Measurement Errors and Monetary Policy: Then and Now
APPENDIX

Pooyan Amir-Ahmadi*
University of Illinois at Urbana-Champaign
Christian Matthes
Federal Reserve Bank of Richmond
Mu-Chun Wang
University of Hamburg

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*The views expressed in this document are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

This appendix first discusses how our choice of variables for the VAR can be motivated by a DSGE model. We then present various robustness exercises:

1. a model with employment growth instead of GDP growth
2. using a recursive identification scheme instead of sign restrictions to infer monetary policy shocks and their effects
3. using an alternative definition of 'final data' due to Aruoba (2008)
4. restricting both the responses of real-time and final data in a sign restriction approach
5. using the Wu-Xia shadow rate (Wu & Xia (2016)) instead of the Federal Funds Rate in our VAR to avoid issues related to the zero lower bound at the end of our sample

1 The Choice of Variables for Our VAR - A DSGE-based Motivation

In this section, we describe one (admittedly, along many dimensions, very simple) DSGE model that delivers, as its reduced form, a VAR in the same state variables that we use in our empirical analysis. We do this to motivate our choice of variables, but also to highlight that we can indeed identify monetary policy shocks using the observables described above. This is not immediately obvious since our sign restrictions imply that the nominal interest rate is the monetary policy instrument. The nominal interest rate in our VAR reacts to final data lagged once and twice, which is not in any central bank's information set. The DSGE model in this section shows how our approach can be justified. The DSGE model does not feature time-varying dynamics or stochastic volatility - those features could be added by introducing learning along the lines of Cogley, Matthes & Sbordone (2015), for example. For simplicity, the DSGE model presented here has a VAR of order 1 as its reduced form, whereas we work with a VAR of order 2 in the empirical analysis. Additional lags could easily be introduced in the DSGE model, but would not add any insight to the exposition. We directly present the linearized version of the model. The first two equations give the dynamics of our real variable x_t and inflation π_t, conditional on iid shocks z_t and g_t as well as the nominal interest rate i_t:

$$x_t = a_x E_t x_{t+1} + b_x(i_t - E_t \pi_{t+1}) + g_t + c_x x_{t-1} \tag{1}$$

$$\pi_t = a_\pi E_t \pi_{t+1} + b_\pi x_t + z_t + c_\pi \pi_{t-1} \tag{2}$$

Following the literature cited in the previous section, the private sector in this model has access to final data and thus the IS and Phillips curves are straight out of standard New Keynesian models. The variable x_t in micro-founded DSGE models is usually the output gap. Real-time measures of the output gap are unfortunately chronically unreliable (Orphanides & van Norden (2002)), so we choose to use alternative measures of real activity in our empirical analysis instead. This lack of reliability does not come from the real-time nature of output data, but rather from the large uncertainty surrounding trend estimates of GDP in real time. Thus, it seems likely that real time measures of the output gap do not have a great weight in monetary policy decisions.

Next, we implicitly define the measurement errors:

$$x_t^{real} = x_t + \eta_t^y \tag{3}$$

$$\pi_t^{real} = \pi_t + \eta_t^\pi \tag{4}$$

The two non-standard features of this economy are the monetary policy rule and the dynamics of the measurement error, which are substantially more general than what is usually assumed in the literature:

$$i_t = a_i x_{t-1}^{real} + b_i \pi_{t-1}^{real} + c_i i_{t-1} + d_i E_{t-1} x_t + e_i E_{t-1} \pi_t + \varepsilon_t^i \tag{5}$$

$$\begin{aligned} \eta_t &= [\eta_t^x \ \eta_t^\pi]' \\ &= M_1 [y_t \ \pi_t \ i_t]' + M_2 [x_{t-1} \ \pi_{t-1} \ i_{t-1}]' + \rho \eta_{t-1} + M_3 \varepsilon_t^{real} \end{aligned} \tag{6}$$

Note that the nominal interest rate only reacts to real-time data as well as survey measures of expectations gathered in the previous period. The monetary policy shock is denoted ε_t^i. While this is certainly restrictive (central banks have access to revised data for past periods), this assumption respects the information constraints of actual central banks in that the central bank can not react to final data for recent periods. Data is revised for multiple years (as mentioned before, Aruoba (2008) uses a window of three years to define final data for the United States, for example), whereas there are multiple policy meetings per year for all major central banks.

