Tales of Transition Paths:
Policy Uncertainty and Random Walks

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Abstract

What happens when fiscal or monetary policy change systematically? We construct a DSGE model in which agents have to estimate fiscal and monetary policy rules and assess how uncertainty surrounding the conduct of policymakers influences transition paths after policy changes. We find that policy changes of the magnitude often considered in the literature can lead private agents to hold substantially different views about the nature of equilibrium than would be predicted by a full information analysis. In particular, random walk-like behavior can arise *endogenously* for a large number of periods in equilibrium, even though the models we use admit stationary dynamics under full-information rational expectations.

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1 Introduction

An economic theory asserts how reduced form parameters change when government policies change.

Thomas J. Sargent, *Points of Departure*

What happens when fiscal or monetary policy rules change dramatically? Large changes in fiscal and monetary policy rules are routinely evaluated in micro-founded dynamic equilibrium models (see for example Curdia and Finocchiaro (2013)). Two strong assumptions commonly underlie such exercises: Agents are initially unaware of possible policy changes, and agents become immediately aware of the new policy rule once it is implemented. Work on Markov-switching DSGE models, such as Bianchi (2013) and Liu *et al.* (2011), dispenses of the first assumption but retains the second. As the quote above from Sargent (2015) highlights, these assumptions represent different economic theories about how firms and households in our models react to policy changes. We put forth another economic theory to remove the second assumption and model agents as econometricians who have to estimate coefficients of policy rules. We borrow the assumption of 'anticipated utility' decision-making (Kreps (1998)) that is common in the learning literature (Sargent *et al.* (2006), Primiceri (2005), and Milani (2007) are three examples) and thus keep the first assumption of the standard approach in play.[1]

A key theme of this paper is how random walk-like behavior (i.e. the VAR representation of the linearized equilibrium dynamics has eigenvalues equal to or slightly larger than 1) can endogenously arise in models that would only feature stationary dynamics under rational expectations. In an analytical example, we show that this endogenous persistence does not depend on the persistence of the exogenous shocks in the model. This behavior arises even though agents in our model have substantial knowledge of the economy and act as sophisticated econometricians to uncover those features of the economy that they do not know precisely. Rather, the dynamics of the data in equilibrium make the learning problem hard, leading to differences between beliefs of agents and the true policy rule, which in turn fuel the random walk-like behavior that arises in equilibrium.

We study two stylized examples of policy transitions. One is motivated by the large

[1]Learning has rarely been tackled in the literature on Markov-switching models. One exception is Bianchi and Melosi (2013), who introduce a very specific type of learning into a Markov-switching DSGE model: Their agents do observe the policy rule coefficients currently in play, but are uncertain how persistent the current regime is.

changes in fiscal and monetary policy that occurred following the recent financial crisis: a transition from a regime of active monetary policy and passive fiscal policy to a situation where fiscal policy is active and monetary policy is passive. Not only is this an a priori reasonable description of current policy in many developed economies, we also show that this policy scenario implies behavior of inflation and debt that is in line with observations since the financial crisis: Random walk-like behavior for debt, but also low and stable inflation. These outcomes make learning about the true policy coefficients for the monetary policy rule hard for the agents in our model, and they lead to substantial periods of confusion about the nature of equilibrium.

The second example we study is motivated by the Volcker disinflation and studies the change of a monetary policy rule from a rule with parameter values that would imply indeterminacy under full-information rational expectations to a rule with parameter values that imply determinacy. We thus explicitly model the transition that is absent in papers that study separately indeterminate and determinate outcomes for the United States, such as Lubik and Schorfheide (2004). Throughout this example we keep fiscal policy passive in the language of Leeper (1991) and Clarida *et al.* (1999). We revisit findings by Cogley *et al.* (2015), namely that even if agents think the economy is stable, unstable behavior can arise if beliefs are different enough from the actual policy rule coefficients. We build on their findings, but instead use a model that allows for fiscal and monetary policy interaction along the lines of Leeper (1991) and show that the economy in this scenario will most likely feature random walk-like behavior rather than outright explosive behavior that would be rejected by the data. This is particularly true for government debt, which Cogley *et al.* (2015) do not study. We also show that this behavior can persist for many periods. It is important to emphasize that our focus throughout this paper is on unusual (when considered against the backdrop of recent policy behavior) policy changes. We are not studying small (or 'modest') policy changes such as those emphasized by Leeper and Zha (2003).

The agents in our model do not place any restrictions on the kind of equilibria they consider (their 'perceived law of motion', or PLM): Both determinate and indeterminate as well as temporarily explosive equilibria are considered as possible outcomes by our agents. This is in contrast to the previous literature, where the nature of the perceived equilibrium has not been studied (often due to the assumed nature of the learning problem, where agents use statistical models to form expectations).

Once we remove arbitrary restrictions on the kind of equilibria considered by agents, they can easily find themselves in situations where they misperceive the nature of the equilibrium: In the first example, they might believe that equilibrium indeterminacy persists for substantial periods and, in the second example, agents are led to believe that the economy is temporarily explosive. Temporarily explosive dynamics can also be a feature of a Markov-switching rational expectations model, an outcome highlighted by Bianchi and Ilut (2013).

We endow our agents with substantial knowledge of the economy. They are only uncertain about the finite dimensional policy rule parameter vector. Furthermore, we endow them with the same knowledge of the timing of the structural change that agents in the standard approach have: Once policy changes, firms and households suspect that policy has indeed changed. In the standard approach, agents immediately learn the new policy rule coefficients, so their suspicions are instantaneously confirmed. In our model, agents instead have to estimate the new policy rule coefficients.[2] We do so to minimize the differences between our approach and the standard approach outlined above. Notwithstanding, even with our agents having substantial knowledge of the economy, equilibrium outcomes in our environment are substantially different from those determined using the standard approach. The agents in our model use the Kalman Filter to estimate coefficients. We thus make them sophisticated econometricians who could not easily improve upon their estimation routine. Private agents in our model do *not* have a suspicion about the direction of the change in the policy rule coefficients. Any announcement about the magnitude or direction of the policy change is thus viewed as incredible by our agents. We view this as a useful starting point on which models of policy communication could be built.

Our focus on agents who only need to learn about the coefficients of policy rules sets this paper apart from the earlier literature that studied monetary-fiscal policy interaction under learning such as Eusepi and Preston (2011) and Eusepi and Preston (2013), who instead endow their agents with less knowledge about the structure of the economy.

The interaction of fiscal and monetary policy under rational expectations in DSGE models was pioneered by Leeper (1991). Empirical analyses include Traum and Yang (2011) and Bhattarai *et al.* (2012). We use similar models and borrow parameter estimates from that literature, but refrain from using full-information rational expectations.

The effects of regime changes in monetary policy on beliefs and economic dynamics have been studied in Andolfatto and Gomme (2003). Changes in fiscal policy and their effects in an RBC economy with distortionary taxation have been studied in Hollmayr and Matthes (2015). Finally, policy uncertainty more generally has recently received substantially more attention due to the work of Baker *et al.* (2012), who quantify policy

[2]We model this suspicion by exogenously increasing the uncertainty about policy rule coefficients in the period in which policy actually changes. Section 4 describes this in detail.

uncertainty using various measures and show its impact using VARs.

Our approach is different from most of the previous literature in that we explicitly try to only use minimal departure from a full information rational expectations approach, restricting the private agents' model uncertainty to the policy rule parameters and focus on the transition dynamics rather than asymptotic results (i.e. whether equilibria are 'learnable'). Previous papers that have studied transition dynamics such as Andolfatto and Gomme (2003) and Giannitsarou (2006) do not analyze the nature of perceived equilibria. No other paper in the literature that we are aware of shows how random walk-like behavior can arise endogenously as a result of agents' learning.

It is important to emphasize, however, that this is not the first paper to show that learning can lead to additional persistence - Milani (2007) makes that point convincingly. We also do not claim that we are the first to show that learning can lead to temporarily explosive behavior - that point is first made by Cogley *et al.* (2015). The innovation in this paper is to show that a specific (but empirically relevant) kind of persistence - random walk-like behavior - arises naturally (and endogenously) in learning models of fiscal-monetary policy interaction. Our paper is also the first to link this behavior to misconceptions held by private agents about the nature of equilibrium (i.e. whether the equilibrium law of motion is unique, indeterminate or unstable).

The next section uses the model in Leeper (1991) and its analytical solution to shed some light on how changes in beliefs alter equilibrium dynamics and can lead to random walk-like behavior. We then turn to our benchmark DSGE model before describing the learning methodology in detail and discussing the differences between outcomes under learning and using the standard approach.

2 An Analytical Example: Beliefs and Equilibrium Outcomes in Leeper (1991)

In this section we highlight the key forces underlying our main results in a simpler context that allows for analytical results - the model in Leeper (1991). We do not explicitly model learning in this section, but instead we ask a simpler question - what happens if beliefs about policy rule coefficients held by private agents happen to be wrong? In the language of the learning literature, the agents' perceived law of motion (PLM) differs from the actual law of motion (ALM). Since agents in our benchmark DSGE model will be wrong about the policy rules while they are learning about policy coefficients after a policy change, this exercise will shed some light on the forces driving

	passive fiscal policy	active fiscal policy
active monetary policy	unique equilibrium	no stable equilibrium
passive monetary policy	multiple equilibria	unique equilibrium

Table 1: The nomenclature of equilibria and policies in Leeper (1991).

our result in the learning model described in later sections. Here we focus on one example: a situation in which agents believe that monetary policy is active and fiscal policy is passive, but the actual policy rule coefficients imply active fiscal policy and passive monetary policy. We explicitly show how and in what variables random walk-like behavior can occur. We will revisit the example in this section later for our benchmark DSGE model. For analytical results that speak to the existence of non-stationary dynamics when fiscal policy stays passive throughout, we refer the reader to Cogley *et al.* (2015).[3]

To start, it is useful to first revisit the nomenclature introduced by Leeper (1991) in order to discriminate between different classes of equilibria under rational expectations. Table 1 does this. We describe below exactly what parameter values constitute active or passive policy, but, broadly speaking, passive fiscal policy describes a situation in which the fiscal authority acts to stabilize debt, whereas active monetary policy describes a situation in which monetary policy tries to counteract inflation sufficiently. We now turn to the economy studied in Leeper (1991). The underlying nonlinear model is described in that paper. To summarize, it is an endowment economy that features lump-sum taxation, money in the utility function and monetary policy that follows a Taylor rule. For the sake of brevity, we will directly focus on the linearized system in inflation π_t and debt b_t:[4]

$$E_t\pi_{t+1} = \alpha\beta\pi_t + \beta\theta_t \tag{1}$$

$$\varphi_1\pi_t + b_t + \varphi_2\pi_{t-1} - (\beta^{-1} - \gamma)b_{t-1} + \varphi_3\theta_t + \psi_t + \varphi_4\theta_{t-1} = 0 \tag{2}$$

$$\theta_t = \rho_\theta\theta_{t-1} + \varepsilon_t^\theta \tag{3}$$

$$\psi_t = \rho_\psi\psi_{t-1} + \varepsilon_t^\psi \tag{4}$$

[3] Cogley *et al.* (2015) do not explicitly model fiscal policy in their analytical example, but the underlying assumption is passive fiscal policy, as is common in textbook New Keynesian models.

