Drifts, Volatilities, and Impulse Responses Over the Last Century ONLINE APPENDIX

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1 Introduction

This online appendix presents robustness checks and additional results for our benchmark specification. Given the plethora of results we generated, we decided to show only representative robustness checks, but we list all specifications we ran (those results not in the appendix are available upon request). Section (3) provides a discussion of all specifications we consider, such as different prior parametrizations, different measures for the money aggregate and all details regarding the different variations we consider for the identification of the monetary transmission mecha-

nism.

2 Data

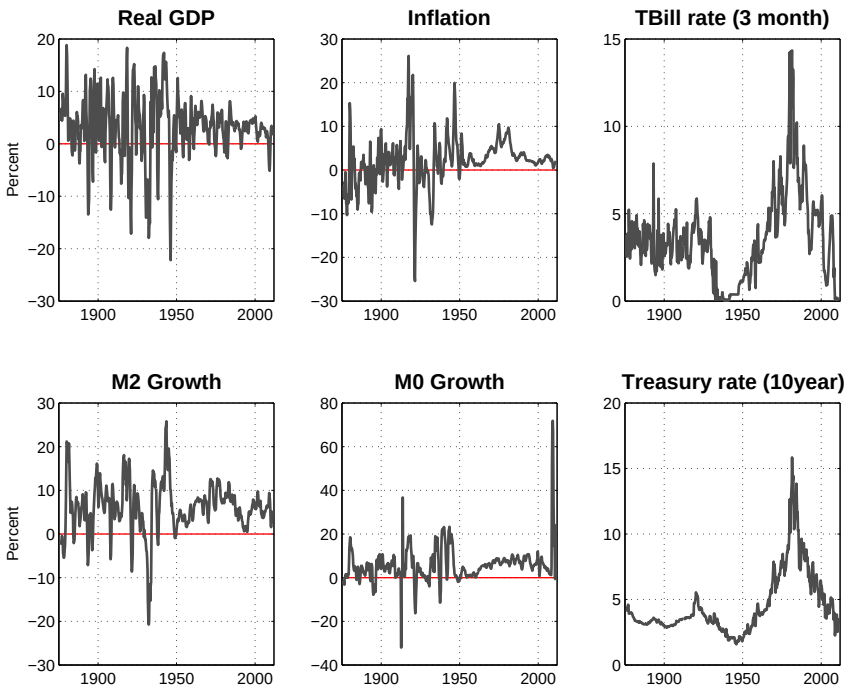

Figure 1: Data

This figure depicts the time series we use as endogenous variables in our econometric model.

3 Specification Overview

3.1 Model Overview

We essentially employ two different VAR models. Our benchmark model consists of the following vector of observables:

$$y_t = (\Delta gdp_t, \pi_t, i_t^s, spread_t, \Delta m_t)'$$

where m_t is represented by money base growth. Alternatively we also estimate our model setting m_t to M2 growth considering a broader monetary aggregate. This can be of interest particularly discussing the evolution of correlation among pertinent U.S. macroeconomic variables over the last century. However, we think that money base growth is a better and more credible time series for a controllable policy instrument than the more broad money aggregate M2 growth.

3.2 Prior Discussion

We use the first 38 years (152 observations, from 1876:I to 1913:IV) as a training sample to calibrate the respective prior distributions following Primiceri (2005). The corresponding mean and the variance of of our time varying AR coefficients (coefficients states, covariance states and log volatility states) are chosen to be the OLS point estimates and six times its variance in a time invariant VAR, estimated on the small initial subsample. Summarizing, the priors take the form:

$$
\begin{aligned}
A_0 &\sim N\left(\hat{A}_{OLS}, 4 \times V(\hat{A}_{OLS})\right) \\
\Lambda_0 &\sim N\left(\hat{\Lambda}_{OLS}, 4 \times V(\hat{\Lambda}_{OLS})\right) \\
log(\sigma_0) &\sim N\left(log(\hat{\sigma}_{OLS}), I_M\right) \\
Q &\sim IW\left(\kappa_Q^2 \times 152 \times V(\hat{A}_{OLS}), 152\right) \\
W &\sim IW\left(\kappa_W^2 \times 4, 4\right) \\
S_i &\sim IW\left(\kappa_S^2 \times (i+1) \times V(\hat{A}_{i,OLS}), (i+1)\right)
\end{aligned}
$$

where S_i denotes the corresponding block of S, while $\hat{A}_{i,OLS}$ stand for the two correspondent blocks of $\hat{A}_{OLS}$. The specific choices for κ_Q,κ_W and κ_S are important. To remain comparable with the literature we choose our benchmark specification follow Primiceri setting $\kappa_Q = 0.01$,$\kappa_W = 0.01$ and $\kappa_S = 0.1$. We also consider variations with respect to κ_Q as this tuning parameter can put discipline on the time variation formulating a prior belief regarding the amount of time variation. The discussion of all major institutional changes, wars, greater macroeconomic and financial disruptions one could assume a potentially a higher degree of time variation a-priori. We consider variations along these dimensions by setting the following values for $\kappa_Q = \{0.01, 0.05, 0.1\}$.

3.3 Summary of Model Space

The following table summarizes all models we estimated. The benchmark model in the main paper is model $\mathcal{M}_1$. For the sake of brevity, we will present results that either change the prior amount of time variation $\mathcal{M}_3$ or the monetary aggregate $\mathcal{M}_4$.

Table 1: **All model specifications**

This table summarizes the all models estimated in the model. Variations are considered along two dimensions. First, we include different monetary aggregates and second, consider different a-priori beliefs regarding the degree of time variation in the law of motion of A_t.

Model Set	Money Aggregate	κ_Q	κ_W	κ_S
$\mathcal{M}_1$	Monetary Base	0.01	0.01	0.1
$\mathcal{M}_2$	Monetary Base	0.05	0.01	0.1
$\mathcal{M}_3$	Monetary Base	0.10	0.01	0.1
$\mathcal{M}_4$	M2	0.01	0.01	0.1
$\mathcal{M}_5$	M2	0.05	0.01	0.1
$\mathcal{M}_6$	M2	0.10	0.01	0.1

3.4 Time Variation in the Benchmark Model

The paper describes various ways to summarize the amount of time variation we find. As a first pass, though, we find it helpful to look at the raw results to see what patterns of time variation emerge. In Figure 2 we plot the median estimates of all elements in μ_t, $A_{1,t}$, and $A_{2,t}$. Our model is able to capture very different patterns of time variation: fixed coefficients, small (relative to the size of the coefficient) time variation, or large shifts in parameters throughout time after periods in which those parameters have been stable.

