, our results capture in part the transition of the economy from periods of high volatility towards the so-called "Great Moderation" where the aggregate volatilities are much lower. The rolling estimates indicate that the driving force behind the transition is the decline in volatilities of aggregate shocks. This is also in line with Wang (2009) where subsample estimates of a small-scale New Keynesian model are reported.

5.2 Forecasting Results

In our out-of-sample forecasting experiments, we focus our attention on three key real variables: output growth, investment growth and consumption growth. We also include VAR models to disentangle two different possible sources of forecast improvements. The first one is whether including *nominal variables* in the VAR alone helps to predict real variables. The second one is whether restrictions derived from economic theory per se help to improve forecasts. In other words, we want to find out whether any improvement in forecasts stems from the theoretical restrictions we impose or from the statistical information contained in the nominal variables we consider. Hence we consider two different VAR specifications. The first VAR model is a seven-variable VAR incorporating the same observables as the New Keynesian model, and the second VAR model is a four-variable VAR with the same observables as in the RBC model. In addition to the unrestricted VAR models, we also consider Bayesian VARs (BVAR) and impose atheoretical restrictions in the form of priors. Following Smets & Wouters (2007) we use a Minnesota prior, which assumes that individual variables in the VAR follow random walks. Hence, the BVARs are estimated in level in contrast to the unrestricted VARs. Both VAR and BVAR models are reestimated at each rolling sample like their DSGE counterparts.[7] The focus

[7] We follow the specification of Smets & Wouters (2007) and set the lag length of unrestricted VARs to one and BVARs to four. The estimation procedure and prior specification follow Lütkepohl (2005). We also experiment with various hyperparameter specifications of the BVARs. Setting the "own lags" tightness parameter to 1 and "other lags" to 0.01 seems to yield overall the best forecasting performance for the BVARs.

of this paper however is *not* to find the best forecasting model per se but to link the forecasting ability of structural macroeconomic models to certain properties of those models that have a clear macroeconomic interpretation.

Figure 3 shows the $rRMSFE(h, i)$ of the New Keynesian model, the RBC model with real frictions and the two unrestricted VAR models. In terms of output growth, the New Keynesian model dominates other competing models and is able to provide informative forecasts for all forecast horizons considered. The RBC model with real frictions can only outperform the New Keynesian model at the one-quarter-ahead horizon and is generally not informative beyond the two quarter horizon. The four-variable VAR has nearly the same forecasting performance as the RBC model, whereas the seven-variable VAR has uniformly the worst predictive ability for output growth. Regarding investment growth, all models can generate fairly informative forecasts at short horizons. The New Keynesian model remains the dominating forecasting model. The RBC model with real frictions is able to outperform the corresponding four-variable VAR when it comes to investment, indicating that the underlying structural restrictions imposed on the RBC model indeed have some predictive content, at least for investment. In terms of consumption growth, none of the model considered can generate informative forecasts.The RBC model is especially bad in this regard, yielding forecasts which have more than 20 percent higher root mean squared forecast errors than the *unconditional mean* forecasts.[8]

Figure 4 shows the $rRMSFE(h, i)$ of the same DSGE models as in figure 3 and the associated BVAR models. First, the four-variables and the seven-variables BVAR have almost the same predictive performance. This is likely due to the tight prior specification of our BVARs. The use of of the Minnesota prior greatly improves the predictive performance of the large seven-variables VAR in terms of consumption and output growth. In contrast to the unrestricted VAR, the seven-variable BVAR is able to provide informative short-horizon output growth forecasts, whereas the predictive performance of the smaller four-variable BVAR is similar to that of the 4 variable VAR estimated using a frequentist approach. Nevertheless, the the New Keynesian model and the RBC model are still able to outperform their BVAR counterparts for output and investment growth. Summing up, at least for output growth and investment, it seems that the structural restrictions imposed by economic theory lead to better out-of-sample forecasting

[8]We checked the sensitivity of our results by increasing the prior standard deviations of the DSGE models. In particular, all priors standard deviations except beta priors are increased by 50 percent. All beta priors are set to be symmetric with mean 0.5 and standard deviation 0.25. The overall results remain very similar.