The definition of the measurement error dynamics (we denote the vector of measurement errors by η_t) allows for dependence on lagged measurement errors as well as lagged and contemporaneous final data. M_j $(j \in \{1, 2, 3\})$ and ρ are conformable matrices containing parameters that determine the properties of the measurement error process. We do not state a theory that delivers these dynamics, but instead want to show that even with general dynamics like these, the dynamics are fully captured by the variables we use in our VAR. ε_t^{real} is a vector of iid shocks, which we assume to be of the same dimension as η_t.

To solve the model, we can first reduce the system and use the implicit definition of the measurement errors to eliminate η. We are then left with a system that does include $E_{t-1}\pi_t$ and $E_{t-1}x_t$ as state variables. Solving this model using standard methods for linear rational expectations models such as Gensys (Sims (2002)), we get the following reduced form:

$$\begin{bmatrix} x_t \\ \pi_t \\ x_t^{real} \\ \pi_t^{real} \\ i_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = A \begin{bmatrix} x_{t-1} \\ \pi_{t-1} \\ x_{t-1}^{real} \\ \pi_{t-1}^{real} \\ i_{t-1} \\ E_{t-1} x_t \\ E_{t-1} \pi_t \end{bmatrix} + B \begin{bmatrix} g_t \\ z_t \\ \varepsilon_t^{real} \\ \varepsilon_t^i \end{bmatrix} \tag{7}$$

where A and B are matrices that are returned by the solution algorithm for linear rational expectations models. If we take time $t-1$ conditional expectations of this system, the resulting first two equations of that system give $E_{t-1}x_t$ and $E_{t-1}\pi_t$ as a linear function of x_{t-1}, π_{t-1}, x_{t-1}^{real}, π_{t-1}^{real}, i_{t-1}, and $E_{t-1}x_t$ and $E_{t-1}\pi_t$. Those two equations can thus be solved for $E_{t-1}x_t$ and $E_{t-1}\pi_t$ as a function of x_{t-1}, π_{t-1}, x_{t-1}^{real}, π_{t-1}^{real} and i_{t-1}. Doing this and replacing the expectation terms in the system above gives the reduced form dynamics in terms of the variables that we use in our VAR:

$$\begin{bmatrix} y_t \\ \pi_t \\ y_t^{real} \\ \pi_t^{real} \\ i_t \end{bmatrix} = A_2 \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ y_{t-1}^{real} \\ \pi_{t-1}^{real} \\ i_{t-1} \end{bmatrix} + B_2 \begin{bmatrix} g_t \\ z_t \\ \varepsilon_t^{real} \\ \varepsilon_t^i \end{bmatrix} \tag{8}$$

where A_2 and B_2 are the matrices corresponding to A and B in the original solution after the lagged expectations terms have been substituted out.

In this model, the forecast error in the nominal interest rate equation is the monetary policy shock (all other right-hand side variables in the monetary policy rule are predetermined), even though some of the right hand-side variables in the reduced form nominal interest rate equation are not in the central bank's information set. If we took this model literally, we could be tempted to use this insight to directly estimate the monetary policy error. Instead, we see this DSGE model as one possible model that delivers a reduced form in line with our empirical specification. Thus, we choose to use sign restrictions to identify the monetary policy shock instead.

2 Robustness Checks

2.1 Results for the Model With Employment Growth

Turning to a model with employment growth rather than GDP growth, we use annualized quarterly real-time and final employment growth (based on nonfarm payroll employment).[1] We find that the forecasted measurement error in inflation is very similar across the two specifications, as shown in figure 1. The error bands for the forecast error in employment growth do not contain 0 for more periods than in the GDP growth case. The case for non-zero measurement errors is thus at least as strong for the employment growth case as for the GDP growth case. Turning to second moments in figure 2, we see the same pattern for the volatility of the inflation measurement error as in the case of the model with GDP growth. The evolution of volatility of the measurement error in employment growth is broadly similar to that of GDP growth. The correlation structure, on the other hand, is quite different from the case of the GDP growth VAR. There is no substantial trend in the correlation; the correlation is smaller in magnitude and is very close to zero for substantial periods of time. While the assumption of uncorrelated measurement errors is less of a problem when using employment and inflation data, our results for this case still show that the dynamics of measurement errors call for a more complex model than what is often assumed. We find that stochastic volatility and a bias in the measurement errors are present.