The form of the monetary policy rule in Leeper (1991) (where monetary policy reacts contemporaneously to inflation) means that changes in beliefs about monetary policy (while keeping fiscal policy fixed) will not affect the matrix multiplying lagged state variables in the equilibrium law of motion. Thus, Leeper's model is less suited to showing how random walk-like behavior can arise in the example inspired by the Volcker disinflation for our benchmark model.

[4] We will present the non-linear model for our benchmark case later.

The φ parameters are convolutions of parameters and steady states and are defined in Leeper (1991). Equation (1) is the result of combining the representative households consumption Euler equation and the monetary policy rule. Equation (2) is the government's budget constraint, where we have plugged in the optimality condition for money holdings and the fiscal and monetary policy rules. A key parameter for our results is the discount factor β. θ_t and ψ_t are exogenous policy shocks. The policy rules used in deriving these equations are

$$R_t = \alpha\pi_t + \theta_t \tag{5}$$

$$\tau_t = \gamma b_{t-1} + \psi_t \tag{6}$$

where R_t is the nominal interest rate and τ_t are lump-sum taxes (those variables are substituted out in equations (1) to (4)).

To analyze the dynamics of the system, we can stack the first two equations above:[5]

$$\begin{pmatrix} 1 & 0 \\ \varphi_1 & 1 \end{pmatrix} \begin{pmatrix} \pi_t \\ b_t \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -\varphi_2 & \beta^{-1} - \gamma \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ b_{t-1} \end{pmatrix} + \begin{pmatrix} -\beta/(\alpha\beta) & 0 \\ -\varphi_3 & -1 \end{pmatrix} \begin{pmatrix} \theta_t \\ \psi_t \end{pmatrix} \tag{7}$$
$$+ \begin{pmatrix} 0 & 0 \\ -\varphi_4 & 0 \end{pmatrix} \begin{pmatrix} \theta_{t-1} \\ \psi_{t-1} \end{pmatrix} + \begin{pmatrix} 1/(\alpha\beta) \\ 0 \end{pmatrix} E_{\pi_{t+1}}$$

To solve the model, we have to plug in above an expression for the one-step ahead expectations of inflation as a function of pre-determined endogenous variables or exogenous shocks. Once we do that, the system above gives the equilibrium dynamics. The equilibrium dynamics under full-information expectations and under arbitrary beliefs about the policy coefficients differ in how expectations are formed - under full information rational expectations agents know the true values α and γ, whereas under misspecified beliefs the agents act as if policy is governed by parameters $\widehat{\alpha} \neq \alpha$ and $\widehat{\gamma} \neq \gamma$. Just as in the larger benchmark model that we use for our numerical work later, we assume here that agents observe all past endogenous variables but not the policy shocks, which the agents have to infer based on the observable data and their

[5]For the sake of simplicity, we do not stack the exogenous laws of motion for the shocks in the system. Our results hold for generic laws of motion of the exogenous process as long as those processes are stationary. Also, to determine the properties of the equilibrium dynamics, we focus on the autoregressive matrix in the equilibrium law of motion. That matrix determines stability properties as long as there is no root cancellation with the roots of the moving average polynomial. This turns out to be the case here.

beliefs about policy rule coefficients. If policy shocks were observed by agents, private agents could learn the policy coefficients immediately.

The dynamics of the system and the nature of the prevailing equilibrium under full-information rational expectations are governed by $\alpha\beta$ and $\beta^{-1} - \gamma$. If the first term is larger than one in absolute value, while the second term is not, we are in a world of monetary activism and passive fiscal policy, whereas, in the reverse case, we are in a world of active fiscal policy (fiscal policy does not use taxes to stabilize debt) and passive monetary policy.

Leeper gives the full-information rational expectations solution for inflation and expected inflation in the 'standard' case of active monetary policy and passive fiscal policy:

$$E_t \pi_{t+1} = -\frac{\beta \rho_\theta}{\alpha\beta - \rho_\theta} \theta_t \tag{8}$$

$$\pi_t = -\frac{\beta}{\alpha\beta - \rho_\theta} \theta_t \tag{9}$$

Note that fiscal shocks or the coefficient γ in the fiscal policy rule do not enter either the expected or actual inflation. This happens precisely because fiscal policy is passive. Under rational expectations and active fiscal policy, the fiscal shocks would play a role in determining inflation, as documented by Leeper.

2.1 The Dynamics Under a PLM When Agents Are Confused About Which Policy is Active

In this section we assume that agents have a degenerate prior on the policy rule coefficients given by $\widehat{\alpha}$ and $\widehat{\gamma}$ such that $\widehat{\alpha}\beta > 1$ and $\beta^{-1} - \widehat{\gamma} < 1$, so agents are certain that fiscal policy is passive and monetary policy is active, but the ALM satisfies $\alpha\beta < 1$ and $\beta^{-1} - \gamma > 1$. Given that agents think monetary policy is governed by a policy rule with coefficient $\widehat{\alpha}$, their perceived policy shock is $\widehat{\theta}_t = (\alpha - \widehat{\alpha})\pi_t + \theta_t$ under the assumption that θ is not directly observable. Their conditional expectations of inflation next period are then given by

$$E_t^* \pi_{t+1} = -\frac{\beta \rho_\theta}{\widehat{\alpha}\beta - \rho_\theta} \widehat{\theta}_t = -\frac{\beta \rho_\theta}{\widehat{\alpha}\beta - \rho_\theta} ((\alpha - \widehat{\alpha})\pi_t + \theta_t) \tag{10}$$

Plugging these expectations into the matrix equation that describes the endogenous dynamics of the system, we get

$$\underbrace{\begin{pmatrix} 1+\frac{1}{\alpha\beta}\frac{\beta\rho_\theta}{\widehat{\alpha}\beta-\rho_\theta}(\alpha-\widehat{\alpha}) & 0 \\ \varphi_1 & 1 \end{pmatrix}}_{A}\begin{pmatrix} \pi_t \\ b_t \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 \\ -\varphi_2 & \beta^{-1}-\gamma \end{pmatrix}}_{B}\begin{pmatrix} \pi_{t-1} \\ b_{t-1} \end{pmatrix} \tag{11}$$

$$+ \begin{pmatrix} -\beta/(\alpha\beta)-\frac{1}{\alpha\beta}\frac{\beta\rho_\theta}{\widehat{\alpha}\beta-\rho_\theta} & 0 \\ -\varphi_3 & -1 \end{pmatrix}\begin{pmatrix} \theta_t \\ \psi_t \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\varphi_4 & 0 \end{pmatrix}\begin{pmatrix} \theta_{t-1} \\ \psi_{t-1} \end{pmatrix}$$

The key insight to take away here is that π_t is still a stationary process (it is perfectly correlated with the stationary shock θ_t). On the other hand, b_t is now non-stationary because inflation expectations and thus actual inflation do not move to offset the effects of $|\beta^{-1}-\gamma| > 1$ since inflation expectations are formed according to the dynamics for the case in which monetary policy is active and fiscal policy is passive. To see this more clearly, we can invert the matrix on the left-hand side of the equation above (call this matrix A) and multiply by the matrix multiplying the lagged endogenous variables (call this matrix B). This gives us the matrix governing the endogenous dynamics in the actual law of motion. The inverse of A will also be lower triangular. To see this, we can apply the formula for the inverse of a 2-by-2 matrix to A. Doing so gives us the result that the $(2,2)$ element of A^{-1} is 1 and the $(1,2)$ element is 0. Thus, we get

$$A^{-1}B = \begin{pmatrix} 0 & 0 \\ -\varphi_2 & \beta^{-1}-\gamma \end{pmatrix} \tag{12}$$

The eigenvalues of that matrix are 0 and $\beta^{-1}-\gamma$. The second eigenvalue is thus larger than 1 in absolute value by our assumption. In general, we assume that $\gamma \geq 0$, so the second eigenvalue is bounded from above by $\frac{1}{\beta}$, which is around 1.01 with standard quarterly calibrations of β. This hints at the possibility that even if agents learn, in finite samples debt can behave much like a random walk. We will see this in our learning economy.

This example, while abstracting from many important channels through which monetary and fiscal policy can influence economic outcomes, already shows that beliefs of private agents can have a large impact on economic outcomes and substantially alter the properties of the equilibrium. In particular, it will turn out that the random walk-like behavior of debt coupled with stationary inflation dynamics will also be an outcome of our larger model that we discuss next for the second policy experiment. Even though both Leeper's model and our larger DSGE model that we turn to next abstract from issues such as the zero-lower bound on nominal interest rates, the behavior of debt and inflation from both models resembles actual data for developed economies since the global financial crises and the subsequent changes in fiscal policies.

While Leeper's model is very useful for developing intuition and obtaining further insights into the issues we want to study, it also has its limitations: An insight that we can only infer from our benchmark economy (as we will see later) and not from Leeper's model is that random walk-like behavior can also occur when agents know that monetary policy is active and fiscal policy passive, but agents do not know the exact policy parameters[6]

Whenever eigenvalues of absolute value 1 or larger appear in arguments based on local approximations (as is the case here and will be the case in our benchmark model) we know that we can not use simulations to approximate unconditional moments. Nonetheless, for simulations or calculations based on finite time periods such as those presented in this paper, local approximation methods can be reliably used to approximate equilibrium dynamics when there are eigenvalues larger than 1 in play (especially when these eigenvalues are not much larger than 1 in absolute value). This point has been made by Kim *et al.* (2003).[7]

3 Model

Our model is a standard a medium-scale New-Keynesian model along the lines of, for example, Smets and Wouters (2007) and Christiano *et al.* (2005). It incorporates nominal frictions, habits, capital utilization and, additionally, a fiscal sector. The government accumulates debt if its income from distortionary labor and capital taxation does not match outlays for government spending, transfers, and debt repayments and interest payments. First-order conditions and the complete log-linearized model may be found in the Appendix. The calibration for all parameters is standard in the literature and is mostly taken from Traum and Yang (2011), who estimate a similar model using U.S. data. We thus relegate the numerical values of the parameters to Table 7 in the Appendix.