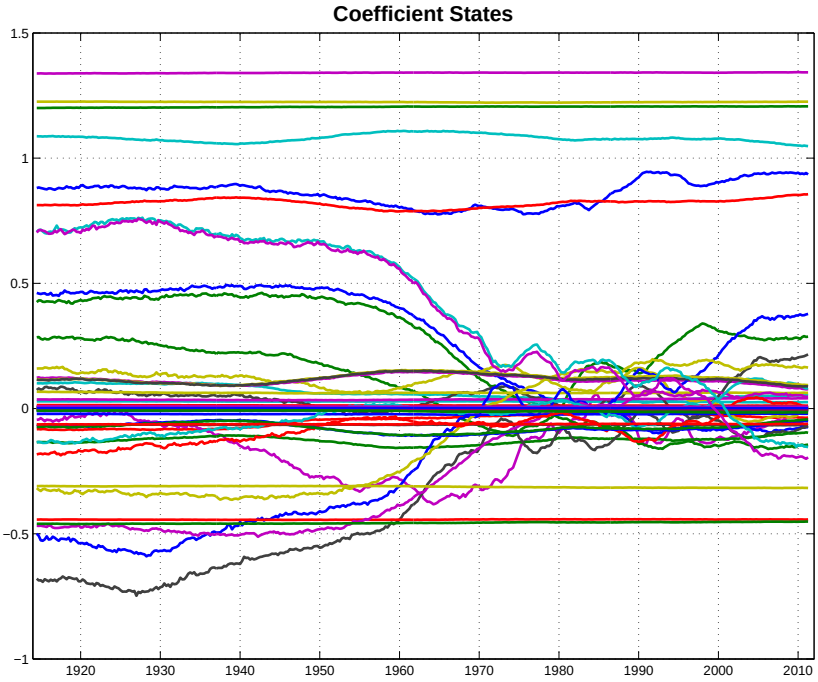

Figure 2: Time variation in coefficients

3.5 Sign Restriction Overview

In table (2) we summarize the sign restrictions we impose to identify a contractionary monetary policy shock. We employ sign restrictions a-la Canova-Faust-Uhlig. The identification strategy used in the paper is $\mathcal{SR} - \mathcal{M}_1$. In this appendix we present error bands for our benchmark specification as well as results for the case in which our sign restrictions are only imposed on impact. Those results confirm our benchmark results. If we were to not restrict the impulse response of output (as in Uhlig(2005), we would get insignificant responses of output growth (even though they median response would point to the same conclusions as in our benchmark case)[1]. An output response of a monetary policy shock is part of most models macroeconomists use and we base our identification restriction on those models, but it is important to remember that this is indeed an identification restriction that matters for our results.

Table 2: **Sign restrictions imposed for identification**
This table summarizes the sign restriction we impose for the identification of Monetary Policy shocks for the different model specifications employed.

	$\mathcal{SR} - \mathcal{M}_1$	$\mathcal{SR} - \mathcal{M}_2$	$\mathcal{SR} - \mathcal{M}_3$	$\mathcal{SR} - \mathcal{M}_4$	$\mathcal{SR} - \mathcal{M}_5$	$\mathcal{SR} - \mathcal{M}_6$
Δgdp	$0 \geq$	X	X	$0 \geq$	X	X
π	$0 \geq$	$0 \geq$	$0 \geq$	$0 \geq$	$0 \geq$	$0 \geq$
i^s	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0	≤ 0
spread	X	X	X	X	X	X
Δ mbase	$0 \geq$	$0 \geq$	$0 \geq$	-	-	-
Δ m2	-	-	-	$0 \geq$	$0 \geq$	$0 \geq$
$\mathcal{SR}$-Horizon	2	1	0	2	1	0

[1]We do not present those results here to not make the appendix even larger, but we feel it is important to point this feature out.

4 Descriptive Results

4.1 Evolving Reduced Form Standard Deviations

In this subsection we show the evolving innovation standard deviations for all model $\mathcal{M}_3$ and model $\mathcal{M}_4$ detailed earlier. Note that sampling the posterior of the evolving volatilities is insensitive to the specific models and prior calibration we considered. Results for the remaining four model specifications are available on request.

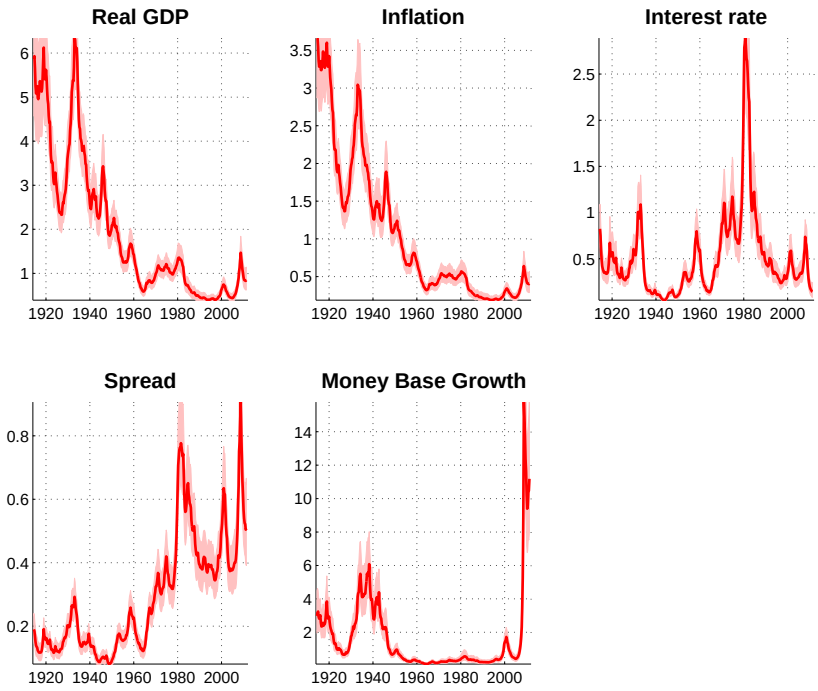

Figure 3: Evolving innovation standard deviations of **Model 3**

This figure depicts the evolving innovation standard deviations of **Model 3**.

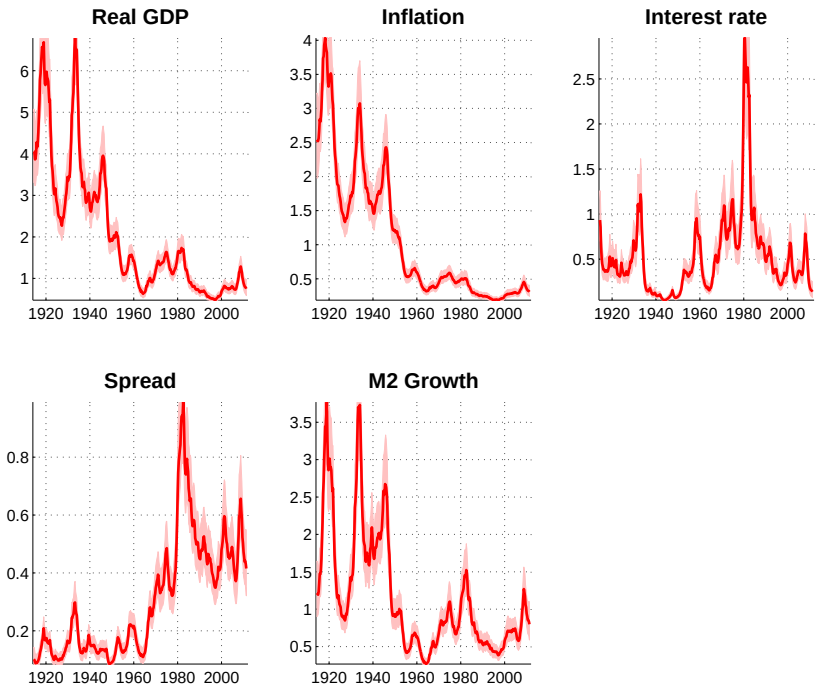

Figure 4: Evolving innovation standard deviations of **Model 4**

This figure depicts the evolving innovation standard deviations of **Model 4**.