We next turn to studying the effects of monetary policy shocks on employment

[1] The employment series is originally monthly. We define employment within a quarter as the average employment over the three months belonging to that quarter. The real-time (or first available) estimate is defined as the estimate available in the middle month of the following quarter (in line with the definition of the other quarterly variables). We also estimated versions of our model using the real-time and final unemployment rate, but there are substantially fewer revisions in the unemployment rate by the very nature in which the data for the unemployment rate is collected - it is a survey-based measure. More details on the difference between real-time and final data for a broader set of variables can be found in Aruoba (2008).

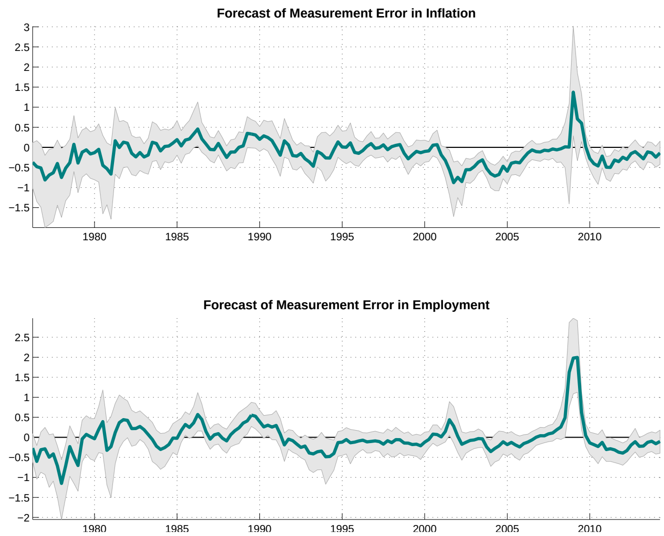

Figure 1: Forecasted measurement errors for employment growth model

growth data. For the model with employment growth, we impose that, in addition to the restrictions on inflation and the nominal interest rate, final employment growth can not increase after a positive monetary policy shock. The responses of the nominal interest rate as well as real-time and final inflation are very similar to the benchmark model, so we omit them here.

For the impulse response of employment growth to a monetary policy shock, a qualitatively similar picture to real GDP growth emerges in figure 3. The largest differences appear on impact. Those differences are economically significant and the final data responses are larger in absolute value than those of the real-time data. Plotting the differences between contemporaneous responses of real-time and final employment growth over time, a similar picture to real GDP growth emerges. Figure 4 shows that the median difference is negative throughout and the 85th percentile band is hovering around 0, which implies that there is a substantial probability at any point in time that the difference in responses is negative and that the values of that difference are economically meaningful.

Finally, we return to using regressions on draws for the real-time and final data contemporaneous responses. Since the responses to inflation turned out to be similar

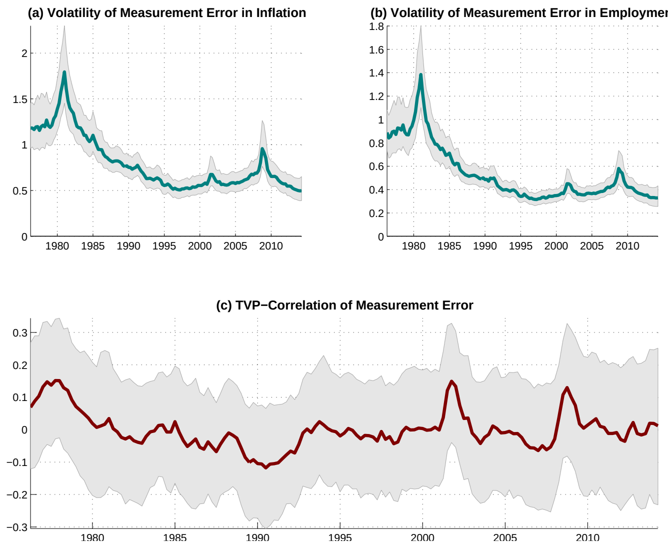

Figure 2: Volatility and correlation of forecasted measurement errors for employment growth model

to those obtained using the VAR with GDP growth (with even smaller differences between real-time and final data responses), we focus on the responses of employment growth.

Figure 5 shows that the movements in the slope and intercept coefficients are smaller than for GDP growth. The slope coefficient is not too far from 1, but the intercept is always larger than 0, so there is still a bias in the relationship of real-time and final data responses on impact. There is a break in the 1980s toward less bias in the relationship between the responses, but the break happens later than in the case of the GDP growth VAR (around 1985).

Most importantly, the responses of final employment growth are larger in magnitude than those of the real-time data, just as we found in the model with GDP growth.