[6] One can use other models that can be solved analytically to show that explosive behavior can occur in the related case where fiscal policy is not explicitly modeled - see Cogley *et al.* (2015).

[7] The fact that the behavior of a very persistent, but stationary, process can be very similar to the behavior of a unit root process in finite samples is at the heart of many issues connected with unit-root test, for example.

3.1 Households

The economy is populated by a continuum of households that each have access to a full set of state-contingent securities, which leads them to make the same investment and consumption decisions, even though they each supply a different labor input to the labor market. Households maximize their expected utility, [8] where the instantaneous utility function of household j takes the following form:

$$U_t^j = U_t^b \left[\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{L(j)_t^{1+\phi}}{1+\phi} \right] \tag{13}$$

The household derives utility from consumption C_t and disutility from hours worked $L(j)_t$. U_t^b is a preference shock that follows in its log-linearized form an AR(1) process[9]:

$$u_t^b = Const_u^b + \rho_{u^b} u_{t-1}^b + \epsilon_t^{u^b} \tag{14}$$

Each period, households can choose either to consume, invest (I_t), or save in the form of government bonds (B_t). Households face the following budget constraint each period:

$$\begin{aligned} C_t + B_t + I_t \quad = \quad & W_t(l)L_t(j,l)(1-\tau_t) + (1-\tau_t)R_t^K V_t \bar{K}_{t-1} + \psi(V_t)\bar{K}_{t-1} + \\ & + \frac{R_{t-1}B_{t-1}}{\pi_t} + Z_t + Pr_t \end{aligned} \tag{15}$$

and the law of motion for private capital:

$$\bar{K}_t = (1-\delta)\bar{K}_{t-1} + U_t^i \left[1 - s\left(\frac{I_t}{I_{t-1}} \right) \right] I_t \tag{16}$$

The household's income stems from working at the wage W_t and interest payments on their savings at the rate R_t. Z_t represents lump-sum transfers and Pr_t are the profits households obtain from the intermediate firm. τ_t denotes the distortionary tax rate that the government levies equally on labor and capital [10]. Effective capital $\bar{K}_t$ is

[8] Agents treat the coefficients both in the fiscal and in the monetary policy rule as fixed when making their decisions. We thus use an *anticipated utility* assumption along the lines of Kreps (1998), which is common in the literature on adaptive learning. A more thorough description follows in the section where the learning algorithm is described in more detail.

[9] Note that, throughout, lower-case letters are used for log-linearized variables of the corresponding uppercase variables when the log is not explicitly stated in a formula. Steady state values will be denoted by variables with a 'hat'.

[10] In using only one tax rate for both input factors we follow Traum and Yang (2011). For our application, this assumption allows us to keep the learning problem of the private agents more parsimonious. The forces that drive our results would also be in play in a model with more than one tax rate.

rented by households to firms at the rate R_t^K and is related to physical capital by its utilization rate V_t along the lines of

$$K_t = V_t \bar{K}_{t-1} \tag{17}$$

The cost of the utilization rate is denoted by $\psi(V_t)$ where the functional form follows standard assumptions in the literature: V is 1 in steady state and $\psi(1) = 0$. Additionally $\psi \in [0, 1]$ is defined, so that the following equation is satisfied: $\frac{\psi''(1)}{\psi'(1)} = \frac{\psi}{1-\psi}$. Capital is subject to a certain depreciation rate δ, but is accumulated over time via investment I_t. Investment is subject to adjustment costs $s(.)$ and to the shock U_t^i, which captures an exogenous disturbance as to how efficiently investment can be turned into effective capital. It also follows a simple AR(1) process in its log-linearized form:

$$u_t^i = Const_u^i + \rho_{u^i} u_{t-1}^i + \epsilon_t^{u^i} \tag{18}$$

3.2 Wages

A composite labor service L_t is produced by labor packers and is given by

$$L_t = \left[\int_0^1 l_t(l)^{\frac{1}{1+\eta_t^w}} dl \right]^{1+\eta_t^w} \tag{19}$$

The demand function of the labor packers stems from the profit maximization problem, which yields (with L_t^d as the composite demand for labor services).

$$l_t(l) = L_t^d \left(\frac{W_t(l)}{W_t} \right)^{-\frac{1+\eta_t^w}{\eta_t^w}} \tag{20}$$

where η_t^w is an exogenous markup shock to wages. It follows in its log-linear form an AR(1) process:

$$log(\eta_t^w) = Const_{\eta^w} + \rho_{\eta^w} log(\eta_{t-1}^w) + \epsilon_t^{\eta^w} \tag{21}$$

The nominal aggregate wage evolution is then given by

$$W_t = \left[(1-\theta_w) \tilde{W}_t^{-\frac{1}{\eta_t^w}} + \theta_w \left(\pi^{1-\chi_w} \pi_{t-1}^{\chi_w} \right)^{-\frac{1}{\eta_t^w}} W_{t-1}^{-\frac{1}{\eta_t^w}} \right]^{-\eta_t^w} \tag{22}$$

We assume that wages are sticky, which we model using the Calvo (1983) approach. The fraction θ_ω of households that cannot re-optimize index their wages to past inflation by the rule:

$$W_t(j) = W_{t-1}(j)(\pi_{t-1}^{\chi_\omega} \pi_{ss}^{1-\chi_\omega}) \tag{23}$$

3.3 Firms

The production function of firm i is linear in technology and labor:

$$Y_t(i) = \exp(A_t) K_{t-1}(i)^{\alpha} L_t(i)^{1-\alpha} \tag{24}$$

where Y_t denotes the output produced with a certain level of technology A_t, labor input $L_t(i)$, and capital K_t. The exogenous process for technology is given by an AR(1):

$$\log(A_t) = \rho_a \log(A_{t-1}) + \epsilon_t^A \tag{25}$$

In terms of price setting, we assume that retailers set their prices according to the Calvo (1983) mechanism, i.e. each period the fraction $(1 - \theta_i)$ of all firms are able to reset their prices optimally. Furthermore, we assume that firms that cannot reoptimize their prices in period t index their prices to the past inflation rate following the equation:

$$P_t(i) = P_{t-1}(i)^{\chi_p} \pi_t^{\chi_p} \pi_{ss}^{1-\chi_p} \tag{26}$$

Profits of firm i (in nominal terms) are then equal to

$$\Pi_t(i) = (P_t(i) - MC_t(i)) \left(\frac{P_t(i)}{P_t} \right)^{-\eta_t^p} Y_t(i) \tag{27}$$

with real marginal costs given by

$$MC_{i,t} = (1-\alpha)^{\alpha-1} \alpha^{-\alpha} (R_t^k)^{\alpha} W_t^{\alpha} A_t^{-1}$$

and the demand for good i $Y_t(i)$ as

$$Y_t(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\frac{1+\eta_t^p}{\eta_t^p}} \tag{28}$$

where η_t^p is an exogenous markup shock to the intermediate good's price that also follows an AR(1) process:

$$log(\eta_t^p) = Const_{\eta_p} + \rho_{\eta^p} log(\eta_{t-1}^p) + \epsilon_t^{\eta^p} \tag{29}$$

3.4 Government

The government budget constraint takes the following form:

$$B_t = B_{t-1}\frac{R_{t-1}}{\pi_t} - R_t^K K_t \tau_t - W_t L_t \tau_t + G_t + Z_t \tag{30}$$

We let the labor tax rule react to past levels of debt. For simplicity, all other fiscal variables follow Gaussian AR(1) processes. We assume that agents know all fiscal policy rules except for the tax rule. Government spending is given by

$$\log(G_t) = Const_G + \rho_g \log(G_{t-1}) + \epsilon_t^G \tag{31}$$

Z_t denotes transfers, which behave as follows:

$$\log(Z_t) = Const_Z + \rho_z \log(Z_{t-1}) + \epsilon_t^Z, \tag{32}$$

As mentioned above, the tax rate is modeled as a rule with the feedback coefficient ρ_b, which the agents do not know for sure and have to infer:

$$\log(\tau_t) - log(\tau_{ss}^L) = \rho_b \log(B_{t-1}) - \rho_b log(B_{ss}) + \epsilon_t^\tau \tag{33}$$

The subscript ss denotes steady state values. Using the debt-to-GDP ratio as the right-hand side variable instead does not alter our results. Monetary policy is conducted via a simple Taylor-type rule, which is only reacting to lagged inflation.

$$\log(R_t) - log(R_{ss}) = \phi_\pi \log(\pi_{t-1}) - \phi_\pi log(\pi_{ss}) + \epsilon_t^R \tag{34}$$

The firms and households in our model know the form of the labor tax rule and the monetary policy rule as described above, but they do not know the coefficients, which they have to estimate. They also know that the government budget constraint has to hold in every period. We model the government and the central bank as reacting to lagged endogenous variables so as to circumvent endogeneity issues in the learning problem of private agents that would occur if the government and the central bank were to react to contemporaneous endogenous variables. Given the information and implementation lags present in economic policymaking, we do not think that this is an overly strong assumption.

3.5 Market Clearing

Demand from the government and households in the form of investment and consumption in addition to the adjustment costs related to the utilization costs must fully absorb the output of the firms:

$$Y_t = C_t + I_t + G_t + \psi(V_t)K_{t-1}$$

Market clearing in the bond market implies that all bonds issued by the government are bought by the households in the economy.

4 Learning Mechanism

Our approach to model learning is borrowed from our earlier work, Hollmayr and Matthes (2015), which in turn builds on Cogley *et al.* (2015). The private agents in our model observe all state variables in the economy and all exogenous innovations except for the true policy shocks. Instead of the true policy shocks in the tax and monetary policy rules, they observe the perceived policy shocks, which are the residuals in their corresponding estimated policy rules. They use those observations to estimate the coefficients of the policy rules. Firms and households know all other aspects of the model. All private agents share the same beliefs and carry out inference by using the Kalman filter. The choice of the Kalman filter as the agents' estimation procedure is motivated by the fact that we want to stay as close as possible to a (limited information) rational expectations setup, and the Kalman filter returns the posterior distribution for conditionally linear models with Gaussian innovations such as ours.[11] The one departure from limited information rational expectations in our setup is the use of

[11] For a comparison of learning based on the Kalman Filter and learning based on recursive least squares algorithms that are also common in the literature, see Sargent and Williams (2005).

the anticipated utility assumption (Kreps (1998)), which is common in the literature on learning in macroeconomics (see, for example, Milani (2007)). This assumption amounts to the private agents using a point estimate each period to form their beliefs (rather than integrating over the posterior) and not contemplating future changes in beliefs when making decisions. We will follow another standard assumption in the learning literature by assuming that agents that make decisions at time t use parameter estimates that were formed at the end of the previous period (and, as such, use data up to and including time $t-1$ variables).