4.2 Conditional Moments

In this section we show results of conditional forecast means for all models at horizon 20-years ahead.

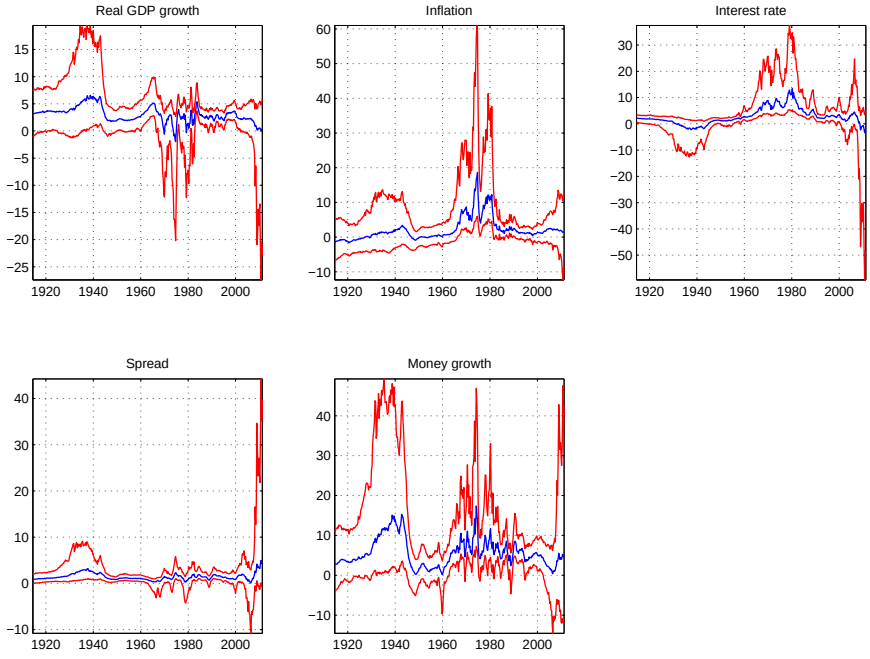

Figure 5: Evolving forecast means 20 years ahead for **Model 3**

This figure shows in blue the posterior median estimates of the time varying forecast means of our **Model 3**. The red lines are the posterior 68% error bands.

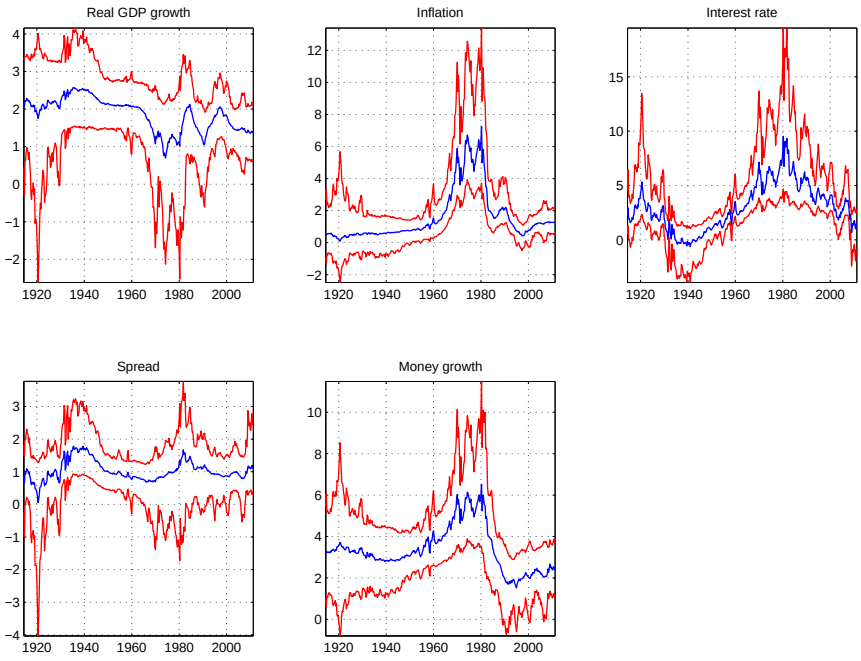

Figure 6: Evolving forecast means 20 years ahead for **Model 4**

This figure shows in blue the posterior median estimates of the time varying forecast means of our **Model 4**. The red lines are the posterior 68% error bands.

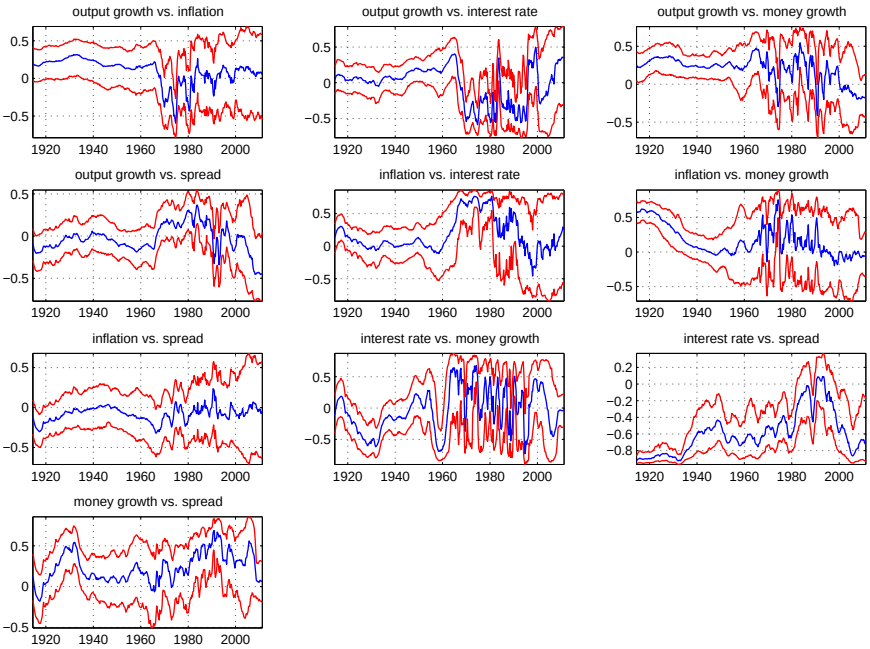

Figure 7: Evolving forecast correlations 1 years ahead for **Model 3**

This figure shows in blue the posterior median estimates of the time varying forecast correlations of our **Model 3**. The red lines are the posterior 68% error bands.

