Learning about Fiscal Policy and the Effects of Policy Uncertainty - Online Supplementary Material

Josef Hollmayr*and Christian Matthes**

First-Order Conditions

Households:

$$
\begin{aligned}
\frac{C_t^{-\sigma}}{1+\tau_t^C} &= E_t \frac{\beta R_t C_{t+1}^{-\sigma}}{1+\tau_{t+1}^C} \\
L_t^{1+\phi}(1+\tau_t^C) &= C_t^{-\sigma}(1-\tau_t^L)(1-\alpha)Y_t \\
1 &= \beta E_t \frac{C_{t+1}^{-\sigma}(1+\tau_t^C)}{C_t^{-\sigma}(1+\tau_{t+1}^C)} \left((1-\tau_{t+1}^K)\frac{\alpha Y_{t+1}}{K_t} + (1-\delta) \right)
\end{aligned}
$$

Firms:

$$
\begin{aligned}
W_t &= \frac{(1-\alpha)Y_t}{L_t} \\
R_t^K &= \frac{\alpha Y_t}{K_{t-1}}
\end{aligned}
$$

*Deutsche Bundesbank, Frankfurt am Main (e-mail: josef.hollmayr@bundesbank.de).

**Federal Reserve Bank of Richmond (email: christian.matthes@rich.frb.org). The views expressed in this paper are those of the authors and do not necessarily reflect those of the Bundesbank, the Federal Reserve Bank of Richmond, or the Federal Reserve System.

Log-Linearized Model

Households:

$$
\begin{aligned}
(1+\phi)log(L_t) &+ \left(\frac{\tau_c^C}{1+\tau_c^C}\right)log(\tau_t^C) = Const^L + log(Y_t) - \left(\frac{\tau_c^L}{1-\tau_c^L}\right)log(\tau_t^L) - \sigma log(C_t) \\
log(C_t) &= Const^C - \frac{1}{\sigma}\left(\frac{\tau_c^C}{1+\tau_c^C}\right)log(\tau_t^C) + \frac{1}{\sigma}\left(\frac{\tau_c^C}{1+\tau_c^C}\right)log(\tau_{t+1}^C) + log(C_{t+1}) - \frac{1}{\sigma}log(R_t) \\
log(K_t) &= Const^{LoM} + (1-\delta)log(K_{t-1}) + \delta log(I_t) \\
log(Y_t) &= Const^Y + log(A_t) + \alpha log(K_{t-1}) + (1-\alpha)log(L_t) \\
\sigma E_t log(C_{t+1}) &= Const^K + \sigma log(C_t) - \frac{\tau_c^C}{(1+\tau_c^C)}E_t log(\tau_{t+1}^C) + \frac{\tau_c^C}{(1+\tau_c^C)}log(\tau_t^C) \\
&+ \beta(1-\tau_c^K)\alpha\frac{Y_{ss}}{K_{ss}}E_t log(Y_{t+1}) - \beta(1-\tau_c^K)\alpha\frac{Y_{ss}}{K_{ss}}log(K_t) - \beta\tau_c^K\alpha\frac{Y_{ss}}{K_{ss}}E_t log(\tau_{t+1}^K)
\end{aligned}
$$

Firms:

$$
\begin{aligned}
log(Y_t) &= Const^{Agg} + \frac{C_{ss}}{Y_{ss}}log(C_t) + \frac{I_{ss}}{Y_{ss}}log(I_t) + \frac{G_{ss}}{Y_{ss}}log(G_t) \\
log(A_t) &= Const^A + \rho_a log(A_{t-1}) + \epsilon_{t^A}
\end{aligned}
$$

Policy Rules:

$$
\begin{aligned}
log(B_t) &+ \tau_c^K\alpha\frac{Y_{ss}}{B_{ss}}(log(\tau_t^K) + log(Y_t)) + \tau_c^L(1-\alpha)\frac{Y_{ss}}{B_{ss}}(log(\tau_t^L) + log(Y_t)) + \tau_c^C\frac{C_{ss}}{B_{ss}}(log(\tau_t^C) + log(C_t)) \\
&= Const^B + \frac{1}{\beta}log(R_{t-1}) + \frac{1}{\beta}log(B_{t-1}) + \frac{G_{ss}}{B_{ss}}log(G_t) + \frac{Z_{ss}}{B_{ss}}log(Z_t) \\
log(G_t) &= G_c - \rho_{g,y}log(Y_{t-1}) - \rho_{g,b}log(B_{t-1}) + \epsilon_t^G \\
log(Z_t) &= Z_c - \rho_{z,y}log(Y_{t-1}) - \rho_{z,b}log(B_{t-1}) + \epsilon_t^Z \\
log(\tau_t^C) &= \tau_c^c + \epsilon_t^C \\
log(\tau_t^L) &= \tau_c^l + \rho_{L,y}log(Y_{t-1}) + \rho_{L,b}log(B_{t-1}) + \epsilon_t^L \\
log(\tau_t^K) &= \tau_c^l + \rho_{K,y}log(Y_{t-1}) + \rho_{K,b}log(B_{t-1}) + \epsilon_t^K
\end{aligned}
$$

with the constants given by:

Constant	Expression
G_c	$log(G_c) + \rho_{g,y} log(Y_{ss}) + \rho_{g,b} log(B_{ss})$
Z_c	$log(Z_c) + \rho_{z,y} log(Y_{ss}) + \rho_{z,b} log(B_{ss})$
τ_c^l	$log(\tau_c^L) - \rho_{L,y} log(Y_{ss}) - \rho_{L,b} log(B_{ss})$
τ_c^k	$log(\tau_c^K) - \rho_{K,y} log(Y_{ss}) - \rho_{K,b} log(B_{ss})$
τ_c^c	$log(\tau_c^C)$
$Const^B$	$log(B_{ss})(1 - \frac{1}{\beta}) + \tau_c^K \alpha \frac{Y_{ss}}{B_{ss}} (log(\tau_c^K) + log(Y_{ss})) + \tau_c^L (1-\alpha) \frac{Y_{ss}}{B_{ss}} (log(Y_{ss}) + log(\tau_c^l))$
	$+\tau_c^C \frac{C_{ss}}{B_{ss}} (log(\tau_c^C) + log(C_{ss})) - \frac{1}{\beta} log(R_{ss}) - \frac{G_{ss}}{B_{ss}} log(G_{ss}) - \frac{Z_{ss}}{B_{ss}} log(Z_{ss})$
$Const^{LoM}$	$\delta(log(K_{ss}) - log(I_{ss}))$
$Const^L$	$(1+\phi) log(L_{ss}) + \frac{\tau_c^C}{1+\tau_c^C} log(\tau_c^C) - log(Y_{ss}) + \frac{\tau_c^L}{1+\tau_c^L} log(\tau_c^L)$
$Const^C$	$\frac{1}{\sigma} log(R_{ss})$
$Const^Y$	$log(Y_{ss}) - log(A_{ss}) - \alpha log(K_{ss}) - (1-\alpha) log(L_{ss})$
$Const^A$	$log(A_{ss})$
$Const^{Agg}$	$log(Y_{ss}) - \frac{C_{ss}}{Y_{ss}} log(C_{ss}) - \frac{G_{ss}}{Y_{ss}} log(G_{ss}) - \frac{I_{ss}}{Y_{ss}} log(I_{ss})$
$Const^K$	$-\beta(1-\tau_c^K)\alpha \frac{Y_{ss}}{K_{ss}} log(Y_{ss}) + \beta(1-\tau_c^K)\alpha \frac{Y_{ss}}{K_{ss}} log(K_{ss}) + \beta(\tau_c^K)\alpha \frac{Y_{ss}}{K_{ss}} log(\tau_c^K)$

Parameters

Calibrated Parameters

Description	Parameter	Value
Impatience	β	0.99
Capital share	α	0.33
Depreciation rate	δ	0.025
CES utility consumption	σ	2
CES utility labor	ϕ	2
Coeff. on Y in gov. exp. rule	$\rho_{g,y}$	0.034
Coeff. on B in gov. exp. rule	$\rho_{g,b}$	0.23
Coeff. on Y in transfer rule	$\rho_{z,y}$	0.13
Coeff. on B in transfer rule	$\rho_{z,b}$	0.5
Coeff. on Y labor tax rule	$\rho_{L,y}$	0.36
Coeff. on B labor tax rule	$\rho_{L,b}$	0.049
Coeff. on Y capital tax rule	$\rho_{K,y}$	1.7
Coeff. on B capital tax rule	$\rho_{K,b}$	0.39
AR parameter technology	ρ_a	0.9
Std. deviation technology	σ_a	0.0062
Std. deviation gov. spending	σ_g	0.031
Std. deviation transfers	σ_z	0.034
Std. deviation cons.tax	σ_c	0.04
Std. deviation labor tax	σ_l	0.03
Std. deviation capital tax	σ_k	0.044

Table 1: Calibrated parameters of the model

Initial Steady State Values of the Actual Law of Motion

Description	Parameter	Value
Output	Y_{ss}	2.0601
Consumption	C_{ss}	1.5010
Cons. tax rate	τ_c^C	0.0287
Capital tax rate	τ_c^K	0.2452
Labor	L_{ss}	0.7847
Investment	I_{ss}	0.3655
Capital	K_{ss}	14.6195
Debt	B_{ss}	0.5623
Labor tax rate	τ_c^L	0.1886
Government spending	G_c	0.1936
Transfers	Z_c	0.2709
Technology	A_{ss}	1
Interest rate	R_{ss}	1.01