performance. Increasing the number of variables in the VAR alone does not improve forecasts, except in combination with reasonable restrictions on the parameter space. Using structural restrictions based on economic theories tends to provide better forecasts than atheoretical restrictions like the Minnesota prior. Moreover, introducing *nominal* rigidities tends to improve forecasts of *real* variables, given that the New Keynesian model is superior to the RBC model in terms of the forecasts presented here. [9]

5.3 Impact of different specifications

In this subsection, we further examine the impact of different structural restrictions on the predictive ability of the DSGE models. We consider the two special cases of our models described in section 2: The first model is the baseline RBC model without real frictions (denoted *plain flex RBC* in the figures). It is essentially the same RBC model as in sections above but without all real rigidities. It should serve as an assessment of the predictive content of the real rigidities on the real variables. Second, in Smets & Wouters (2007), it is documented that the price and wage indexations to lagged inflation are empirically quite unimportant *in-sample*. Therefore, the second model is the same New Keynesian model as in sections above but without the price and wage indexations to evaluate their impact out-of-sample. Figure 5 reports the $rRMSFE(h,i)$ of all DSGE models.[10]

From the figure, it is obvious that price and wage indexations to lagged inflation are irrelevant in terms of forecasting real variables. Forecasts of the New Keynesian model both with and without price and wage indexations to lagged inflation have virtually the same performance for all variables and forecast horizons. Turning to the results of the RBC models, on the one hand, it seems that the real frictions can help improve short horizons forecasts of output and investment growth (up to approximately the three quarter horizon). On the other hand, albeit still uninformative, the baseline

[9] One might wonder why the VAR models do poorly in our forecasting exercises. Note that we use a small sample size, which could be a culprit for performance of the VARs. Also, it is worth noting that our results on the forecasting performance of VARs vs. DSGE models are in line with the results in Smets & Wouters (2007).

[10] Details of the specifications of the VARs are given in an technical appendix available upon request. The prior specifications remain unchanged and all models are re-estimated at each subsample with the procedure described in section 3. Figure 5 does not include the $rRMSFE(h,i)$ for investment in the case of the plain RBC model since investment is not an observable variable when we estimate that model. We could still calculate forecasts for investment with that model, but since investment is not included as an observable this would not be a fair comparison.

RBC model is clearly superior to the model with real frictions in consumption growth forecasts. We argue that the results confirm our view that nominal frictions add useful information when forecasting real variables, since both RBC models are inferior to the New Keynesian models in terms of $RMSFE$.

5.4 Forecast dynamics and uncertainty

In previous sections, we have demonstrated the usefulness of nominal frictions to predict real variables. In order to assess the reason for this phenomenon, we take a closer look at the forecasts dynamics of the underlying models. Figure 6 shows the out-of-sample forecasts of few selected subsamples for output growth. The means of the posterior forecasts of the models are plotted against the actual values. The pictures show that real frictions seem to help little to predict output growth. The h-step ahead forecasts of the baseline RBC model are almost flat, hence, miss all out-of-sample dynamics. By introducing real frictions, we can observe little additional short run persistence in the forecasts that help to capture the dynamics of output growth out-of-sample. It is the nominal frictions which are really successful in generating enough persistence in the forecasts, thus mimicking the actual movements of the variables. Our results are in line with the impulse responses analyses of New Keynesian models such as Christiano et al. (2005) where standard RBC models fail to provide adequate empirical responses after fundamental shocks. Our results seem to confirm that nominal frictions have merits in out-of-sample forecasting.

Thus far we have focused on point forecasts. Naturally a researcher would also be interested in the uncertainty associated with the forecast generated by a certain model. As is usually the case in econometric applications a researcher is faced with a trade-off between bias and variance when choosing an estimator. However in our case the inclusion of nominal frictions did not change the standard deviation of forecasts considerably so as a first step we focus on mean forecasts. This is illustrated in figure 7, where we report the 90% HPDIs (dashed lines) of posterior forecasts from the upper left subplot of figure 6. The forecast uncertainty (the width of the HPDI) of the baseline RBC model is somewhat smaller than the New Keynesian model and the RBC model with real frictions, whereas the degree of uncertainties of the latter models are very similar. The 90% HPDIs we report here only take into account *parameter uncertainty*, but not the uncertainty associated with future shocks and the uncertainty associated with the last estimated unobserved state we condition our forecasts on (we follow the dynare package in using the last available smoothed estimate of the state for each parameter draw). Thus those confidence bands are confidence bands for the minimum mean squared error forecast conditional on a particular value for the state x_t.