We first describe the model that agents use to form estimates of policy rule coefficients. Then, we will describe what those estimates imply for the agents' view about the dynamics of the economy - we derive their perceived law of motion (PLM). Finally, we ask what those perceptions imply for actual equilibrium dynamics - we derive the actual law of motion (ALM) for the model.

We denote by $\mathbf{\Omega_t}$ the vector of policy rule coefficients ϕ_π and ρ_b that agents want to estimate. In order for agents to be able to use the Kalman Filter for inference, we need to build a state-space system that encompasses our assumptions about the learning behavior of agents. The observation equation is obtained by stacking the monetary policy rule and the fiscal policy rule for taxes, whereas the state equation represents the perceived dynamics in policy rule coefficients. For simplicity, we assume that the steady states are known to the private agents (Cogley *et al.* (2015) highlight the fact that the differences between dynamics under learning and the full information case emerge mainly from different views held by agents on policy rule response coefficients, not intercepts).[12] Agents know that the policy rules they have to estimate are specified in deviations from the known steady state values, so any intercepts disappear from these equations. The agents can back out the implied intercepts after the estimation using the known values for the steady states and the latest estimates of the policy rule coefficients.

The vector of observables $\xi_\mathbf{t}$ is given by

$$\xi_\mathbf{t} = \begin{bmatrix} \log(R_t) - log(R_{ss}) \\ \log(\tau_t) - log(\tau^L_{ss}) \end{bmatrix} \tag{35}$$

The observation equation is then:

$$\xi_\mathbf{t} = \mathbf{X_{t-1}\Omega_t} + \eta_\mathbf{t} \tag{36}$$

[12]Since the steady states are not affected by the policy changes we consider, agents can use all available data to learn about the steady states by computing sample averages.

where $\eta_{\mathbf{t}}$ collects the i.i.d. disturbances in the policy rules. $\mathbf{X_{t-1}}$ collects the right-hand side variables in the two policy rules, lagged inflation and lagged debt (minus their respective steady state values). What is left to specify then is the perceived law of motion for Ω_t - how do firms and households in the economy think policy rule coefficients change over time? We study two assumptions: Agents either know when the policy rule changes and take into account that policy rule coefficients before and after the break date *are fixed*, or they suspect that policy changes every period. The following law of motion for the coefficients encodes these assumptions, inspired by the literature on time-varying coefficient models in empirical macroeconomics (such as Cogley and Sargent (2005) or Primiceri (2005)) [13]:

$$\mathbf{\Omega_t} = \mathbf{\Omega_{t-1}} + \mathbf{1}_t \nu_t \tag{37}$$

If we set the variance of $\nu_{\mathbf{t}}$ to a conformable matrix of zeroes, then the private agents in our model believe that policy rule coefficients do not change and they estimate unknown constant coefficients. The indicator function $\mathbf{1}_t$ selects in what periods agents perceive there to be a change in policy. We will assume that this indicator function is 0 unless the policy rule actually changes, i.e. the agents think that the policy coefficient changes only in the period in which it actually changes.

Given beliefs for $\mathbf{\Omega_t}$, agents in our model will adhere to the anticipated utility theory of decision-making: They will act as if $\mathbf{\Omega_t}$ is going to be fixed at the currently estimated level forever onwards. This is a common assumption in the literature on learning, see for example Milani (2007). We use the posterior mean forecast $E_{t-1}(\mathbf{\Omega_t}) = \mathbf{\Omega_{t|t-1}}$ calculated via the Kalman Filter as a point estimate that the agents in the model condition on when forming expectations. By the random walk assumption on the parameters that agents use in the Kalman filter, this also implies that $\mathbf{\Omega_{t|t-1}} = E_{t-1}(\mathbf{\Omega_{t-1}})$.

If we denote the vector of all variables (plus a constant intercept) in the model economy by $\overline{\mathbf{Y}}_{\mathbf{t}}$, then we can stack the log-linearized equilibrium conditions (approximated around the steady state, which we assume is known to agents) and the estimated policy rules to get the log-linearized perceived law of motion in the economy:

$$\overline{\mathbf{A}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{Y}}_{\mathbf{t}} = \overline{\mathbf{B}}(\mathbf{\Omega_{t|t-1}})\mathbf{E}^*_{\mathbf{t}}\overline{\mathbf{Y}}_{\mathbf{t+1}} + \overline{\mathbf{C}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{Y}}_{\mathbf{t-1}} + \overline{\mathbf{D}}\varepsilon^*_{\mathbf{t}} \tag{38}$$

$\varepsilon^*_{\mathbf{t}}$ contains the standardized (i.e. variance 1) actual shocks that agents observe as well as the standardized perceived policy shocks (the residuals in the estimated policy rules). Those residuals are standardized because we choose to include the standard

[13] This assumption has been applied in the learning literature by Sargent *et al.* (2006), for example.

deviations of the shocks in the system matrices. Because we use the anticipated utility assumption, agents act as if their beliefs will not change in the future and this system can be solved using a number of standard algorithms for the solution of linearized rational expectations models such as Gensys (Sims (1994)).[14] The resulting reduced form perceived law of motion is given by

$$\overline{\mathbf{Y}}_{\mathbf{t}} = \overline{\mathbf{S}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{Y}}_{\mathbf{t-1}} + \overline{\mathbf{G}}(\mathbf{\Omega_{t|t-1}})\varepsilon^*_{\mathbf{t}} \tag{39}$$

$\mathbf{S}(\mathbf{\Omega_{t|t-1}})$ solves the following matrix quadratic equation[15]:

$$\overline{\mathbf{S}}(\mathbf{\Omega_{t|t-1}}) = (\overline{\mathbf{A}}(\mathbf{\Omega_{t|t-1}}) - \overline{\mathbf{B}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{S}}(\mathbf{\Omega_{t|t-1}}))^{-\mathbf{1}}\overline{\mathbf{C}}(\mathbf{\Omega_{t|t-1}}) \tag{40}$$

and $\overline{\mathbf{G}}(\mathbf{\Omega_{t|t-1}})$ is given by

$$\overline{\mathbf{G}}(\mathbf{\Omega_{t|t-1}}) = (\overline{\mathbf{A}}(\mathbf{\Omega_{t|t-1}}))^{-\mathbf{1}}\overline{\mathbf{D}} \tag{41}$$

To derive the ALM, we replace the perceived policy rule coefficients in $\overline{\mathbf{C}}(\mathbf{\Omega_{t|t-1}})$ with the actual policy rule coefficients and use the actual innovation vector $\varepsilon_{\mathbf{t}}$:

$$\overline{\mathbf{A}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{Y}}_{\mathbf{t}} = \overline{\mathbf{B}}(\mathbf{\Omega_{t|t-1}})\mathbf{E}^*_{\mathbf{t}}\overline{\mathbf{Y}}_{\mathbf{t+1}} + \overline{\mathbf{C}}^{\mathbf{actual}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{Y}}_{\mathbf{t-1}} + \overline{\mathbf{D}}\varepsilon_{\mathbf{t}} \tag{42}$$

To solve the model, we can plug the PLM into the ALM twice to get

$$\overline{\mathbf{A}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{Y}}_{\mathbf{t}} = \overline{\mathbf{B}}(\mathbf{\Omega_{t|t-1}})(\overline{\mathbf{S}}(\mathbf{\Omega_{t|t-1}})^{\mathbf{2}}\overline{\mathbf{Y}}_{\mathbf{t-1}} + \overline{\mathbf{S}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{G}}(\mathbf{\Omega_{t|t-1}})\varepsilon^*_{\mathbf{t}}) + \overline{\mathbf{C}}^{\mathbf{actual}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{Y}}_{\mathbf{t-1}} + \overline{\mathbf{D}}\varepsilon_{\mathbf{t}} \tag{43}$$

Note that there are two types of shocks appearing in the last equation: the true and the perceived shocks. We can solve for the dynamics of $\overline{\mathbf{Y}}_{\mathbf{t}}$ by only inverting $\overline{\mathbf{A}}(\mathbf{\Omega_{t|t-1}})$ as long as we can derive an expression for the perceived shocks that only depends on pre-determined and exogenous variables. Fortunately enough, this is true in our case:

$$\varepsilon^*_{\mathbf{t}} = \varepsilon_{\mathbf{t}} + \mathbf{I}^{\mathbf{P}}(\overline{\mathbf{C}}^{\mathbf{actual}}(\mathbf{\Omega_{t|t-1}}) - \overline{\mathbf{C}}(\mathbf{\Omega_{t|t-1}}))\overline{\mathbf{Y}}_{\mathbf{t-1}} \tag{44}$$

where $\mathbf{I}^{\mathbf{P}}$ denotes a selection matrix that selects those rows of the vector it multiplies, which are associated with the policy instruments about whose dynamics the agents

[14] If the estimated policy rule coefficients imply an indeterminate equilibrium, we use the equilibrium returned by Gensys.