Table 2: Calibrated parameters of the model

Perceived Steady States

The perceived steady states in the updating algorithm of the agents are given by the following twelve equations:

$$\begin{aligned}
R &= \frac{1}{\beta} \\
\frac{\alpha Y}{K} &= \frac{\frac{1}{\beta} - (1-\delta)}{1-\tau^K} \\
L^{1+\phi}(1+\tau^C) &= C^{-\sigma}(1-\tau^L)(1-\alpha)Y \\
Y &= AK^{\alpha}L^{1-\alpha} \\
Y &= C + I + G \\
I &= \delta K \\
B &= B\frac{1}{\beta} - \tau^K \alpha Y - \tau^L(1-\alpha)Y - \tau^C C + G + Z \\
G_c &= log(G) + \rho_{g,y} log(Y) + \rho_{g,b} log(B) \\
Z_c &= log(Z) + \rho_{z,y} log(Y) + \rho_{z,b} log(B) \\
\tau_c^L &= log(\tau_c^L) - \rho_{L,y} log(Y) - \rho_{L,b} log(B) \\
\tau_c^K &= log(\tau_c^K) - \rho_{K,y} log(Y) - \rho_{K,b} log(B) \\
\tau_c^C &= log(\tau_c^C)
\end{aligned}$$

for the twelve variables: $Y, K, L, C, G, Z, \tau^L, \tau^K, \tau^C, B, I, R$, which are solved numerically.

Robustness Check: Preferences

Do our results hold when agents have different preferences? To address this issue with a particular focus on the behavior of labor supply, we redo our benchmark analysis for two classes of preferences that imply very different wealth effects on labor supply: the preferences of Greenwood *et al.* (1988) and those of King *et al.* (1988). Figures 1 and 2 show the results for these two cases. While the dynamics differ from our benchmark case for both preferences, the big picture remains the same: We see substantial differences in average outcomes and increases in volatility relative to rational expectations.

Robustness Check: Capital Tax Change

After a negative shock hits the economy, government spending is not the only instrument the fiscal sector can change to boost the economy. In figure 3 we study a capital tax decrease equivalent to 1 percent of GDP. This is calculated along the lines of Leeper *et al.* (2010) and our own calculations for the government spending case, so that the decrease of total capital tax revenues approximately equals one percent of overall pre-policy-change steady state GDP. Qualitatively the results are the same as under the scenario of an increase of government spending. Cumulated GDP is lower by about 5 percent after the end of our simulation horizon while cumulated debt is around 15 percent higher in the case of learning compared to the rational expectations outcome. Investment and therefore also capital are decreasing constantly throughout. Volatility increases are quite small for all variables.

Equations for Different Utility Function Specifications

A: First-order conditions of households: As robustness checks we consider the following utility function (compare Jaimovich and Rebelo (2009)):

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - \psi N_t^{\theta} X_t\right)^{1-\sigma} - 1}{1-\sigma} \tag{1}$$

with $X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}$ which nests both the King *et al.* (1988) Preferences ($\gamma = 1$) and the Greenwood *et al.* (1988) preferences ($\gamma = 0$).

$$\begin{aligned}
\left(C_t - \psi N_t^{\theta} X_t\right)^{-\sigma} &+ \mu_t \gamma C_t^{\gamma-1} X_t^{1-\gamma} = \lambda_t (1+\tau_t^c) \\
\left(C_t - \psi N_t^{\theta} X_t\right)^{-\sigma} \psi N_t^{\theta} &+ \mu_t = \beta E_t \left[\mu_{t+1}(1-\gamma) C_{t+1}^{\gamma} X_t^{-\gamma}\right] \\
\left(C_t - \psi N_t^{\theta} X_t\right)^{-\sigma} \psi \theta N_t^{\theta-1} X_t &= \lambda_t (1-\tau_t^l) W_t \\
1 &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left((1-\tau_{t+1}^K) R_{t+1}^K + (1-\delta)\right)
\end{aligned}$$

B: First-order conditions in the GHH case:

$$\begin{aligned}
\frac{\left(C_t - \psi N_t^{\theta}\right)^{-\sigma}}{(1+\tau_t^C)} &+ \beta E_t \frac{R_t \left(C_{t+1} - \psi N_{t+1}^{\theta}\right)^{-\sigma}}{(1+\tau_{t+1}^c)} \\
\psi \theta N_t^{\theta} (1+\tau_t^C) &= (1-\tau_t^L)(1-\alpha) Y_t \\
1 &= \beta E_t \frac{\left(C_{t+1} - \psi N_{t+1}^{\theta}\right)^{-\sigma} (1+\tau_t^C)}{\left(C_t - \psi N_t^{\theta}\right)^{-\sigma} (1+\tau_{t+1}^C)} \left((1-\tau_{t+1}^K) \alpha \frac{Y_{t+1}}{K_t} + (1-\delta)\right)
\end{aligned}$$