Naturally, this pattern could be a consequence of our decision to estimate the New Keynesian models with more variables than the RBC model. Otherwise the uncertainty associated with the forecasts coming form the New Keynesian model should be larger since it is a larger model.
As a robustness check we re-estimate the New-Keynesian model with the 4 observables from the RBC specification ($\Delta \log GDP$, $\Delta \log C$, $\Delta \log INV$, and $\Delta \log W$). Figure 8 shows the error bands from Figure 7 and the corresponding error band for the 4 variable New Keynesian model. Clearly, the New Keynesian model with only 4 observables has more uncertainty associated with it than the same New Keynesian model with more observables and the smaller RBC model with the same observables. To confirm this point we calculate the width of the error bands for output growth across time and forecasting horizons for our models and plot them in Figure 9.

6 Conclusion

This paper tries to shed some light on what specific features of modern macroeconomic models contribute to their forecasting performance. To do so we use a standard medium scale macroeconomic model as our workhorse and show that nominal frictions are indeed important when it comes to forecasting real economic variables. However our results indicate that while nominal frictions are helpful, this is not true uniformly across all real variables and nominal frictions included in the Smets & Wouters (2007) model. In particular, price and wage indexation to lagged inflation seem to contribute little to the (real) forecasting ability of the Smets & Wouters (2007) model. Furthermore, all economic models considered here do not provide a substantial improvement over a naive mean forecast when it comes to real consumption.

By focusing on a subset of variables and a well-known forecasting statistic ($RMSFE$) we take a narrower approach to model fit comparison than the standard marginal likelihood comparison. The marginal likelihood encodes all the information of the one step ahead forecasting density for all observables. We think by concentrating on a subset of observables and one statistic of the forecasting distribution a researcher can gain additional insights that are blurred by the amount of information contained in the marginal likeli-

hood.
While our results are obviously partly driven by our choice of models and forecasting variables we think that our approach can nonetheless highlight how nominal features of macroeconomic models contribute to the forecasting performance of those models, which is hopefully of interest to researchers in both academia and at central banks.

References

An, S. & Schorfheide, F. (2007), 'Bayesian analysis of DSGE models', *Econometric Reviews* **26**(2-4), 113–172.

Christiano, L., Eichenbaum, M. & Evans, C. (2005), 'Nominal rigidities and the dynamic effects of a shock to monetary policy', *Journal of Political Economy* **113**(1), 1–45.

Clarida, R., Gali, J. & Gertler, M. (2000), 'Monetary policy rules and macroeconomic stability: Evidence and some theory', *The Quarterly Journal of Economics* **115**(1), 147–180.

Del Negro, M. & Schorfheide, F. (2004), 'Priors from general equilibrium models for VARs', *International Economic Review* **45**, 643–673.

Inoue, A. & Kilian, L. (2002), In-sample or out-of-sample tests of predictability: Which one should we use?, CEPR Discussion Papers 3671.

Kano, T. & Nason, J. M. (2008), Business cycle implications of internal habit formation. Federal Reserve Bank of Atlanta.

Lütkepohl, H. (2005), *New introduction to multiple time series analysis*, Springer.

Sims, C. A. (2002), 'Solving linear rational expectations models', *Computational Economics* **20**(1-2), 1–20.

Smets, F. & Wouters, R. (2003), 'An estimated dynamic stochastic general equilibrium model of the euro area', *Journal of the European Economic Association* **1**, 1123–1175.

Smets, F. & Wouters, R. (2004), 'Forecasting with a Bayesian DSGE model: An application to the euro area', *Journal of Common Market Studies* **42**, 841–867.