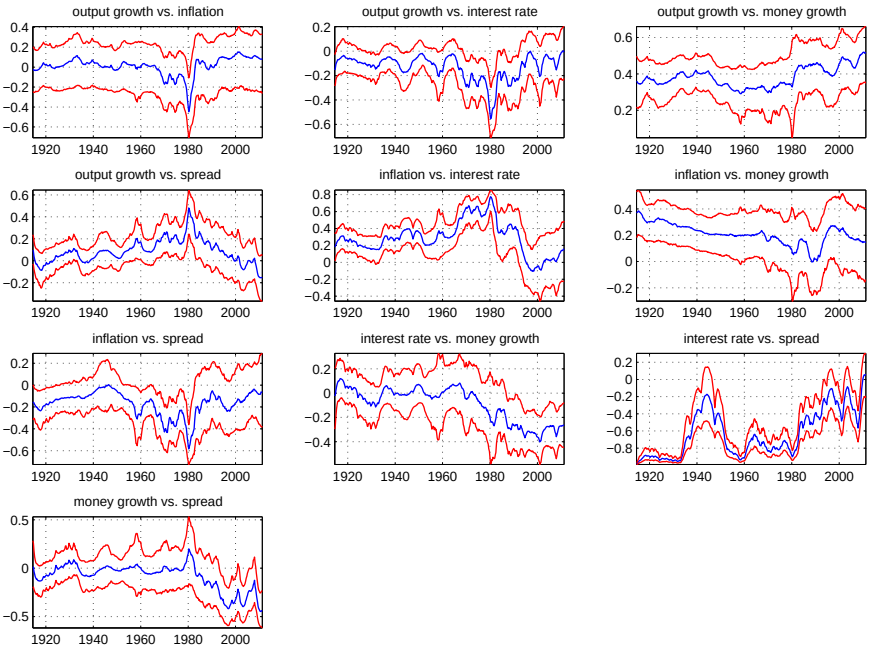

Figure 8: Evolving forecast correlations 1 years ahead for **Model 4**

This figure shows in blue the posterior median estimates of the time varying forecast correlations of our **Model 4**. The red lines are the posterior 68% error bands.

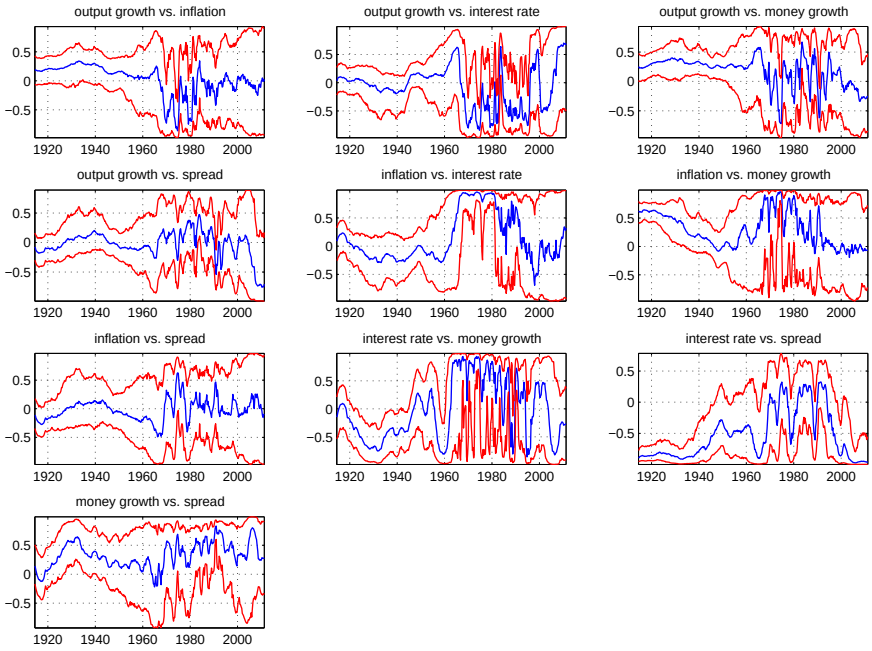

Figure 9: Evolving forecast correlations 20 years ahead for **Model 3**

This figure shows in blue the posterior median estimates of the time varying forecast correlations of our **Model 3**. The red lines are the posterior 68% error bands.

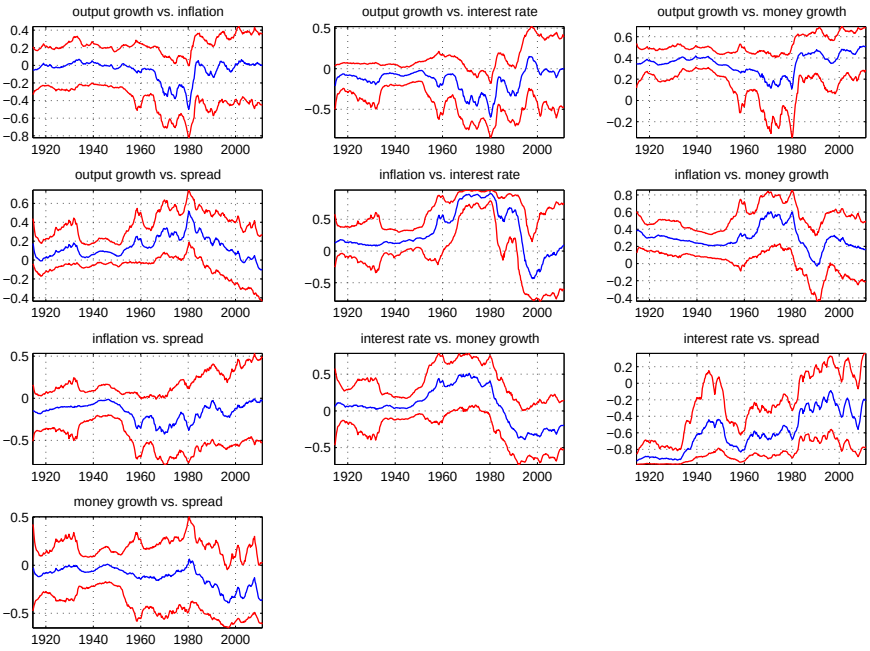

Figure 10: Evolving forecast correlations 20 years ahead for **Model 4**

This figure shows in blue the posterior median estimates of the time varying forecast correlations of our **Model 4**. The red lines are the posterior 68% error bands.

4.3 Analysis of Instability

In this subsection we show the results analyzing the posterior instability of our models discussed earlier. Figures (11)-(12) each depict three panels. Panels (a) show the probability of non-stationarity over time, panels (b) the complete posterior distribution of the maximum absolute eigenvalue of companion matrix of the coefficients for all draws at each point in time and finally panels (c) show the percentage share of draws for each given decile of the duration of non-stationarity.