2.2 A Recursive Identification Scheme

The results so far all use sign restrictions to identify monetary policy shocks. To check whether or not our results are robust to other identification schemes (especially identification schemes that impose the same restriction on real-time and final

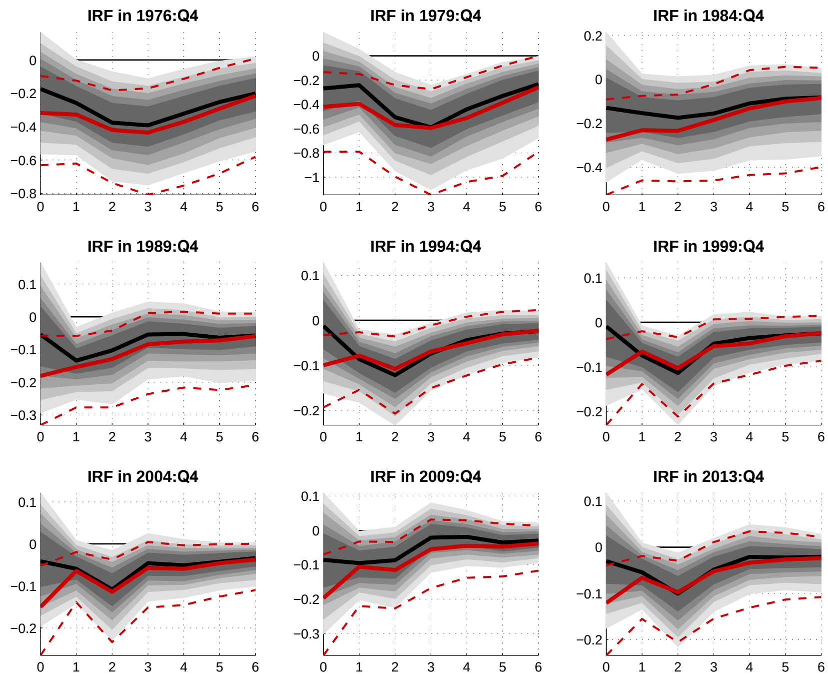

Figure 3: Impulse response functions for real time (gray/black) and final (red) employment growth to a one standard deviation monetary policy shock.

data), the same exercise is carried out using a recursive identification scheme with the nominal interest rate ordered last. Conditional on reduced-form parameter estimates, the exact ordering of the other variables does not matter for the impulse response to a monetary policy shock (Christiano, Eichenbaum & Evans (1999)).[2]. Thus, this recursive identification scheme imposes the same restrictions on the responses of real-time and final data - they are ordered before the monetary policy variable. For the sake of brevity, we focus on the response to real GDP growth. It should be noted, though, that in the case of the recursive identification scheme the response of inflation displays a price puzzle, which is one reason why we prefer the sign restriction approach. Figure 6 displays the responses of real GDP growth. We still see

[2]The ordering of variables can in theory matter for the estimation of the reduced-form parameters in the class of models we use - see Primiceri (2005) for a discussion.

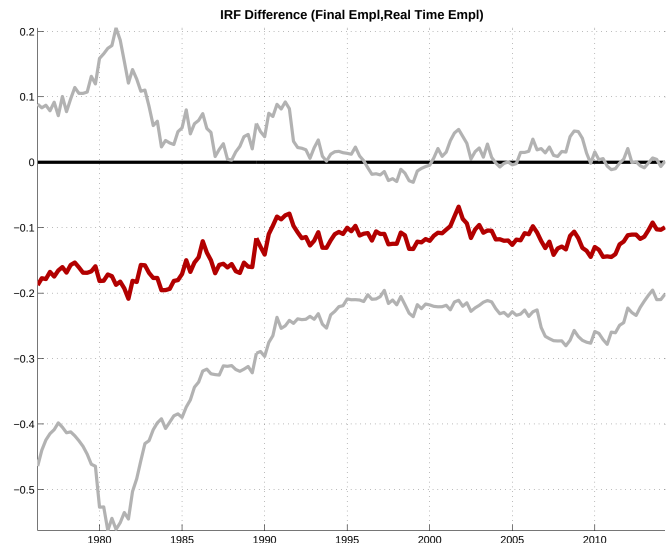

Figure 4: Differences between employment growth impulse responses on impact.