Smets, F. & Wouters, R. (2007), 'Shocks and frictions in US business cycles: A Bayesian DSGE approach', *American Economic Review* **97**(3), 586–606.

Wang, M.-C. (2009), 'Comparing the DSGE model with the factor model: An out-of-sample forecasting experiment', *Journal of Forecasting* **28**(2), 167–182.

A Prior Distributions

Parameter	distribution	mean	standard deviation
$\overline{\gamma}$	Normal	0.40	0.10
h	Beta	0.70	0.10
σ_c	Normal	1.50	0.37
σ_l	Normal	2.00	0.75
$100\beta^{-1}$	Gamma	0.25	0.10
δ	Normal	0.025	0.00
g_y	Normal	0.18	0.00
φ	Normal	4.00	1.50
ϕ_p	Normal	1.25	0.25
α	Normal	0.30	0.05
ψ	Beta	0.50	0.15
ι_p	Beta	0.50	0.15
ξ_p	Beta	0.50	0.10
ξ_w	Beta	0.50	0.10
ι_w	Beta	0.50	0.15
ρ	Beta	0.75	0.10
r_y	Normal	0.12	0.05
r_π	Normal	1.50	0.25
$r_{\Delta y}$	Normal	0.12	0.05
$\overline{\pi}$	Gamma	0.62	0.10
$\overline{L}$	Normal	0.00	2.00
σ_a	Inverse Gamma	0.10	2.00
σ_b	Inverse Gamma	0.10	2.00
σ_i	Inverse Gamma	0.10	2.00
σ_g	Inverse Gamma	0.10	2.00
σ_w	Inverse Gamma	0.10	2.00
σ_r	Inverse Gamma	0.10	2.00
σ_p	Inverse Gamma	0.10	2.00
ρ_a	Beta	0.50	0.20
ρ_b	Beta	0.50	0.20
ρ_i	Beta	0.50	0.20
ρ_g	Beta	0.50	0.20
ρ_w	Beta	0.50	0.20
ρ_r	Beta	0.50	0.20
ρ_p	Beta	0.50	0.20
ρ_{ga}	Beta	0.50	0.20
μ_p	Beta	0.50	0.20
μ_w	Beta	0.50	0.20