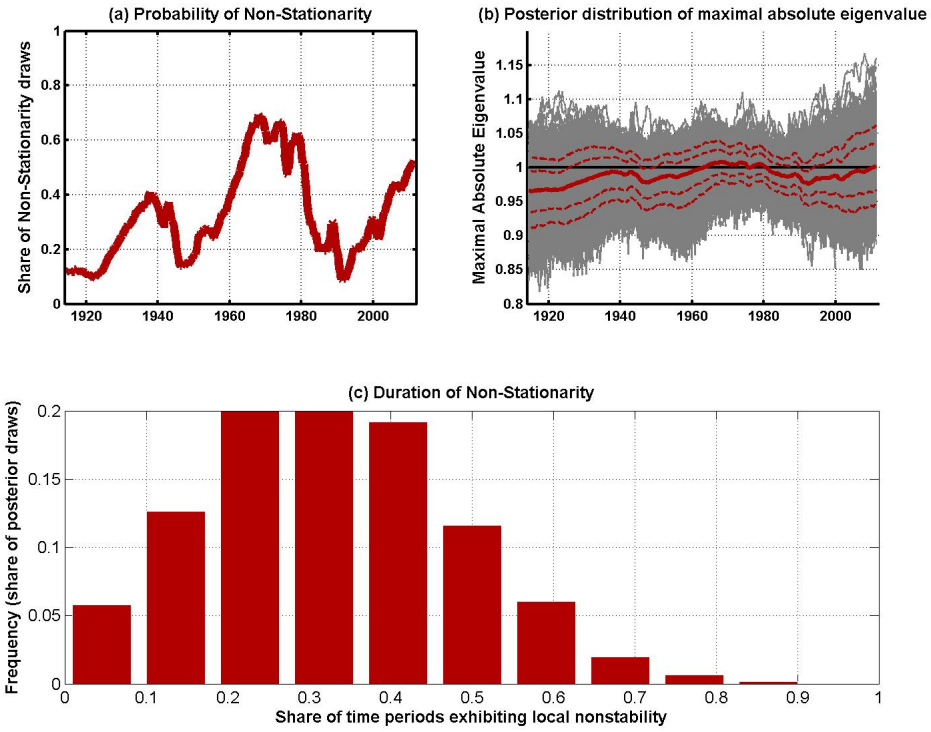

Figure 11: Explosive behavior Model 1 with Prior 3

Results in this figure are based on **Model 3**. Panel (a) shows the probability of non-stationarity over time, panel (b) shows the complete posterior distribution of the maximum absolute eigenvalue of the companion matrix of coefficients for all draws at each point in time in gray, the red dashed lines cover the posterior 68% and 90% error bands respectively and the red solid line the posterior median, panel (c) shows the percentage share of draws for each given decile of the duration of non-stationarity.

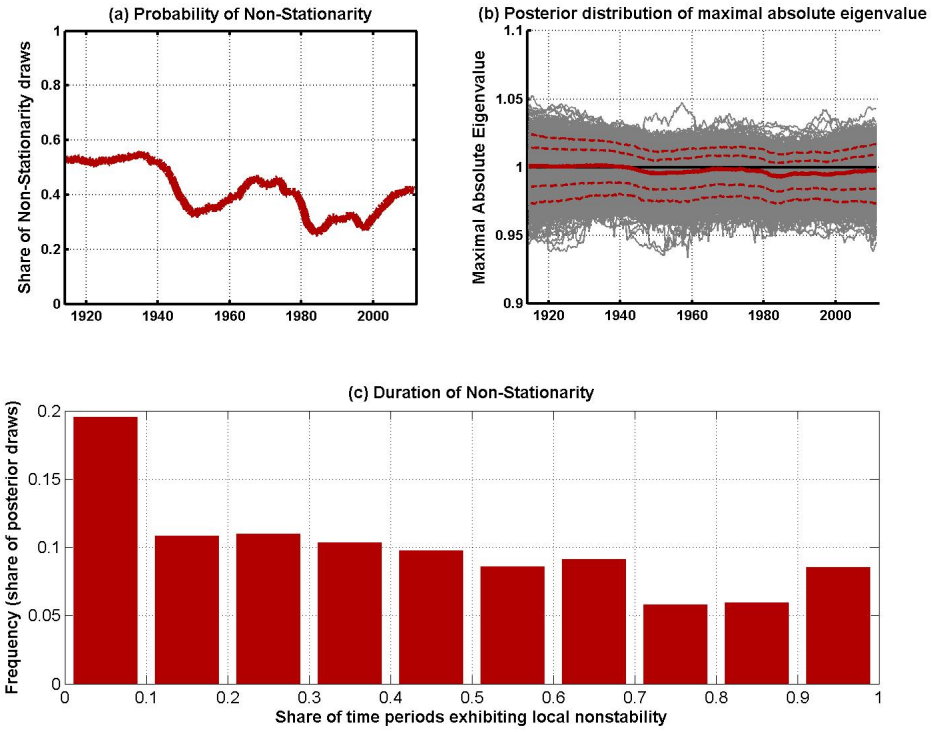

Figure 12: Explosive behavior Model 2 with Prior 1

Results in this figure are based on **Model 4**, substituting **M2 growth** as a monetary aggregate. Panel (a) shows the probability of non-stationarity over time, panel (b) shows the complete posterior distribution of the maximum absolute eigenvalue of the companion matrix of coefficients for all draws at each point in time in gray, the red dashed lines cover the posterior 68% and 90% error bands respectively and the red solid line the posterior median, panel (c) shows the percentage share of draws for each given decile of the duration of non-stationarity.

5 Structural Results based on Sign Restrictions

In this section we present impulse response functions and forecast error variance decomposition based on sign restrictions for models $\mathcal{M}_1$ imposing the restrictions $\mathcal{SR} - \mathcal{M}_3$ and alternatively for the benchmark case $\mathcal{SR} - \mathcal{M}_1$ from table (2). We estimated each combination for a each time t standard deviation and normalized (using the normalization discussed in the main text)at each point in time. For the figures that feature error bands each shade of gray represents 10% posterior probability.

5.1 Impulse Response Functions and FEVD for a Typical Standard Deviation Shock

In the following we will show the results for IRF and FEVD for **Model 1**.