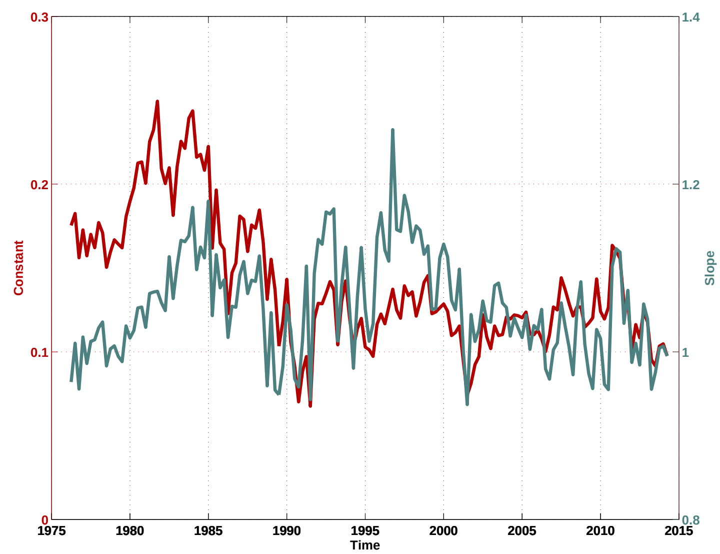

Figure 5: Relationship between real-time and final data employment growth based impact impulse responses over time. Intercept α_t^{emp} in red and slope β_t^{emp} in gray.

substantial differences between real-time and final data responses (with final data responses showing some erratic behavior in 1985 and 2000). While we do not put a lot of faith in a literal interpretation of the results of this recursive identification scheme, it is nonetheless important to out that just as in the case with sign restrictions, the differences between real-time and final data responses are still largest on impact (because of the recursive identification scheme, a monetary policy shock only impacts the other variables at horizon 1) and the response of final data is smaller on impact (except for the first two time periods shown, where the impact responses are similar). Thus, the difference on impact in the sign restriction case thus does not seem to be an artifact of imposing restrictions on only final data responses nor an artifact of using sign restrictions per se.[3]

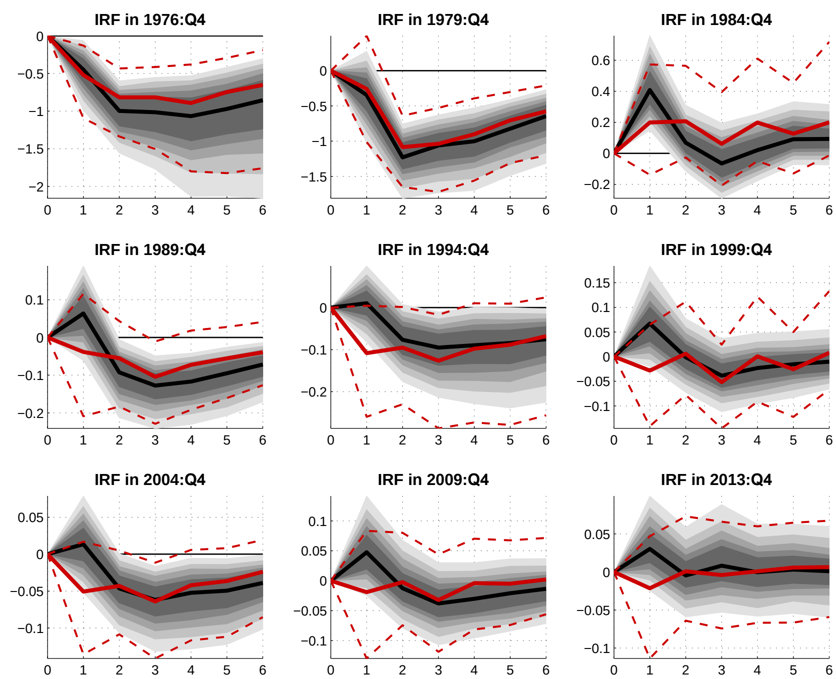

Figure 6: Impulse response functions for real-time (gray/black) and final (red) real GDP growth to a one standard deviation monetary policy shock, using the recursive identification scheme.

[3]Below we show that our results hold (and even become stronger) if we impose sign restrictions on *both* real-time and final data.

2.3 An Alternative Definition of Final Data

Aruoba (2008) argued for a definition of final data that uses data published after a fixed lag (roughly 3 years for most of his variables). We now follow this procedure (with a lag of 3 years) and replicate our benchmark analysis. For the sake of brevity, we focus on the response to monetary policy shocks (the reduced form evidence on the dynamics of measurement errors is in line with the benchmark case).
The responses of both the nominal interest rate as well as both the real-time and final inflation rates are also similar to the benchmark case and are thus omitted here.