[15] The perceived law of motion can be derived by assuming a VAR perceived law of motion of order 1 and then using the method of undetermined coefficients.

are learning and rescales those rows by the inverse of the standard deviation of the corresponding policy shock.
Plugging that expression into equation (43), we can derive the reduced form actual law of motion:

$$\overline{\mathbf{Y}}_{\mathbf{t}} = \mathbf{F}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{Y}}_{\mathbf{t-1}} + \mathbf{R}(\mathbf{\Omega_{t|t-1}})\varepsilon^*_{\mathbf{t}} \tag{45}$$

$$\mathbf{F}(\mathbf{\Omega_{t|t-1}}) = \overline{\mathbf{A}}^{-1}(\mathbf{\Omega_{t|t-1}})(\overline{\mathbf{C}}(\mathbf{\Omega_{t|t-1}}) + \overline{\mathbf{B}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{S}}^{\mathbf{2}}(\mathbf{\Omega_{t|t-1}})) + \overline{\mathbf{A}}^{-1}(\mathbf{\Omega_{t|t-1}})(\overline{\mathbf{B}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{S}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{G}}(\mathbf{\Omega_{t|t-1}})\mathbf{I^P}(\overline{\mathbf{C}}^{\mathbf{actual}}(\mathbf{\Omega_{t|t-1}}) - \overline{\mathbf{C}}(\mathbf{\Omega_{t|t-1}}))) \tag{46}$$

$$\mathbf{R}(\mathbf{\Omega_{t|t-1}}) = \overline{\mathbf{A}}^{-1}(\mathbf{\Omega_{t|t-1}})(\overline{\mathbf{B}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{S}}(\mathbf{\Omega_{t|t-1}}))\overline{\mathbf{G}}(\mathbf{\Omega_{t|t-1}}) + \overline{\mathbf{D}} \tag{47}$$

5 Equilibrium Outcomes Under Learning

We now turn to simulating our learning economy. First, we consider a scenario in which monetary policy becomes passive and distortionary taxes react less to the level of debt. In particular, we consider a one-time switch in the policy rule coefficients α_π and ρ_b from 1.5 and .1 to .8 and .04. Put differently, the economy undergoes a switch from monetary dominance to fiscal dominance. This scenario is meant to approximate the situation in many developed countries after the recent financial crisis.
We then consider a second scenario where we leave the fiscal policy rule constant at all times. It is passive in the sense that it stabilizes debt. Monetary policy is assumed to switch from passive to active: α_π changes from 0.8 to 1.5. This scenario is introduced to broadly mimic the disinflation period during Paul Volcker's chairmanship of the Federal Reserve in the early 1980s in the United States. The numerical values for the policy rule parameters pre- and post-policy change for both scenarios are in line with parameter estimates for the U.S. economy (Traum and Yang (2011)).
To produce the figures in the next section, we run 500 simulations of 100 periods with the policy switch occurring in period 10.[16] To keep the number of additional parameters manageable and to keep those parameters interpretable, we assume agents

[16] We choose not to put the policy switch at the beginning of the simulations in order to minimize the effect of the choice of the initial covariance matrix for the Kalman Filter.

use a covariance matrix of the innovations in the perceived policy rule coefficients of the following form[17]:

$$\mathbf{E}(\nu_\mathbf{t}\nu_\mathbf{t}') = \begin{bmatrix} (scale * (1.5 - 0.8))^2 & 0 \\ 0 & (scale * (0.1 - 0.04))^2 \end{bmatrix} \tag{48}$$

For simplicity, we will use the same calibration for both scenarios. The initial values of the parameters are given by the pre-policy change true values - we can think of this as the agents initially living in a world where any learning process has converged. For *scale*, we use 1 as a benchmark value, but we also show selected results for 2 and 4. Whenever it is not explicitly stated, the value for *scale* is 1 in the sections below. Agents thus experience a shock that is either a 1 standard deviation shock, a shock of size half a standard deviation, or a one-fourth standard deviation shock. Values of *scale* larger than 1 imply that agents were contemplating very large policy changes, which is why we use the value of $scale = 1$ as our benchmark.

6 Results

6.1 The Dawn of Active Fiscal Policy

For our first experiment, the analytical results from Leeper's model already hint at the strong possibility of random walk-like behavior when fiscal policy becomes active, but private agents have incomplete knowledge of this. Note that, even if agents learn quickly about changes in the fiscal policy rule, they can be misled if learning about the monetary policy coefficient is slow. This is exactly what happens in this scenario. Figure 1 plots the probability of a stable perceived equilibrium. While, under rational expectations, agents would know that the equilibrium is always stable, here agents are indeed misled and believe they are in an unstable equilibrium. In contrast to the second example that we discuss later, here even the perceived law of motion is unstable. This was not the case in our analysis of Leeper's model, where we assumed a stable PLM for simplicity. An unstable PLM can lead to instability in the ALM because of the first term on the right-hand side in the equation for $\mathbf{F}(\mathbf{\Omega_{t|t-1}})$. To see this, we revisit that equation here:

[17] We have used covariance matrices of this form in our previous work and found them useful for interpreting the perceived amount of time variation.

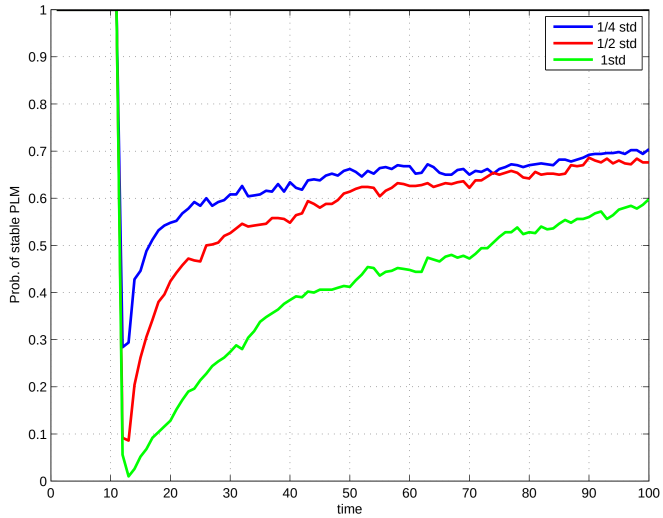

Figure 1: Perceived equilibria in the first experiment

$$\begin{aligned}\mathbf{F}(\mathbf{\Omega_{t|t-1}}) &= \overline{\mathbf{A}}^{-1}(\mathbf{\Omega_{t|t-1}})(\overline{\mathbf{C}}(\mathbf{\Omega_{t|t-1}}) + \overline{\mathbf{B}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{F}}^{2}(\mathbf{\Omega_{t|t-1}})) \\ &+ \overline{\mathbf{A}}^{-1}(\mathbf{\Omega_{t|t-1}})(\overline{\mathbf{B}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{F}}(\mathbf{\Omega_{t|t-1}})\overline{\mathbf{G}}(\mathbf{\Omega_{t|t-1}})\mathbf{I^P}(\overline{\mathbf{C}}^{\mathbf{actual}}(\mathbf{\Omega_{t|t-1}}) - \overline{\mathbf{C}}(\mathbf{\Omega_{t|t-1}})))\end{aligned}$$

The first term on the right-hand side gives the equilibrium dynamics under the perceived law of motion, which can be unstable in this scenario (but will turn out to always be stable in the second scenario). The second term on the right-hand side features the difference between actual and perceived laws of motion. If that difference is large enough, the eigenvalues of the entire matrix can become larger than 1 in absolute value, even though the perceived law of motion is stable. In the standard approach to modeling policy changes, we do not see random walk-like behavior - the economy switches from one stable equilibrium to another. The instability of the PLM creeps into the ALM in our case, as can be seen in Figure 2, which plots the largest eigenvalue in absolute value in the ALM versus the number of periods for which an eigenvalue larger than 1 in absolute value is present in the equilibrium dynamics. We see a negative relationship between the duration of explosive episodes and the absolute magnitude of the eigenvalues. Since this pattern also arises in our second scenario, we discuss the intuition for this finding in the next section. Are all variables affected by this random walk-like behavior? Figure 3 shows some randomly selected sample paths from our simulation for inflation and debt. In spite of the difference in the nature of the PLM, we see the same results as for Leeper's model: Inflation is low and stable, whereas debt is very persistent and displays random walk-like behavior. To see what impact the non-stationarity has on equilibrium outcomes, Figure 4 plots the difference between the outcomes under the standard approach and the learning approach using 500 simulations of 100 periods each.[18] We use the same shock series for both approaches, calculate the levels of the relevant variables and derive the ratio of the median under one approach to the median under the other approach, normalized by the steady state. For any variable at time j, we thus plot

$$Diff_j^W = \frac{(W_j^{learning} - W_j^{standard})}{W} \tag{49}$$

where $W_j^{learning}$ is the median outcome in levels under learning and $W_j^{standard}$ the corresponding outcome for the standard approach. To make the differences in GDP more visible, we plot *cumulative* differences for GDP instead:

$$Diff_j^W = \sum_{t=1}^{j} \frac{(W_t^{learning} - W_t^{RE})}{W} \tag{50}$$

We also plot the average parameter estimates.

It is the stable behavior of inflation that makes learning about the monetary policy coefficient hard, as can be seen by the average estimate in Figure 4. While debt is volatile enough in our economy so that the average coefficient for the fiscal policy rule is learned almost immediately, [19] it takes a long time to learn about the change in monetary policy. In many simulations, agents thus think that both monetary and fiscal policy are active, leading them to believe that the equilibrium is unstable. In this scenario, we see larger differences in (cumulated) GDP and debt between the learning economy and the standard approach to modeling policy changes.

Even though our model abstracts from many issues that are important in the current economic situation, this experiment still reproduces the broad patterns of low and stable inflation and substantial persistent movements in debt. If we were to model a fiscal stimulus along the lines of Drautzburg and Uhlig (2011), Cogan *et al.* (2010), or Hollmayr and Matthes (2015), we would see a larger initial jump in debt. The learning problem that agents face in this scenario is hard because inflation does not move enough to identify the changes in monetary policy. Incorporating the zero-lower bound on nominal interest rates would only make this identification issue harder. The

[18]Thus, when we refer to a median in the description of the result, this means the median across all 500 simulations.

[19]Not all simulations feature fiscal policy coefficient that directly collapse to the true value, but they all move in the right direction and the variation is quite small, so focusing on the average estimate across simulations is not misleading.

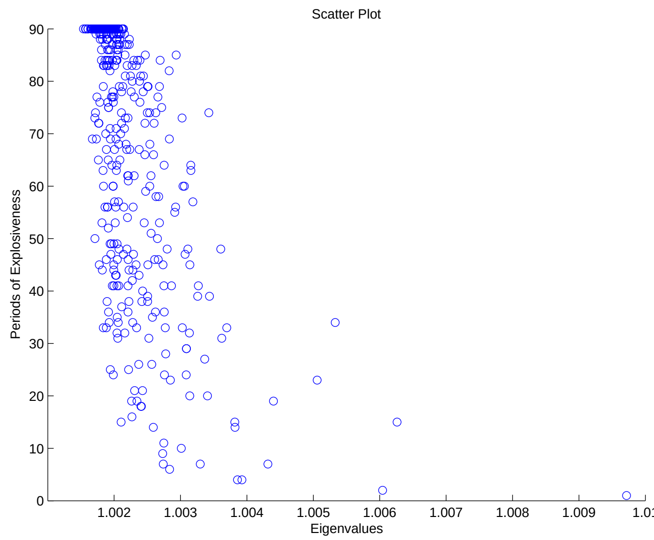

Figure 2: Persistence in the first experiment

issues we highlight here therefore seem very much relevant for the current economic environment.