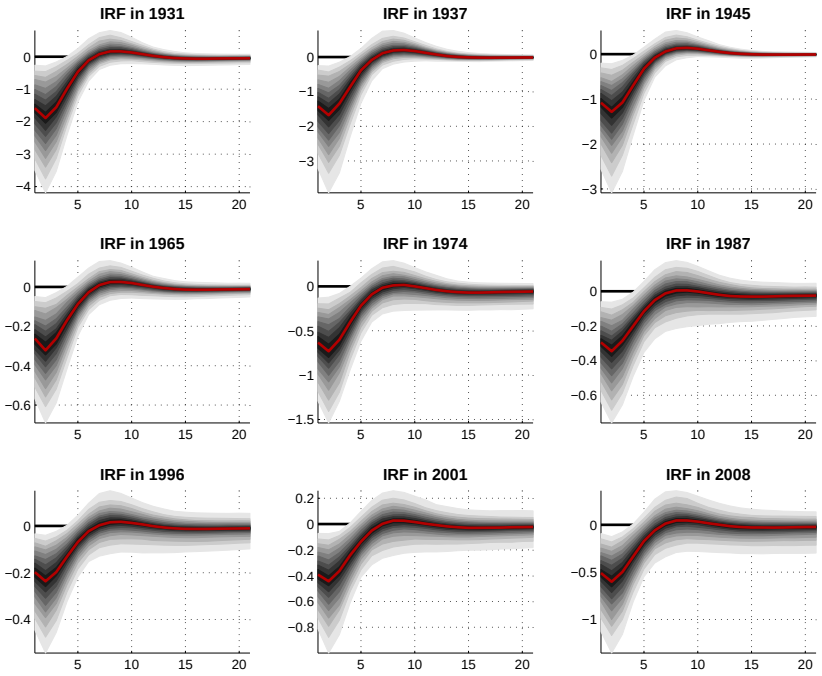

Figure 13: GDP SR-IRF for Model 1, SR Horizon 2, Std Shock

Real GDP growth responses based on SR-IRF for **Model 1** with SR on ouput, SR-horizon set to 2 and the size of the shock is one standard deviation at each point in time. Each shade of grey represents 10% posterior probability.

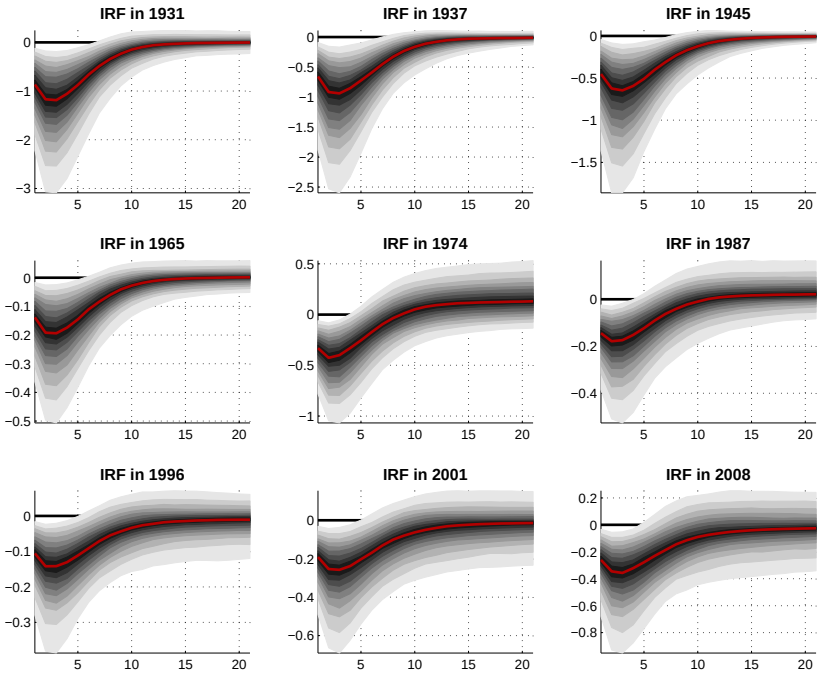

Figure 14: Inflation SR-IRF for Model 1, SR Horizon 2, Std Shock

Inflation responses based on SR-IRF for **Model 1** with SR on ouput, SR-horizon set to 2 and the size of the shock is one standard deviation at each point in time. Each shade of grey represents 10% posterior probability.

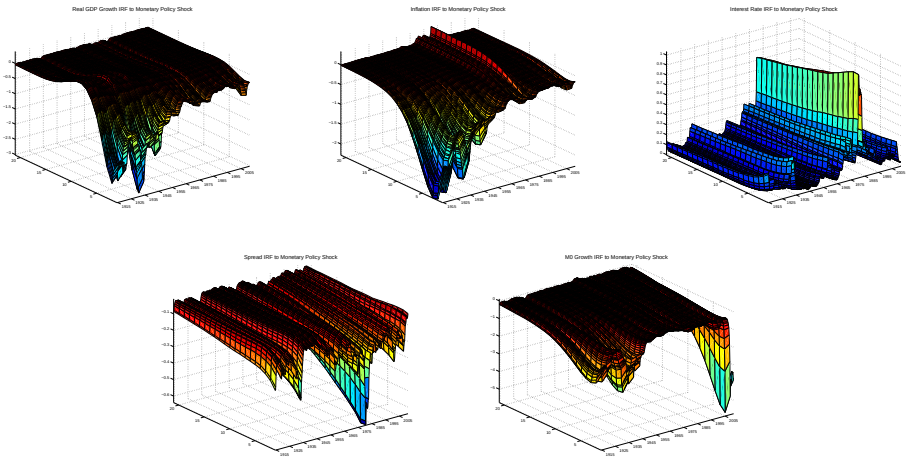

Figure 15: SR-IRF for Model 1, SR Horizon 0, Std Shock

Figure is based on SR-IRF for **Model 1** with SR on ouput, SR-horizon set to 0 and the size of the shock is one standard deviation at each point in time. The corresponding variables are real GDP, inflation, short term interest rate, spread and the money aggregate starting from the upper left panel going clockwise respectively.

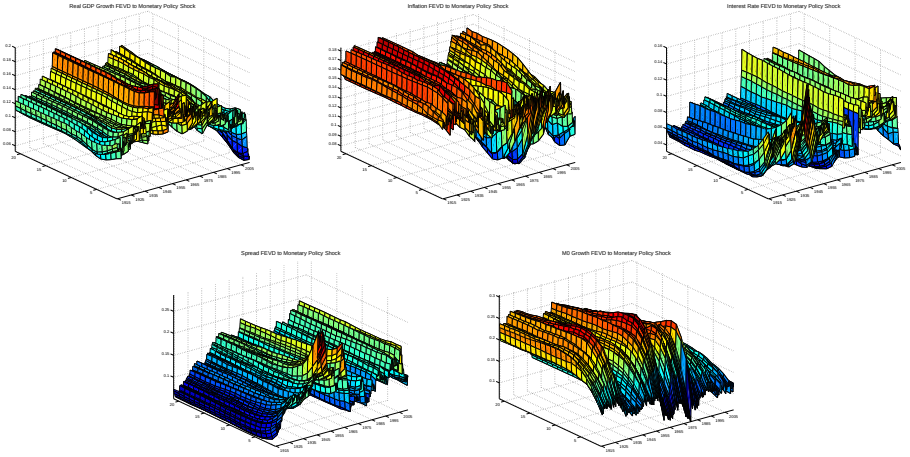

Figure 16: SR-FEVD for Model 1, SR Horizon 0, Std Shock

Figure is based on SR-FEVD for **Model 1** with SR on ouput, SR-horizon set to 0. The corresponding variables are real GDP, inflation, short term interest rate, spread and the money aggregate starting from the upper left panel going clockwise respectively.