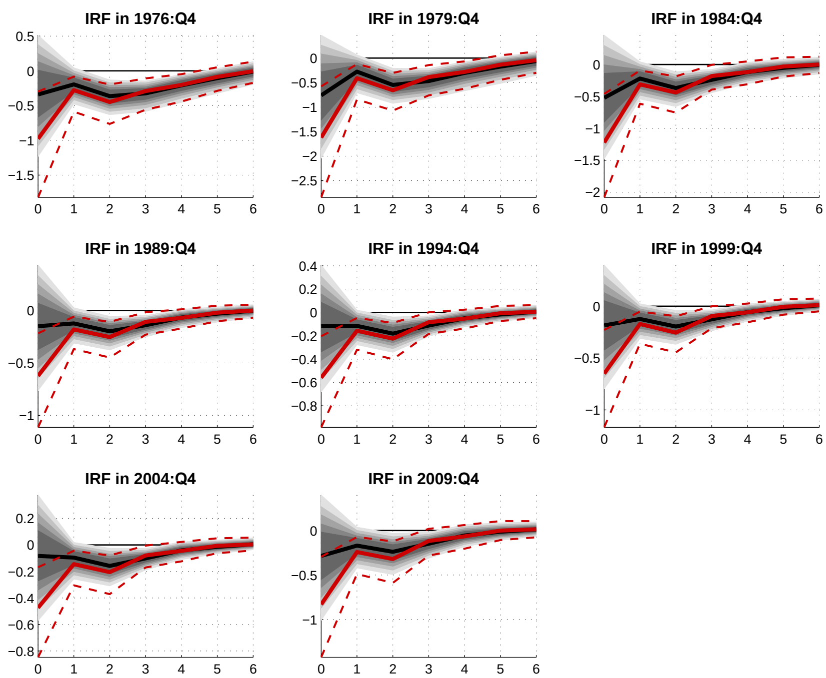

Figure 7: Impulse response functions for the real GDP growth rate to a one standard deviation monetary policy shock, using the alternative definition of final data.

Importantly, the response of real GDP growth (figure 7) shows the same bias as in the benchmark case: On impact, the response of final data is substantially larger in absolute magnitude. The differences in this case are more short-lived, but also more pronounced on impact relative to the benchmark.

2.4 Sign Restrictions on *Both* Real-Time and Final Data

We now impose sign restrictions on both real-time and final data. The sign restrictions (including the horizon) are the same as in our benchmark application. Figures 8 to 10 show that imposing these additional restrictions does not alter our findings of a stronger final-data response of GDP growth (if anything, the differences become more pronounced). To be able to fully appreciate the differences between real-time and final data responses in this case, figure 11 plots the cumulated response of real-time and final real GDP growth (i.e. the responses of the logs of final and real-time real GDP).

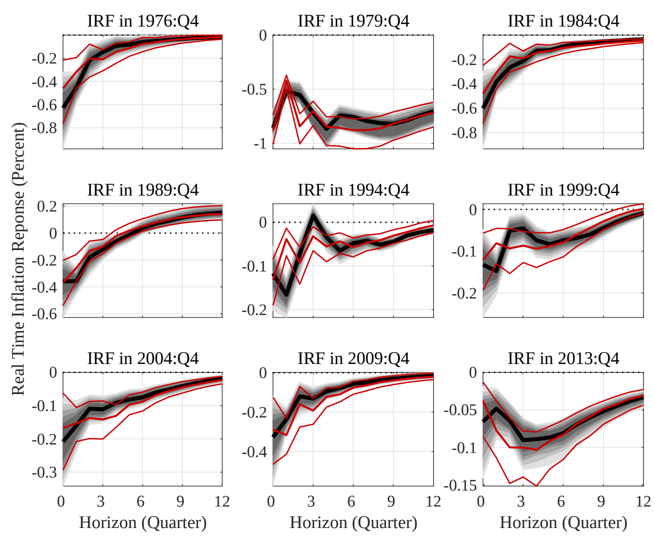

Figure 8: Impulse response functions for the inflation rate to a one standard deviation monetary policy shock, imposing sign restrictions on both real-time and final data. Final data responses are in red.