To check whether or not our results hinge on the specific signal-to-noise structure inherent in our calibration, we next turn to a calibration where we set the standard deviation of the monetary policy error ϵ_t^R to half of the value in our benchmark calibration. This will increase the information value of observations. Nonetheless, as can be seen in Figure 5, the nature of perceived equilibria is basically unchanged. Agents learn somewhat faster that the true equilibrium is stable, but the difference from the standard calibration is not large. These similarities carry over to other equilibrium outcomes, which we omit here for the sake of brevity. Thus, even with a substantially smaller noise term in the policy rule, we still get the same results - inflation in this experiment just does not vary enough to make agents update their beliefs faster.[20] To assess the impact of the information structure in our model relative to the standard approach, we end this section by showing how the impulse responses to positive one standard deviation shocks under the standard approach (titled 'RE' in the figure since the standard approach assumes full information rational expectations after the policy change) to the response under the actual law of motion (titled 'ALM' in the figure) three periods after the policy change with our benchmark calibration.[21] For the sake

[20] In the appendix we show that only once we almost let the noise term vanish do we get much faster learning. In that case, the standard deviation of the noise term in the monetary policy equation is 1/20th of our benchmark number. Even with such a small noise term, learning is not immediate, though.

[21] The impulse responses are constructed assuming the the ALM will not change in the future. Given that learning in this scenario is slow, this assumption seems reasonable to us. Plotting the responses after 10 periods gives very similar pictures.

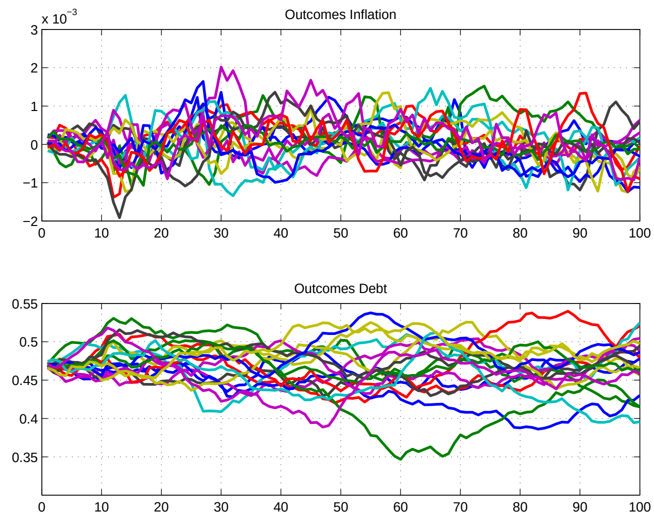

Figure 3: Inflation and debt sample paths in the first experiment

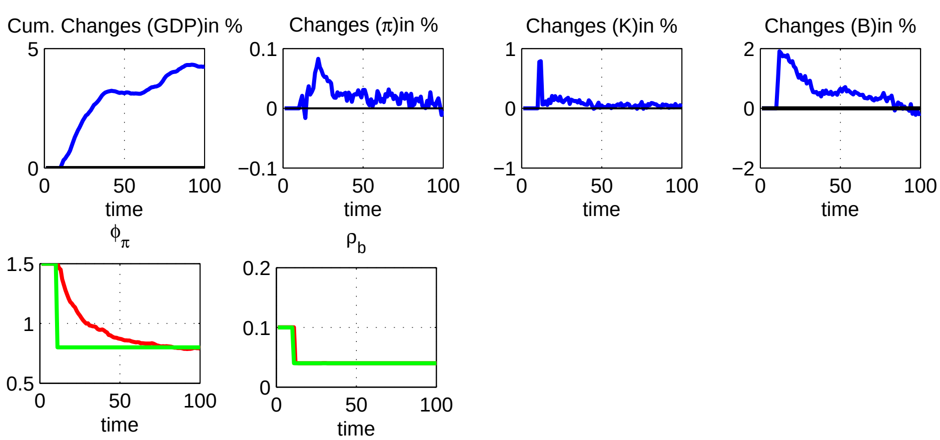

Figure 4: Summary of outcomes for the first experiment

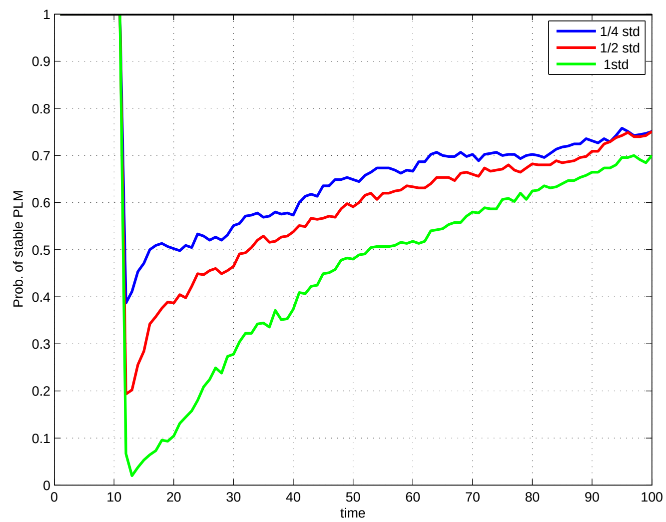

Figure 5: Perceived equilibria in the first experiment when monetary policy error standard deviation is reduced by 50 %

of brevity, we focus on the median response across simulations for quarterly inflation (plotted in annualized percentages) in figure 6 and the log output in deviations from steady state (multiplied by 100) in figure 7.

The responses to shocks under the actual law of motion are substantially different from those under the standard assumption. Since the new policy regime implies passive monetary policy, we get the perverse, but well known result that inflation increases with a positive monetary policy shock under the standard assumption on policy changes (Lubik and Schorfheide (2003)). This is not the case under learning exactly because agents misperceive the nature of the equilibrium. Quantitatively, there are also stark differences - government spending shocks have a substantially larger effect on output under learning. We want to emphasize again that since learning is slow in this scenario, these effects persist for long periods of time.

6.2 A Switch From Passive to Active Monetary Policy - Revisiting Volcker

Figure 8 shows the probability that the perceived equilibrium of the private agents is determinate. The colored lines give the probabilities for the different sizes of perceived policy changes. We see that, depending on the prior of the size of the policy change, initially the probability that agents know that the policy rules actually imply a unique

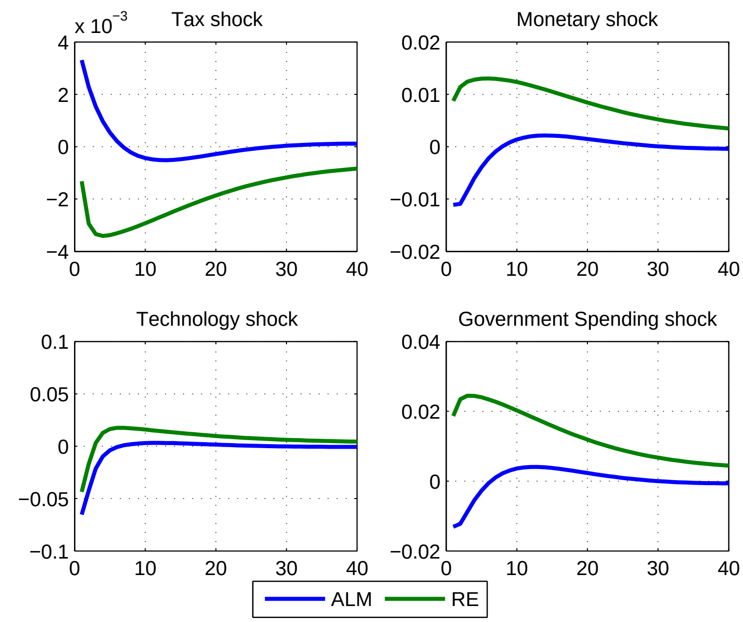

Figure 6: Impulse responses for inflation to various shocks.

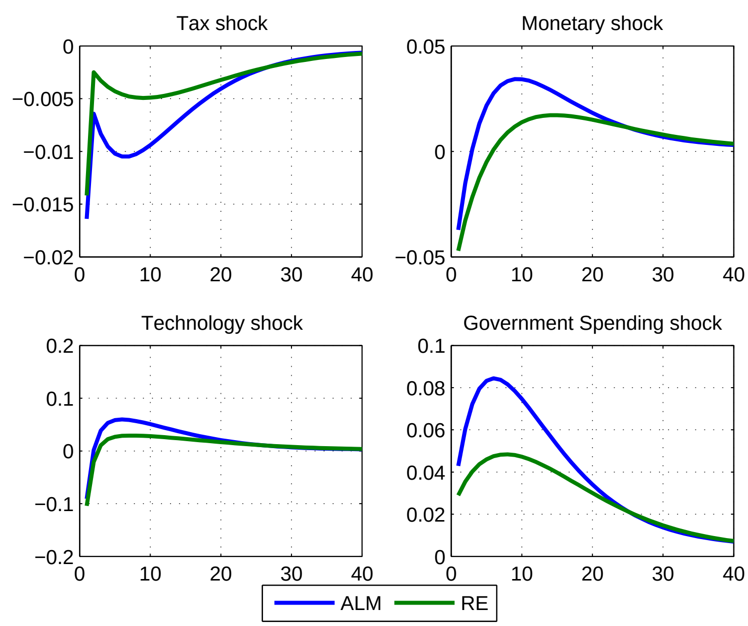

Figure 7: Impulse responses for output to various shocks.

equilibrium after the policy change is only between 20% and 50%. Furthermore, learning about the nature of the equilibrium is slow. Before we turn to figuring out why this is, we want to point out that agents in our simulations for this scenario never think that they are in a situation in which the equilibrium is unstable - therefore the complement of the probability plotted in Figure 8 is indeed the probability that the agents think the equilibrium is still indeterminate.
Does the fact that agents never think that the equilibrium is unstable imply that the actual equilibrium dynamics are stable? Figure 9 answers this. It plots for each simulation the largest eigenvalue in absolute value of $\mathbf{F}(\mathbf{\Omega_t})$ encountered in that simulation versus the number of periods for which an eigenvalue of that matrix is larger than 1 in absolute value. First, we see that large eigenvalues are pervasive in this scenario. All simulations have at least one period with eigenvalues larger than 1. Second, the eigenvalues, while larger than 1, are not substantially larger than 1, so the economy does not explode within a few periods. Rather, eigenvalues of this magnitude imply random walk-like behavior. Third, there are many simulations that feature large eigenvalues for substantial periods of time - the probability of random walk-like behavior is substantial. And finally, there is a negative relationship between the size of the largest eigenvalue during a simulation and the number of periods for which the simulation feature random walk-like behavior. This last feature can be explained by simple econometrics: The larger the eigenvalue is, the more volatile the economy tends to be. If there is more volatility, then the agents can more easily identify the post-policy change parameters since the right-hand side variables in their models become more volatile. But how can it be that the economy features random walk-like behavior when the perceived law of motion is stable? The logic behind this result is given in Cogley *et al.* (2015) (that paper does not focus on the size of the eigenvalues, however, and instead focuses on solving for an optimal monetary policy rule.) and can also be seen by revisiting the equation determining $\mathbf{F}(\mathbf{\Omega_{t|t-1}})$, where the second term on the right hand side features the difference between actual and estimated structural form dynamics. If those differences are large enough, explosive behavior can occur even if the first term on the right-hand side of that equation alone would imply stationarity.
Figure 10 shows the equilibrium outcomes in this scenario, which are constructed just as in picture 4.