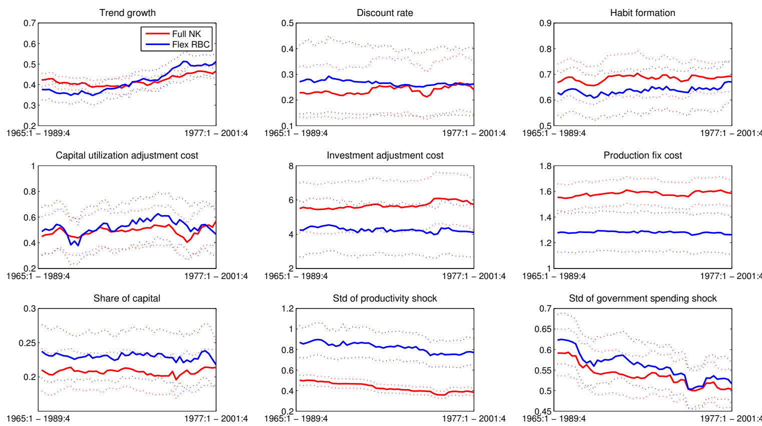

Figure 1: Parameter estimates over subsamples: real side

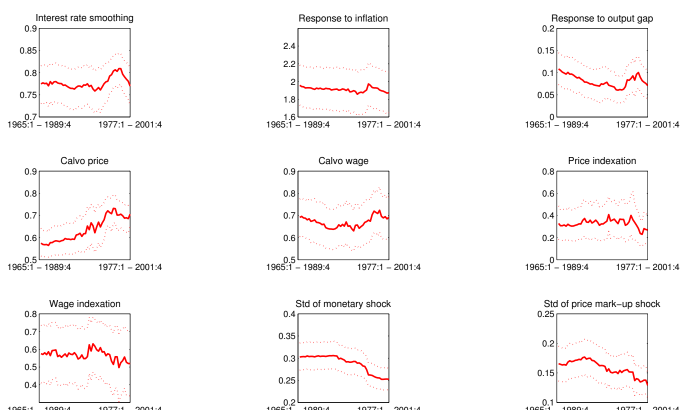

Figure 2: Selected parameter estimates over subsamples: nominal side

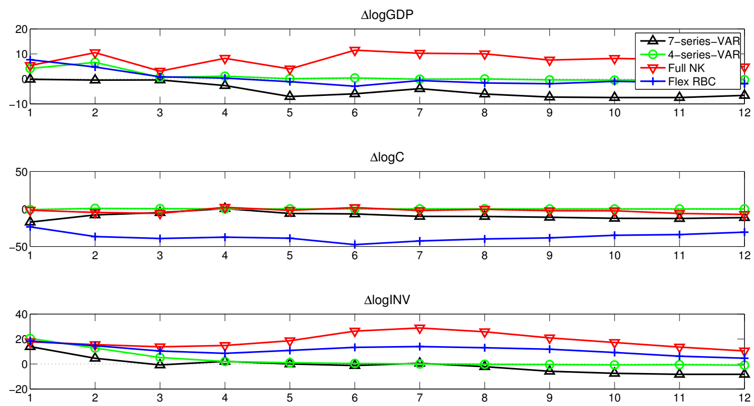

Figure 3: Forecasting performance of DSGE and VAR

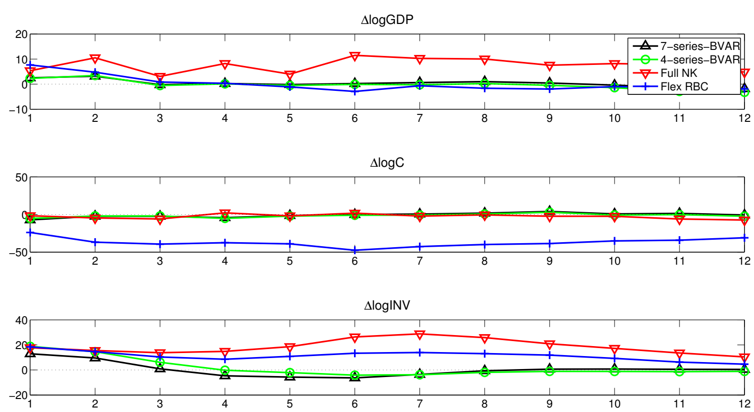

Figure 4: Forecasting performance of DSGE and Bayesian VAR

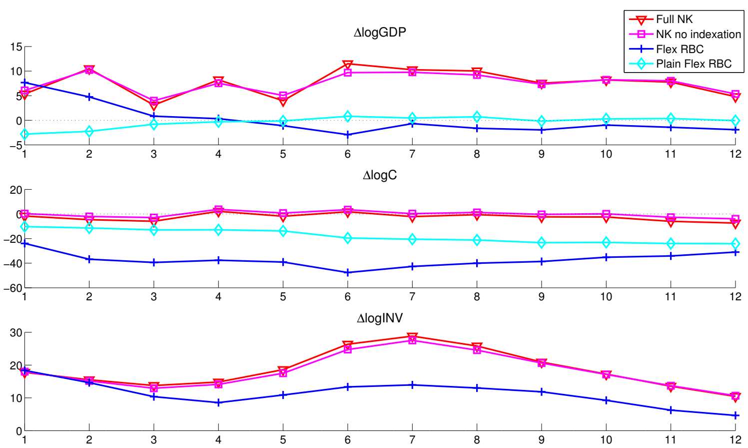

Figure 5: Forecasting performance of different DSGE specifications