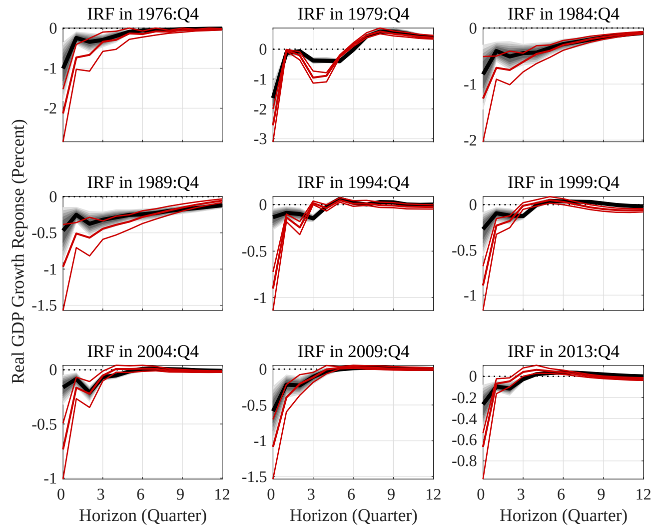

Figure 9: Impulse response functions for the real GDP growth rate to a one standard deviation monetary policy shock, imposing sign restrictions on both real-time and final data. Final data responses are in red.

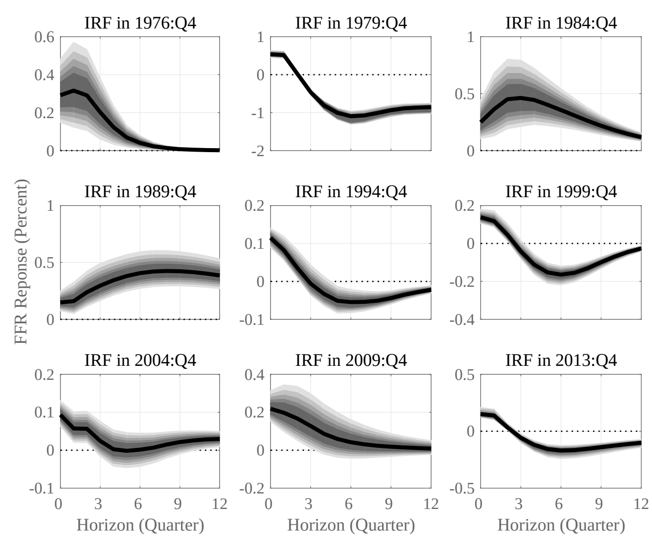

Figure 10: Impulse response functions for the nominal interest rate to a one standard deviation monetary policy shock, imposing sign restrictions on both real-time and final data.

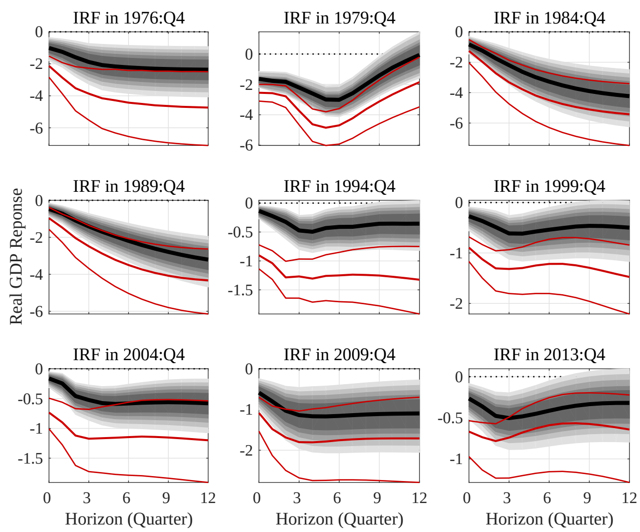

Figure 11: Impulse response functions for the log of real GDP to a one standard deviation monetary policy shock, imposing sign restrictions on both real-time and final data.

2.5 An Alternative Measure of the Monetary Policy Stance

In our benchmark application, we use the Federal Funds rate as our measure of the monetary policy instrument. Since our sample encompasses at the tail end part of the zero lower bound (ZLB) episode in the US, we now replace the Federal Funds rate with the Wu-Xia shadow rate (Wu & Xia (2016)). This shadow rate is basically identical to the Federal Funds rate before the zero-lower bound episode and this is reflected in our results - impulse responses in non-ZLB periods are very similar across the two specifications. We thus focus on two periods in which the ZLB binds and that we have used in other plots. The responses of inflation are, just as in the benchmark, similar for real-time and final data, so we focus on the response of real GDP growth. We can see that the responses do differ across the two specifications for those periods. Figure 13 plots the response of the log of real GDP across the two specifications. We can see that in 2009 there is still a meaningful difference between the response of real-time and final data, whereas this is not the case for 2013. It is interesting to note that the response of the Federal Funds rate itself is also markedly different from the response of the shadow rate during the ZLB period, in particular in 2013, as can be seen in figure 14.