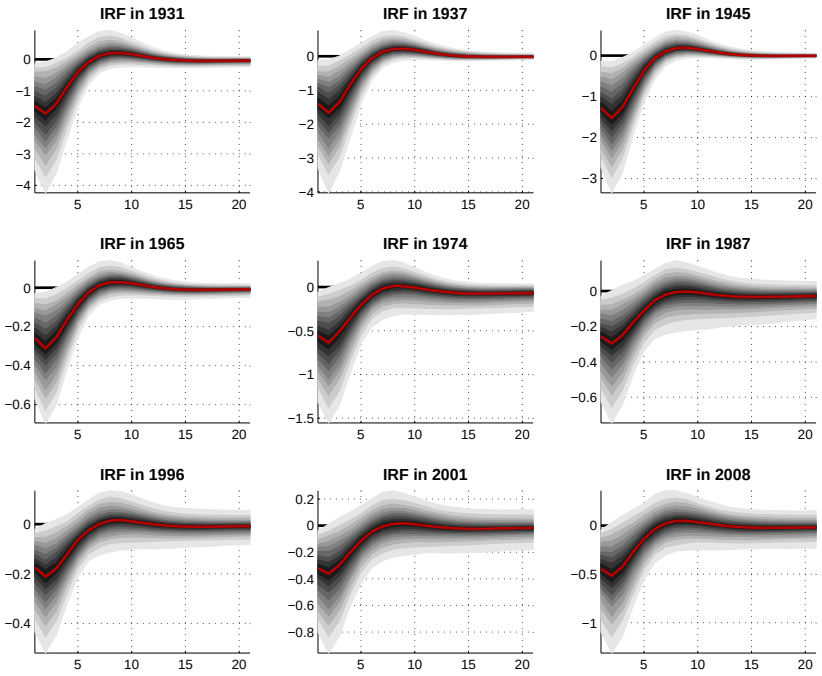

Figure 17: GDP SR-IRF for Model 1, SR Horizon 0, Std Shock

Real GDP grwoth responses based on SR-IRF for **Model 1** with SR on ouput, SR-horizon set to 0 and the size of the shock is one standard deviation at each point in time. Each shade of grey represents 10% posterior probability.

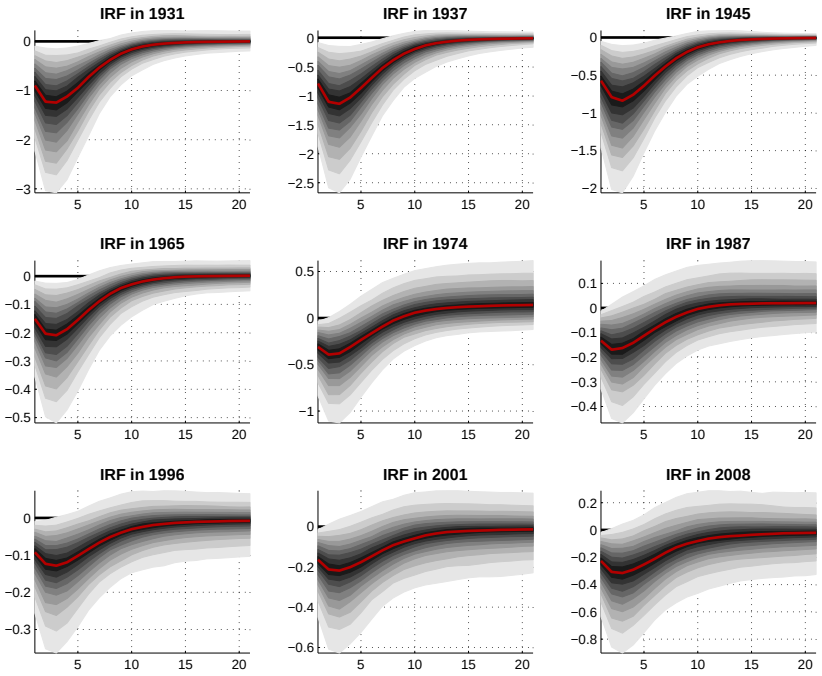

Figure 18: Inflation SR-IRF for Model 1, SR Horizon 0, Std Shock

Inflation responses based on SR-IRF for **Model 1** with SR on ouput, SR-horizon set to 0 and the size of the shock is one standard deviation at each point in time. Each shade of grey represents 10% posterior probability.

5.2 Impulse Response Functions and FEVD for a normalized shock

In the following we will show the results for IRF and FEVD for **Model 1**.

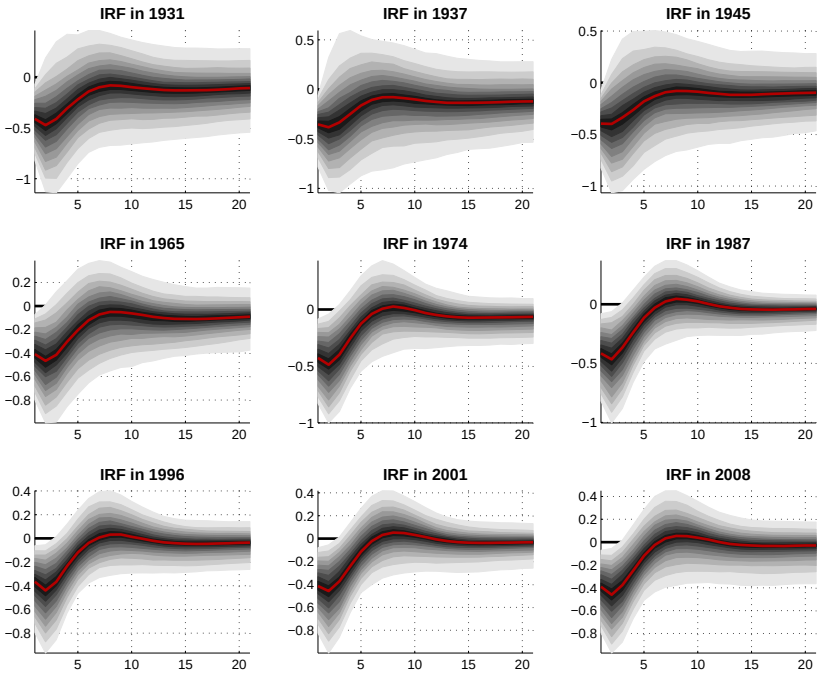

Figure 19: GDP SR-IRF for Model 1, SR Horizon 2, Normalized Shock

Real GDP growth responses based on SR-IRF for **Model 1** with SR on ouput, SR-horizon set to 2. Each shade of grey represents 10% posterior probability.