C: Log-linearized conditions in the GHH case

$$
\begin{aligned}
\theta log(L_t) &+ \left(\frac{\tau_c^C}{1+\tau_c^C}\right) log(\tau_t^C) = Const^L + log(Y_t) - \left(\frac{\tau_c^L}{1-\tau_c^L}\right) log(\tau_t^L) \\
-\frac{\sigma C_{ss}}{C_{ss}-\psi N_{ss}^\theta} log(C_t) &+ \frac{\sigma\psi\theta N_{ss}^\theta}{C_{ss}-\psi N_{ss}^\theta} log(N_t) - \left(\frac{\tau_c^C}{1+\tau_c^C}\right) log(\tau_t^C) = Const^C + R_t - \left(\frac{\tau_c^C}{1+\tau_c^C}\right) log(\tau_{t+1}^C) \\
&- \frac{\sigma C_{ss}}{C_{ss}-\psi N_{ss}^\theta} log(C_{t+1}) + \frac{\sigma\psi\theta N_{ss}^\theta}{C_{ss}-\psi N_{ss}^\theta} log(N_{t+1}) \\
\frac{\sigma C_{ss}}{C_{ss}-\psi N_{ss}^\theta} E_t log(C_{t+1}) &- \frac{\sigma\psi\theta N_{ss}^\theta}{C_{ss}-\psi N_{ss}^\theta} log(N_{t+1}) - \frac{\tau_c^C}{(1+\tau_c^C)} log(\tau_t^C) - \frac{\sigma C_{ss}}{C_{ss}-\psi N_{ss}^\theta} log(C_t) \\
&+ \frac{\sigma\psi\theta N_{ss}^\theta}{C_{ss}-\psi N_{ss}^\theta} log(N_t) + \frac{\tau_c^C}{(1+\tau_c^C)} E_t log(\tau_{t+1}^C) = Const^K \\
&+ \beta(1-\tau_c^K)\alpha\frac{Y_{ss}}{K_{ss}} E_t log(Y_{t+1}) - \beta(1-\tau_c^K)\alpha\frac{Y_{ss}}{K_{ss}} log(K_t) \\
&- \beta\tau_c^K\alpha\frac{Y_{ss}}{K_{ss}} E_t log(\tau_{t+1}^K)
\end{aligned}
$$

D: First-order conditions in the KPR case:

$$
\begin{aligned}
\frac{\left(C_t-\psi N_t^\theta C_t\right)^{-\sigma}\left(1-\psi N_t^\theta\right)}{(1+\tau_t^C)} &= \beta E_t \frac{R_t\left(C_{t+1}-\psi N_{t+1}^\theta C_{t+1}\right)^{-\sigma}\left(1-\psi N_{t+1}^\theta\right)}{(1+\tau_{t+1}^c)} \\
\psi\theta C_t N_t^\theta(1+\tau_t^C) &= (1-\psi N_t^\theta)(1-\tau_t^L)(1-\alpha)Y_t \\
1 &= \beta E_t \frac{\left(C_{t+1}-\psi N_{t+1}^\theta C_{t+1}\right)^{-\sigma}(1-\psi N_{t+1}^\theta)(1+\tau_t^C)}{\left(C_t-\psi N_t^\theta C_t\right)^{-\sigma}(1-\psi N_t^\theta)(1+\tau_{t+1}^C)} \\
&\quad \left((1-\tau_{t+1}^K)\alpha\frac{Y_{t+1}}{K_t} + (1-\delta)\right)
\end{aligned}
$$

E: Log-linearized conditions in the KPR case

$$
\begin{aligned}
\theta log(N_t) &+ \left(\frac{\tau_c^C}{1+\tau_c^C}\right) log(\tau_t^C) + log(C_t) = Const^L + log(Y_t) - \left(\frac{\tau_c^L}{1-\tau_c^L}\right) log(\tau_t^L) \\
&- \frac{\psi\theta N_{ss}^{\theta}}{1-\psi N_{ss}^{\theta}} log(N_t) \\
-\sigma log(C_t) &- \frac{(1-\sigma)\psi\theta N_{ss}^{\theta}}{1-\psi N_{ss}^{\theta}} log(N_t) - \left(\frac{\tau_c^C}{1+\tau_c^C}\right) log(\tau_t^C) = Const^C + R_t - \left(\frac{\tau_c^C}{1+\tau_c^C}\right) log(\tau_{t+1}^C) \\
&- \sigma log(C_{t+1}) - \frac{(1-\sigma)\psi\theta N_{ss}^{\theta}}{1-\psi N_{ss}^{\theta}} log(N_{t+1}) \\
\sigma E_t log(C_{t+1}) &+ \frac{(1-\sigma)\psi\theta N_{ss}^{\theta}}{1-\psi N_{ss}^{\theta}} log(N_{t+1}) - \frac{\tau_c^C}{(1+\tau_c^C)} log(\tau_t^C) - \sigma log(C_t) - \frac{(1-\sigma)\psi\theta N_{ss}^{\theta}}{1-\psi N_{ss}^{\theta}} log(N_t) \\
&+ \frac{\tau_c^C}{(1+\tau_c^C)} E_t log(\tau_{t+1}^C) = Const^K + \beta(1-\tau_c^K)\alpha\frac{Y_{ss}}{K_{ss}} E_t log(Y_{t+1}) - \beta(1-\tau_c^K)\alpha\frac{Y_{ss}}{K_{ss}} log(K_t) \\
&- \beta\tau_c^K\alpha\frac{Y_{ss}}{K_{ss}} E_t log(\tau_{t+1}^K)
\end{aligned}
$$