We can see that sizeable differences arise for GDP, where the differences are most visible. Agents learn about the monetary policy coefficient slowly - inflation is not volatile enough to make them learn faster. Nonetheless, differences are -maybe surprisingly- small. Even under rational expectations, our model features substantial persistence

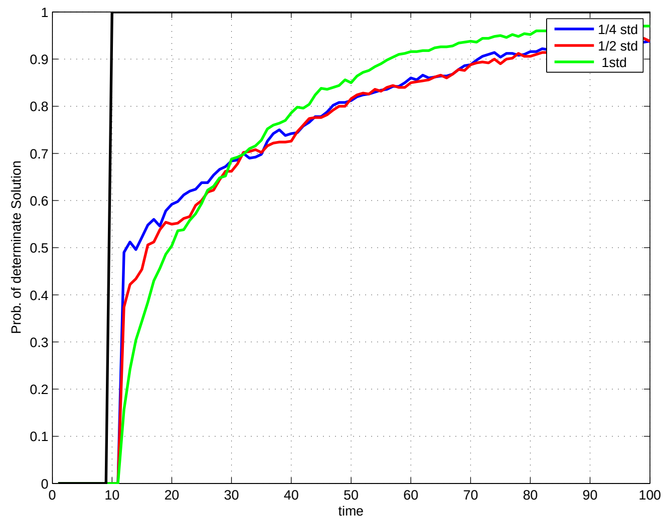

Figure 8: Perceived equilibria in the ‘Volcker’ experiment

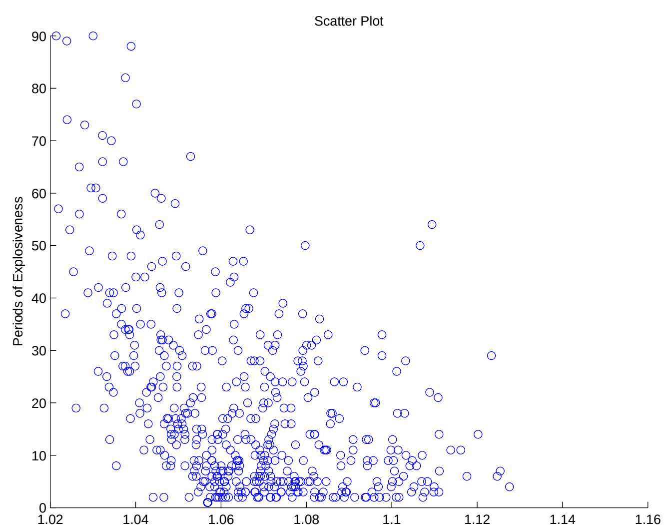

Figure 9: Persistence in the ‘Volcker’ experiment

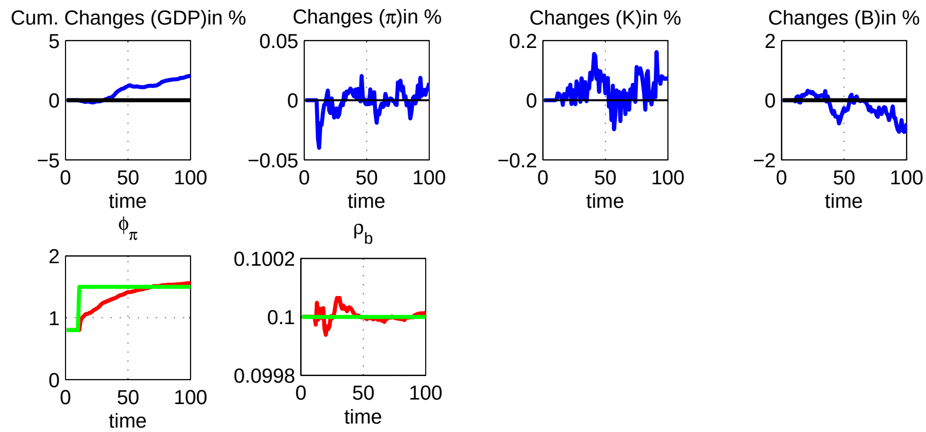

Figure 10: Summary of outcomes for the 'Volcker' experiment

in many variables, so the emergence of random walk-like behavior does not lead to vastly different outcomes in this experiment.[22] Also, even though we do not explicitly model credible policy communication in this paper, our findings highlight that policy communication could be useful if it speeds up learning of the agents.

7 Conclusion

This paper analyzes different modeling strategies for the analysis of discrete changes in economic policy. We show that removing immediate knowledge of the new policy rule from agents can alter the nature of the perceived equilibrium. Persistent, random walk-like behavior occurs *endogenously* when agents have to learn about policy changes. The resulting equilibrium dynamics can make learning the true policy rule coefficients difficult. It is important to remember that the agents in our models are sophisticated econometricians - the properties of observed data in equilibrium make learning hard, not the use of unsophisticated econometric techniques.

[22]This is also evidenced by the standard deviations of the variables (which we omit here for the sake of brevity) under the two approaches - they are not very different from each other for this scenario.

References

Andolfatto, D. and Gomme, P. (2003). Monetary Policy Regimes and Beliefs. *International Economic Review*, **44**(1), 1–30.

Baker, S. R., Bloom, N., and Davis, S. J. (2012). Has economic policy uncertainty hampered the recovery? Technical report.

Bhattarai, S., Lee, J. W., and Park, W. Y. (2012). Monetary-Fiscal Policy Interactions and Indeterminacy in Postwar US Data. *American Economic Review*, **102**(3), 173–78.

Bianchi, F. (2013). Regime Switches, Agents Beliefs, and Post-World War II U.S. Macroeconomic Dynamics. *Review of Economic Studies*, **80**(2), 463–490.

Bianchi, F. and Ilut, C. (2013). Monetary/Fiscal Policy Mix and Agents' Beliefs. Cepr discussion papers, C.E.P.R. Discussion Papers.

Bianchi, F. and Melosi, L. (2013). Dormant shocks and fiscal virtue. In *NBER Macroeconomics Annual 2013, Volume 28*, NBER Chapters. National Bureau of Economic Research, Inc.

Calvo, G. (1983). Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics*, **12**, 383–398.

Christiano, L., Eichenbaum, M., and Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a shock to Monetary Policy. *Journal of Political Economy*, **113**(1), 1–45.

Clarida, R., Gali, J., and Gertler, M. (1999). The Science of Monetary Policy: A new-Keynesian Perspective. *Journal of economic Literature*, **37**, 1661–1707.

Cogan, J. F., Cwik, T., Taylor, J. B., and Wieland, V. (2010). New Keynesian versus old Keynesian government spending multipliers. *Journal of Economic Dynamics and Control*, **34**, 281–295.

Cogley, T. and Sargent, T. J. (2005). Drift and volatilities: Monetary policies and outcomes in the post WWII U.S. *Review of Economic Dynamics*, **8**(2), 262–302.

Cogley, T., Matthes, C., and Sbordone, A. M. (2015). Optimized Taylor rules for disinflation when agents are learning. *Journal of Monetary Economics, forthcoming*.

Curdia, V. and Finocchiaro, D. (2013). Monetary regime change and business cycles. *Journal of Economic Dynamics and Control*, **37**(4), 756–773.

Drautzburg, T. and Uhlig, H. (2011). Fiscal Stimulus and Distortionary Taxation. NBER Working Papers 17111, National Bureau of Economic Research, Inc.

Eusepi, S. and Preston, B. (2011). Learning the fiscal theory of the price level: Some consequences of debt-management policy. Journal of the japanese and international economies, forthcoming.

Eusepi, S. and Preston, B. (2013). Fiscal foundations of inflation: imperfect knowledge. Staff Reports 649, Federal Reserve Bank of New York.

Giannitsarou, C. (2006). Supply-side reforms and learning dynamics. *Journal of Monetary Economics*, **53**(2), 291–309.

Hollmayr, J. and Matthes, C. (2015). Learning about fiscal policy and the evolution of policy uncertainty. *Journal of Economic Dynamics & Control*, **59**, 142–162.

Kim, J., Kim, S., Schaumburg, E., and Sims, C. A. (2003). Calculating and using second order accurate solutions of discrete time dynamic equilibrium models. Finance and Economics Discussion Series 2003-61, Board of Governors of the Federal Reserve System (U.S.).

Kreps, D. (1998). *Anticipated Utility and Dynamic Choice*, pages 242–274. Frontiers of Research in Economic Theory. Cambridge University Press.

Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of Monetary Economics*, **27**(1), 129–147.

Leeper, E. M. and Zha, T. (2003). Modest policy interventions. *Journal of Monetary Economics*, **50**(8), 1673–1700.

Liu, Z., Waggoner, D. F., and Zha, T. (2011). Sources of Macroeconomic Fluctuations: A Regime-Switching DSGE approach. *Quantitative Economics*, **2**(2), 251–301.

Lubik, T. A. and Schorfheide, F. (2003). Computing sunspot equilibria in linear rational expectations models. *Journal of Economic Dynamics and Control*, **28**(2), 273–285.

Lubik, T. A. and Schorfheide, F. (2004). Testing for Indeterminacy: An Application to U.S. Monetary Policy. *American Economic Review*, **94**(1), 190–217.

Milani, F. (2007). Expectations, learning and macroeconomic persistence. *Journal of Monetary Economics*, **54**(7), 2065–2082.

Primiceri, G. (2005). Time varying structural vector autoregressions and monetary policy. *Review of Economic Studies*, **72**(3), 821–852.

Sargent, T., Williams, N., and Zha, T. (2006). Shocks and government beliefs: The rise and fall of American inflation. *American Economic Review*, **96**(4), 1193–1224.