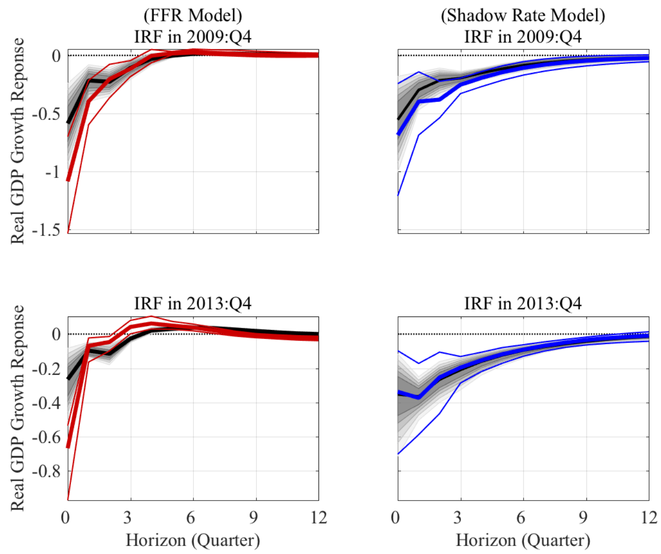

Figure 12: Impulse response functions for the real GDP growth rate to a one standard deviation monetary policy shock, using either the Federal Funds rate or the Wu-Xia shadow rate. Final data responses are in red/blue.

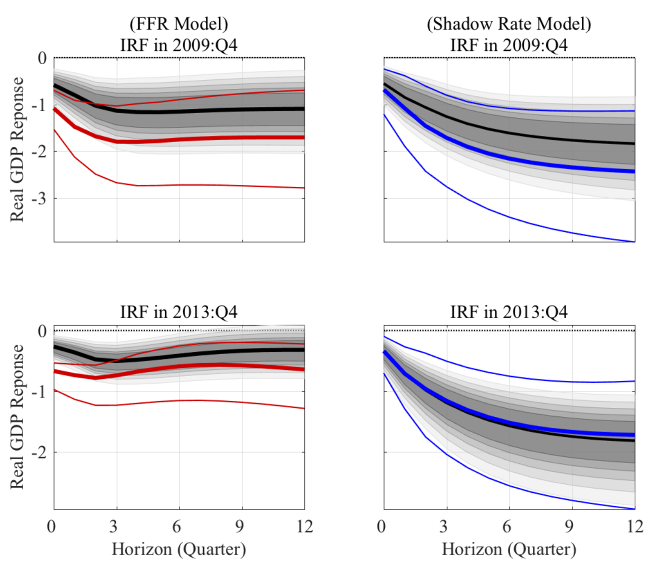

Figure 13: Impulse response functions for the log of real GDP to a one standard deviation monetary policy shock, using either the Federal Funds rate or the Wu-Xia shadow rate. Final data responses are in red/blue.

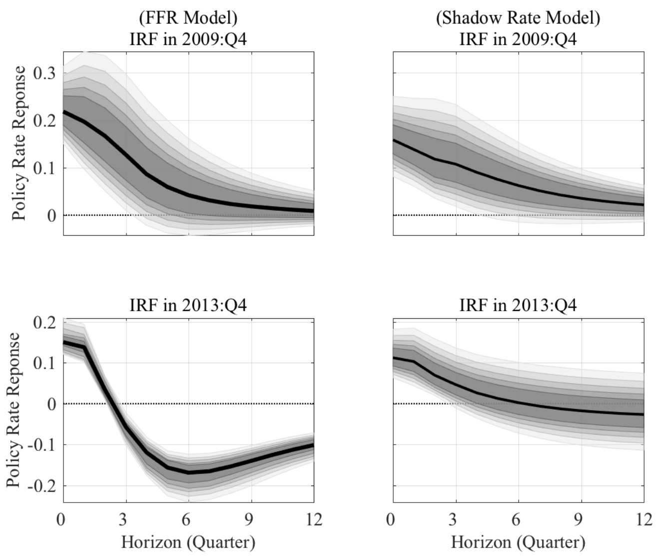

Figure 14: Impulse response functions for the nominal interest rate to a one standard deviation monetary policy shock, using either the Federal Funds rate or the Wu-Xia shadow rate.

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