Simulation

The simulation of our learning economy is carried out via the following steps:

1. We endow agents with initial beliefs Ω_0, which coincide with the true pre-policy-change parameter values.

2. Given the beliefs $\Omega_{t|t-1}$, the perceived steady states are calculated and then used to log-linearize the equilibrium conditions, which together with the estimated policy rules gives the following expectational difference equation:
$A(\Omega_{t|t-1})\mathbb{Y}_t = B(\Omega_{t|t-1})E_t^*\mathbb{Y}_{t+1} + C(\Omega_{t|t-1})\mathbb{Y}_{t-1} + D\varepsilon_t^*$
which yields the perceived law of motion (using the RE solution algorithm Gensys by Sims (2001))
$\mathbb{Y}_t = S(\Omega_{t|t-1})\mathbb{Y}_{t-1} + G(\Omega_{t|t-1})\varepsilon_t^*$.

3. The actual law of motion takes the perceived steady states but uses the true policy parameters $C^{true}(\Omega_{t|t-1})$ to arrive at the system:
$A(\Omega_{t|t-1})\mathbb{Y}_t = B(\Omega_{t|t-1})E_t^*\mathbb{Y}_{t+1} + C^{true}(\Omega_{t|t-1})\mathbb{Y}_t + D\varepsilon_t$

with the actual shock vector ε_t. To solve out for the expectations we use the perceived law of motion to obtain
$\mathbb{Y}_t = H(\Omega_{t|t-1})\mathbb{Y}_{t-1} + G(\Omega_{t|t-1})\varepsilon_t$

4. Shocks are realized by drawing from a multivariate Gaussian distribution, which together with the transition matrices produced by step 3 determine the macroeconomic outcomes for period t.

5. Observing these outcomes, beliefs are updated via the Kalman filter, which gives $\Omega_{t+1|t}$.

We simulate the economy for each setting 1000 times with a sample length of $T = 100$.

Learning and the Kalman Filter

In this section we set up the matrices for the agents' estimation problem and describe the Kalman filter they use. t subscripts on parameters denote estimates at time t.

$$\tau_t = \begin{pmatrix} G_t \\ Z_t \\ \tau_t^C \\ \tau_t^L \\ \tau_t^k \end{pmatrix} \tag{2}$$

$$X_{t-1} = \begin{pmatrix} 1 & -\log(Y_{t-1}) & -\log(B_{t-1}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\log(Y_{t-1}) & -\log(B_{t-1}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \log(Y_{t-1}) & \log(B_{t-1}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \log(Y_{t-1}) & \log(B_{t-1}) \end{pmatrix} \tag{3}$$

$$\Omega_t = \begin{pmatrix} G_{c,t} \\ \rho_{g,y,t} \\ \rho_{g,b,t} \\ Z_{c,t} \\ \rho_{z,y,t} \\ \rho_{z,b,t} \\ \tau_{c,t}^c \\ \tau_{c,t}^l \\ \rho_{L,y,t} \\ \rho_{L,b,t} \\ \tau_{c,t}^k \\ \rho_{K,y,t} \\ \rho_{K,b,t} \end{pmatrix} \tag{4}$$

$$\eta_t = \begin{pmatrix} \epsilon_t^G \\ \epsilon_t^Z \\ \epsilon_t^C \\ \epsilon_t^L \\ \epsilon_t^K \end{pmatrix} \tag{5}$$

In the first step the Kalman K_t gain is computed, where S_{t-1} is the covariance matrix of the state (of the previous period) and I_{NxN} denotes an identity matrix of dimension N.

$$K_t = I_{13x13} S_{t-1} X'_{t-1} \left(X_{t-1} S_{t-1} X'_{t-1} + \Sigma_\eta \right)^{-1} \tag{6}$$

The next step involves updating the initial state:

$$\Omega_t = I_{13x13} \Omega_{t-1} + K_t \left(\tau_t - X_{t-1} \Omega_{t-1} \right) \tag{7}$$