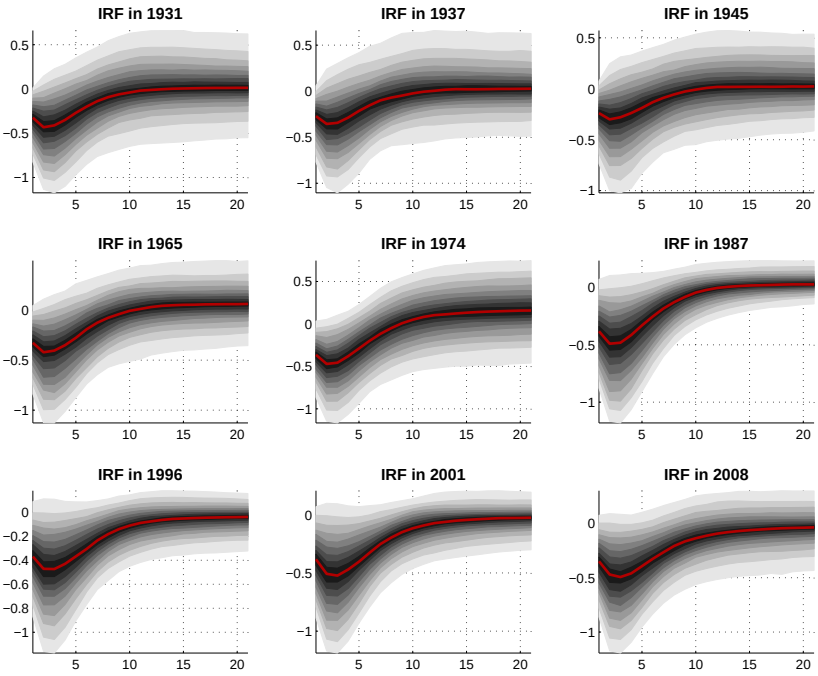

Figure 20: Inflation SR-IRF for Model 1, SR Horizon 2, Normalized Shock

Inflation responses based on SR-IRF for **Model 1** with SR on ouput, SR-horizon set to 2. Each shade of grey represents 10% posterior probability.

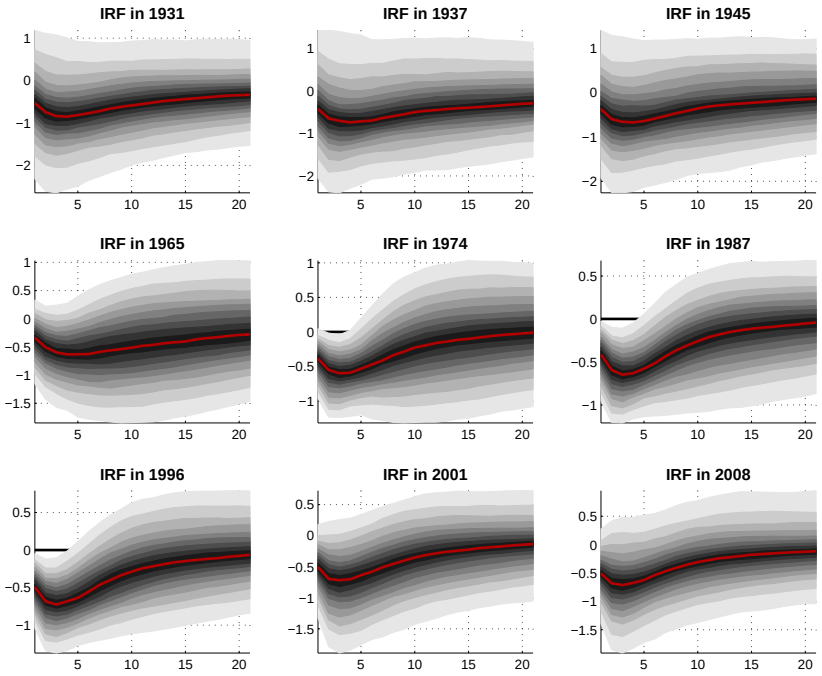

Figure 21: money growth SR-IRF for Model 1, SR Horizon 2, Normalized Shock

Money growth responses based on SR-IRF for **Model 1** with SR on ouput, SR-horizon set to 2. Each shade of grey represents 10% posterior probability.

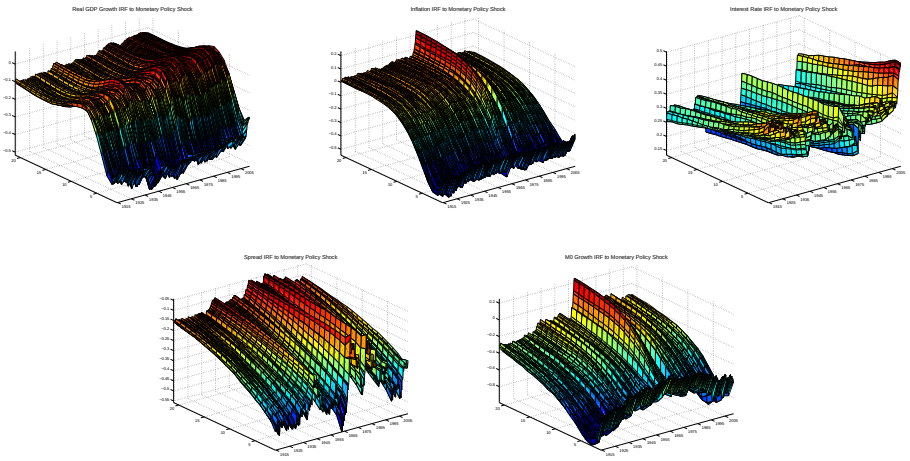

Figure 22: SR-IRF for Model 1, SR Horizon 0, Normalized Shock

Figure is based on SR-IRF for **Model 1** with SR on ouput, SR-horizon set to 0 and the size of the shock is one standard deviation at each point in time. The corresponding variables are real GDP, inflation, short term interest rate, spread and the money aggregate starting from the upper left panel going clockwise respectively.

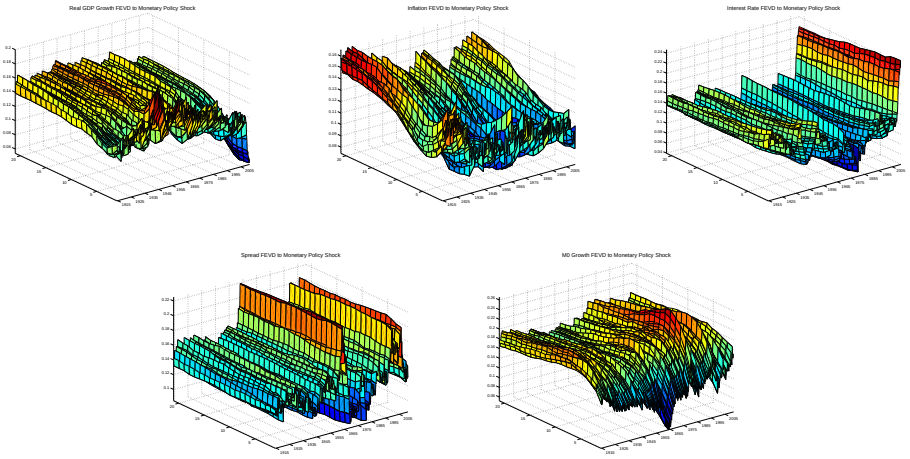

Figure 23: SR-FEVD for Model 1, SR Horizon 0, Normalized Shock

Figure is based on SR-FEVD for **Model 1** with SR on ouput, SR-horizon set to 0. The corresponding variables are real GDP, inflation, short term interest rate, spread and the money aggregate starting from the upper left panel going clockwise respectively.