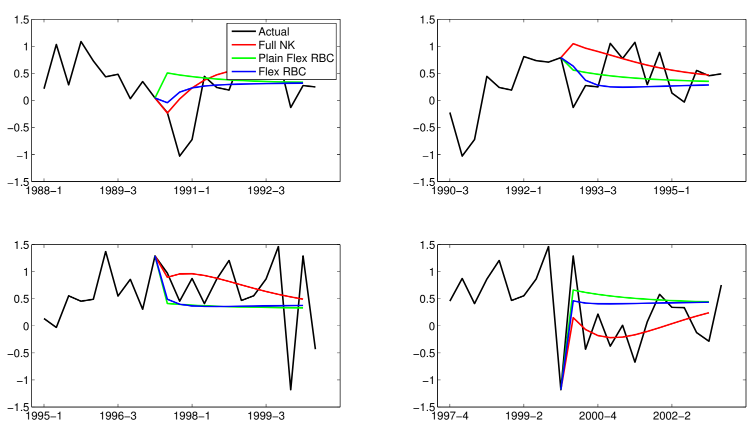

Figure 6: Illustration of forecasting performance: output growth

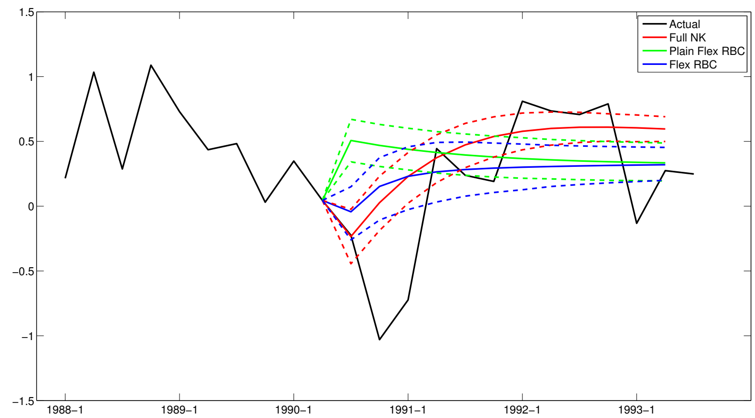

Figure 7: Illustration of forecast uncertainty: output growth

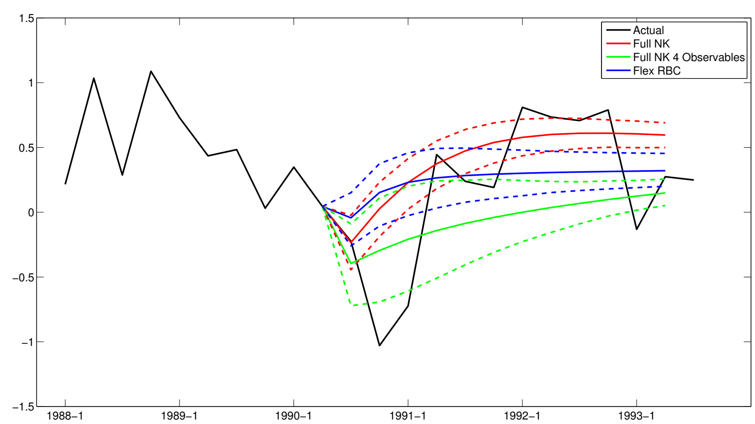

Figure 8: Illustration of forecast uncertainty with 4 variable New Keynesian model: output growth

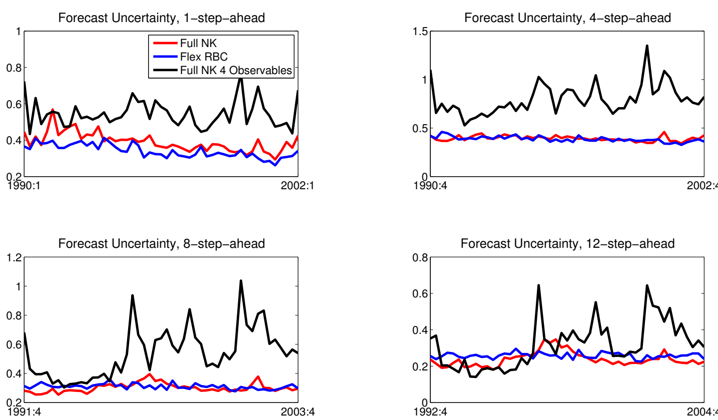

Figure 9: Width of error bands across time and forecasting horizons