In the last step we update the covariance of the state:

$$S_t = I_{13x13} \left(S_{t-1} - S_{t-1} X'_{t-1} \left[X_{t-1} S_{t-1} X'_{t-1} + \Sigma_\eta \right]^{-1} X_{t-1} S_{t-1} \right) I_{13x13} + \mathbf{1}_t \Sigma_\nu \tag{8}$$

The Matrices for the PLM and ALM

In this section t subscripts on parameters denote estimates used in decisions at time t (which are based on data up to and including data from period $t-1$). All variables in $\mathbb{Y}_t$ are in logs (denoted by lowercase letters except for the tax rates) except for the constant. The constant terms in all equations (except for those in the actual policy rules) are functions of the private agents estimates and also vary each period as agents update their beliefs about parameters and thus the steady state of the economy.

$$\mathbb{Y}_t = \tag{9}$$

$$\begin{pmatrix} y_t \\ c_t \\ \log(\tau_t^C) \\ \log(\tau_t^K) \\ l_t \\ i_t \\ k_t \\ b_t \\ \log(\tau_t^L) \\ g_t \\ z_t \\ a_t \\ r_t \\ 1 \end{pmatrix} \tag{10}$$

$$A(\Omega_{t|t-1}) =$$

$$\begin{pmatrix}
1 & 0 & 0 & 0 & -(1-\alpha) & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
-1 & \frac{C_{ss}}{Y_{ss}} & 0 & 0 & 0 & \frac{I_{ss}}{Y_{ss}} & 0 & 0 & 0 & \frac{G_{ss}}{Y_{ss}} & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & \sigma & \frac{\tau_c^C}{1+\tau_c^C} & 0 & 1+\phi & 0 & 0 & 0 & \frac{\tau_c^L}{1-\tau_c^L} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \delta & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\sigma & -\frac{\tau_c^C}{1+\tau_c^C} & 0 & 0 & 0 & \beta(1-\tau_c^K)\alpha\frac{Y_{ss}}{K_{ss}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{Y_{ss}}{B_{ss}}(\tau_c^K\alpha + \tau_c^L(1-\alpha)) & \tau_c^C\frac{C_{ss}}{B_{ss}} & \tau_c^C\frac{C_{ss}}{B_{ss}} & \tau_c^K\alpha\frac{Y_{ss}}{B_{ss}} & 0 & 0 & 0 & 1 & \tau_c^L(1-\alpha)\frac{Y_{ss}}{B_{ss}} & -\frac{G_{ss}}{B_{ss}} & -\frac{Z_{ss}}{B_{ss}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & \frac{1}{\sigma}\frac{\tau_c^C}{1+\tau_c^C} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sigma} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$B(\Omega_{t|t-1}) \quad = \tag{12}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\beta(1-\tau_c^K)\alpha\frac{Y_{ss}}{K_{ss}} & -\sigma & -\frac{\tau_c^C}{1+\tau_c^C} & -\beta\tau_c^K\alpha\frac{Y_{ss}}{K_{ss}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \frac{1}{\sigma}\frac{\tau_c^C}{1+\tau_c^C} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$C(\Omega_{t|t-1}) \quad = \tag{13}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & Const^Y \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Const^{Agg} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau_{c,t}^c \\
\rho_{K,y,t} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{K,b,t} & 0 & 0 & 0 & 0 & 0 & \tau_{t,c}^k \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Const^L \\
0 & 0 & 0 & 0 & 0 & 0 & -(1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 & -Const^{LoM} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\beta} & 0 & 0 & 0 & 0 & \frac{1}{\beta} & Const^K \\
\rho_{L,y,t} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{L,b,t} & 0 & 0 & 0 & 0 & 0 & Const^B \\
-\rho_{g,y,t} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{g,b,t} & 0 & 0 & 0 & 0 & 0 & \tau_{c,t}^l \\
-\rho_{z,y,t} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{z,b,t} & 0 & 0 & 0 & 0 & 0 & G_{c,t} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{c,t} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_a & 0 & Const^A \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Const^C \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$C(\Omega_{t|t-1})^{True} = \tag{14}$$

$$\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & Const^{Y} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -Const^{Agg} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau_c^c \\
\rho_{K,y} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{K,b} & 0 & 0 & 0 & 0 & 0 & \tau_t^k \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Const^{L} \\
0 & 0 & 0 & 0 & 0 & 0 & -(1-\delta) & 0 & 0 & 0 & 0 & 0 & 0 & -Const^{LoM} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\beta} & 0 & 0 & 0 & 0 & \frac{1}{\beta} & Const^{K} \\
\rho_{L,y} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{L,b} & 0 & 0 & 0 & 0 & 0 & Const^{B} \\
-\rho_{g,y} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{g,b} & 0 & 0 & 0 & 0 & 0 & \tau_c^l \\
-\rho_{z,y} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{z,b} & 0 & 0 & 0 & 0 & 0 & G_c \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{c,t} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_a & 0 & Const^{A} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Const^{C} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$D = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\sigma_c & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_k & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_l & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma_g & 0 & 0 \\
0 & 0 & 0 & 0 & \sigma_z & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma_a \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Figures

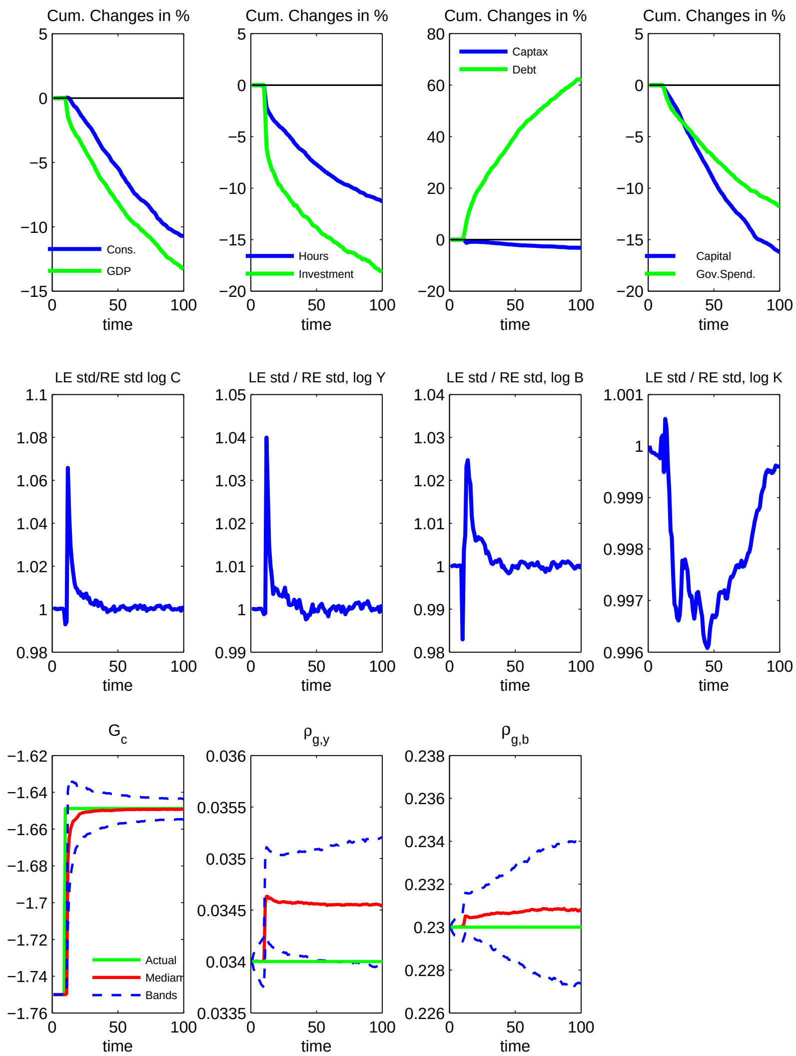

Figure 1: Summary of outcomes under learning when agents have GHH preferences

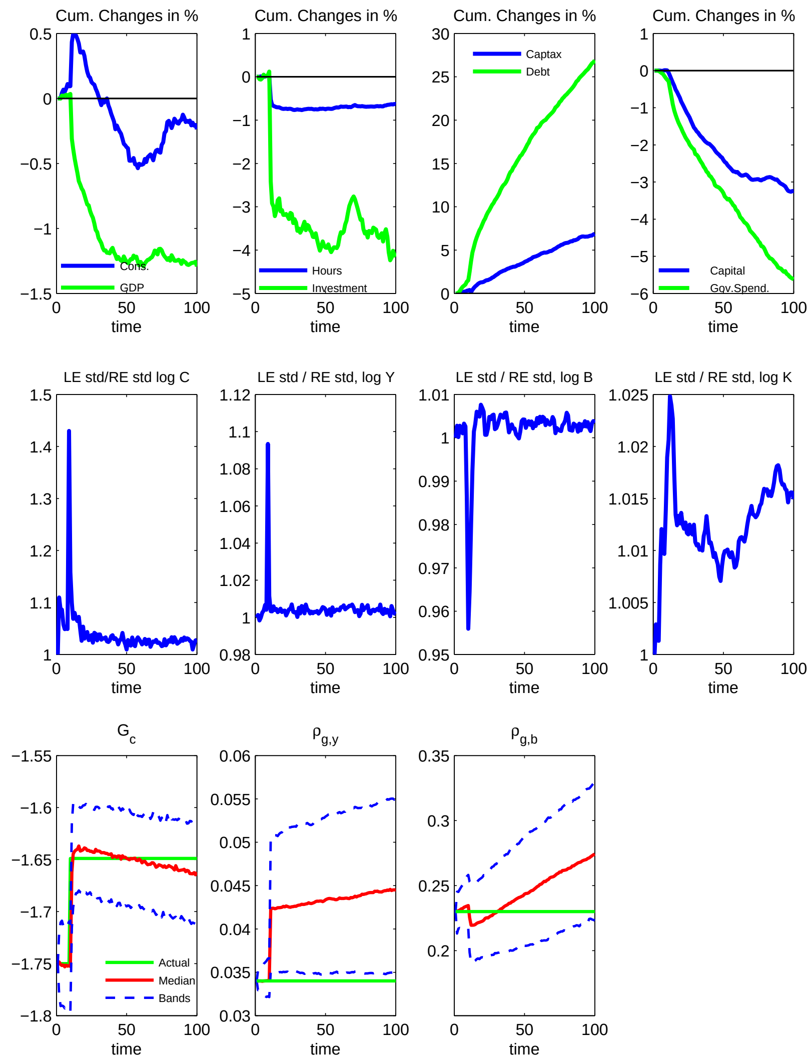

Figure 2: Summary of outcomes under learning when agents have KPR preferences

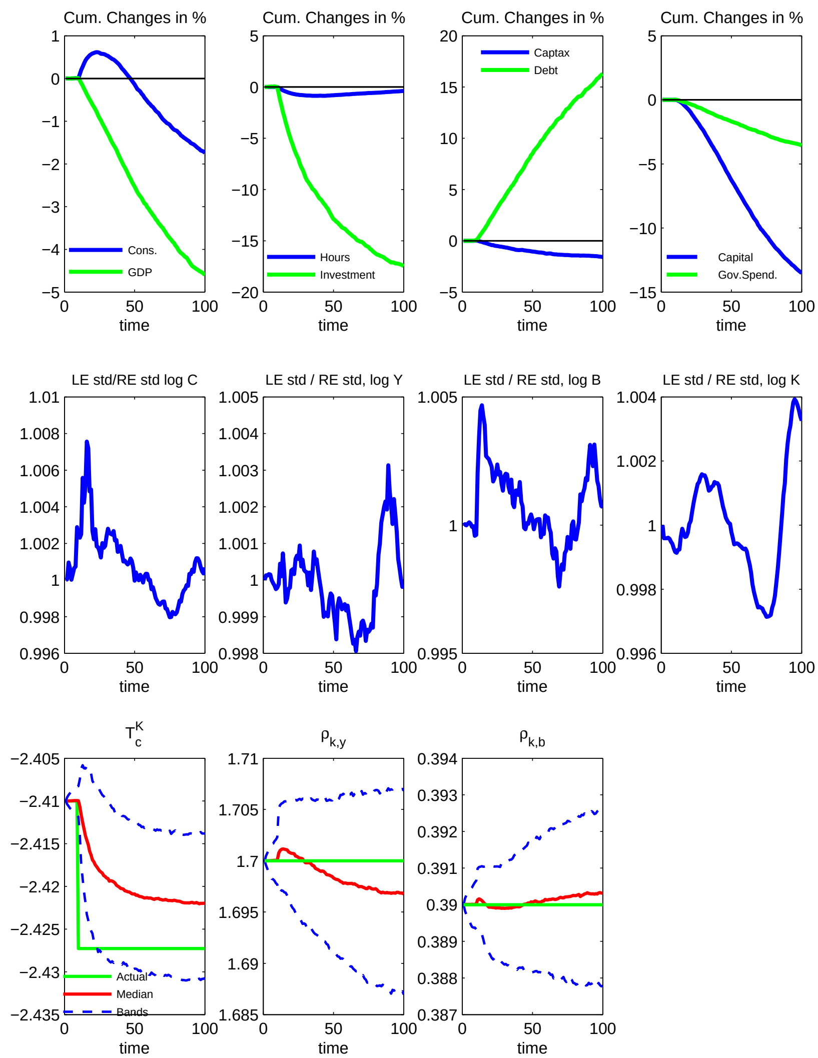

Figure 3: Summary of outcomes under learning when the capital tax policy rule changes

References

Greenwood, J., Hercowitz, Z., and Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. *American Economic Review*, **78**(3), 402–17.

Jaimovich, N. and Rebelo, S. (2009). Can news about the future drive the business cycle? *American Economic Review*, **99**(4), 1097–1118.

King, R. G., Plosser, C. I., and Rebelo, S. T. (1988). Production, growth and business cycles : I. the basic neoclassical model. *Journal of Monetary Economics*, **21**(2-3), 195–232.

Leeper, E. M., Plante, M., and Traum, N. (2010). Dynamics of fiscal financing in the United States. *Journal of Econometrics*, **156**, 304–321.

Sims, C. A. (2001). Solving linear rational expectations models. *Computational Economics*, **20**, 1–20.