Sargent, T. J. (2015). Points of Departure. mimeo, nyu.

Sargent, T. J. and Williams, N. (2005). Impacts of priors on convergence and escapes from Nash inflation. *Review of Economic Dynamics*, **8**, 360–391.

Sims, C. A. (1994). A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy. *Economic Theory*, **4**(3), 381–99.

Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, **97**(3), 586–606.

Traum, N. and Yang, S.-C. S. (2011). Monetary and fiscal policy interactions in the post-war U.S. *European Economic Review*, **55**(1), 140–164.

Appendix

FOCs and Log-linearized Equation

A First-Order Conditions

Households

$$
\begin{aligned}
\lambda_t &= U_t^b \left(C_t - hC_{t-1}\right)^{-\sigma} \\
\lambda_t &= \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \\
(1-\tau_t)R_t^k &= \psi'(V_t) \\
Q_t &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[(1-\tau_{t+1})R_{t+1}^k V_{t+1} - \psi(V_{t+1}) + (1-\delta)Q_{t+1}\right] \\
1 &= Q_t\left[1 - \Gamma\left(\frac{I_t}{I_{t-1}}\right) - \Gamma'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] \\
&+ \beta E_t\left[Q_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\Gamma'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]
\end{aligned}
$$

Firms

$$
\begin{aligned}
W_t &= (1-\alpha)\frac{Y_t MC_t}{L_t} \\
R_t^k &= \alpha\frac{Y_t MC_t}{K_t} \\
0 &= E_t\left[\sum_{s=0}^{\infty}(\beta\omega_p)^s\lambda_{t+s}\bar{y}_{t+s}\left[\bar{p}_t\prod_{k=1}^{s}\left[\left(\frac{\pi_{t+k-1}}{\pi_{ss}}\right)^{\chi_p}\left(\frac{\pi_{ss}}{\pi_{t+k}}\right)\right]-(1+\eta_{t+s}^P)MC_{t+s}\right]\right] \\
\bar{y}_{t+s} &= \left(\bar{p}_t\prod_{k=1}^{s}\left[\left(\frac{\pi_{t+k-1}}{\pi_{ss}}\right)^{\chi_p}\left(\frac{\pi_{ss}}{\pi_{t+k}}\right)\right]\right)^{-\frac{1+\eta_{t+s}^P}{\eta_{t+s}^P}} \\
1 &= \left[(1-\omega_P)\bar{p}_t^{\frac{1}{\eta_t^P}}+\omega_P\left[\left(\frac{\pi_{t-1}}{\pi_{ss}}\right)^{\chi_P}\left(\frac{\pi_{ss}}{\pi_t}\right)^{\frac{1}{\eta_t^P}}\right]\right]^{\eta_t^P} \\
0 &= E_t\left[\sum_{t=0}^{\infty}(\beta\theta_w)^s\lambda_{t+s}\bar{L}_{t+s}\left[\tilde{w}_t\prod_{k=1}^{s}\left(\frac{\pi_{ss}}{\pi_{t+k}}\right)-\frac{(1+\eta_t^w)\psi\bar{L}_{t+s}^{\xi}}{(1-\tau_{t+s})\lambda_{t+s}}\right]\right] \\
\bar{L}_{t+s} &= \left[\tilde{w}_t\prod_{k=1}^{s}\left(\frac{\pi_{ss}}{\pi_{t+k}}\right)\right]^{-\frac{1+\eta_t^w}{\eta_t^w}} L_{t+s} \\
w_t^{\frac{1}{\eta_t^w}} &= (1-\theta_w)\tilde{w}_t^{\frac{1}{\eta_t^w}}+\theta_w\left[\left(\frac{\pi_{ss}}{\pi_t}\right)w_{t-1}\right]^{\frac{1}{\eta_t^w}}
\end{aligned}
$$

B Log-Linearized Model

Households

$$
\begin{aligned}
log(\lambda_t) &= Const_C + log(u_t^b) - \frac{\sigma}{1-h}log(c_t) + \frac{\sigma h}{1-h}log(c_{t-1}) \\
log(\lambda_t) &= Const_\lambda + log(R_t) + E_t log(\lambda_{t+1}) - E_t log(\pi_{t+1}) \\
log(r_t^k) &= Const_V + \frac{\psi}{1-\psi}log(v_t) + \frac{\tau_{ss}}{1-\tau_{ss}}log(\tau_t) \\
log(q_t) &= Const_Q + E_t log(\lambda_{t+1}) - log(\lambda_t) + \beta(1-\tau_{ss})R_{ss}^k E_t log(r_{t+1}^k) - \beta\tau_{ss}R_{ss}^K E_t log(\tau_{t+1}) \\
&\quad +\beta(1-\delta)E_t log(q_{t+1}) \\
log(k_t) &= Const_K + log(v_t) + log(k_{t-1}) \\
log(\bar{k}_t) &= Const_{\bar{K}} + (1-\delta)log(\bar{k}_{t-1}) + \delta(log(u_t^i) + log(i_t)) \\
(1+\beta)log(i_t) &- \frac{1}{s}(log(q_t) + log(u_t^i)) - \beta E_t log(i_{t+1}) = log(i_{t-1}) + Const_I
\end{aligned}
$$

Firms

$$
\begin{aligned}
log(y_t) &= Const_{Agg} + \frac{C_{ss}}{Y_{ss}}log(c_t) + \frac{I_{ss}}{Y_{ss}}log(i_t) + \frac{G_{ss}}{Y_{ss}}log(g_t) + \frac{\psi'(1)K_{ss}}{Y_{ss}}log(v_t) \\
log(y_t) &= Const_Y + log(a_t) + \alpha log(k_t) + (1-\alpha)log(l_t) \\
log(\pi_t) &= Const_\pi + \frac{\beta}{1+\chi^p\beta}E_t log(\pi_{t+1}) + \frac{\chi^p}{1+\chi^p\beta}log(\pi_{t-1}) + \kappa_p log(mc_t) + \kappa_p log(\eta_t^p) \\
log(mc_t) &= Const_{MC} + \alpha log(r_t^k) + (1-\alpha)log(w_t) - log(a_t) \\
log(r_t^k) &= Const_{R^K} + log(l_t) - log(k_t) + log(w_t) \\
log(w_t) &= Const_W + \frac{1}{1+\beta}log(w_{t-1}) + \frac{\beta}{1+\beta}E_t log(w_{t+1}) \\
&- \kappa_w \left[log(w_t) - \nu log(l_t) - log(u_t^b) + \log(\lambda_t) - \frac{\tau_{ss}}{1-\tau_{ss}}log(\tau_t)\right] + \frac{\chi^w}{1+\beta}log(\pi_{t-1}) \\
&- \frac{1+\chi^w\beta}{1+\beta}log(\pi_t) + \frac{\beta}{1+\beta}E_t log(\pi_{t+1}) + \kappa_w log(\eta_t^w)
\end{aligned}
$$

Policy Rules and Shocks

$$
\begin{aligned}
log(b_t) &+ \tau_{ss}\frac{W_{ss}L_{ss}}{B_{ss}}\left(log(\tau_t) + log(w_t) + log(l_t)\right) + \tau_{ss}\frac{R_{ss}^k K_{ss}}{B_{ss}}\left(log(\tau_t) + log(r_t^k) + log(k_t)\right) \\
&= Const_B + \frac{1}{\beta}log(R_{t-1}) + \frac{1}{\beta}log(b_{t-1}) - \frac{1}{\beta}log(\pi_t) + \frac{G_{ss}}{B_{ss}}log(g_t) + \frac{Z_{ss}}{B_{ss}}log(z_t) \\
log(g_t) &= Const_G + \rho_G log(g_{t-1}) + \epsilon_t^G \\
log(z_t) &= Const_Z + \rho_Z log(z_{t-1}) + \epsilon_t^Z \\
log(\tau_t) &= Const_\tau + \rho_b log(b_{t-1}) + \epsilon_t^\tau \\
log(R_t) &= Const_R + \alpha log(\pi_{t-1}) + \epsilon_t^R \\
log(a_t) &= Const_A + \rho_A log(a_{t-1}) + \epsilon_t^A \\
log(u_t^i) &= Const_u i + \rho_{u^i} log(u_{t-1}^i) + \epsilon_t^{u^i} \\
log(u_t^b) &= Const_u b + \rho_{u^b} log(u_{t-1}^b) + \epsilon_t^{u^b} \\
log(\eta_t^p) &= Const_{\eta_p} + \rho_{\eta^p} log(\eta_{t-1}^p) + \epsilon_t^{\eta^p} \\
log(\eta_t^w) &= Const_{\eta_w} + \rho_{\eta^w} log(\eta_{t-1}^w) + \epsilon_t^{\eta^w}
\end{aligned}
$$

with the constants given by [23]

[23] $Const_\tau$ and $Const_R$ are the implied constants in the respective policy rules.

Constant	Expression
$Const_G$	$log(G_{ss})(1-\rho_G)$
$Const_Z$	$log(Z_{ss})(1-\rho_Z)$
$Const_\tau$	$log(\tau^L_{ss}) - \rho_b log(B_{ss})$
$Const_R$	$log(R_{ss}) - \phi_\pi log(\pi_{ss})$
$Const_{u^i}$	$log(U^i_{ss})(1-\rho_{u^i})$
$Const_{u^b}$	$log(U^b_{ss})(1-\rho_{u^b})$
$Const_{\eta^w}$	$log(\eta^w_{ss})(1-\rho_{\eta^w})$
$Const_{\eta^p}$	$log(\eta^p_{ss})(1-\rho_{\eta^p})$
$Const_B$	$log(B_{ss})(1-\frac{1}{\beta}) + \tau_{ss}\frac{L_{ss}W_{ss}}{B_{ss}}(log(\tau_{ss}) + log(W_{ss}) + log(L_{ss})) + \tau_{ss}\frac{K_{ss}R^k_{ss}}{B_{ss}}(log(\tau_{ss}) + log(R^k_{ss}) +$
	$log(K_{ss})) - \frac{1}{\beta}log(R_{ss}) + \frac{1}{\beta}log(\pi_{ss}) - \frac{G_{ss}}{B_{ss}}log(G_{ss}) - \frac{Z_{ss}}{B_{ss}}log(Z_{ss})$
$Const_Y$	$log(Y_{ss}) - log(A_{ss}) - (1-\alpha)log(L_{ss}) - \alpha log(K_{ss})$
$Const_A$	$log(A_{ss})(1-\rho_A)$
$Const_{Agg}$	$log(Y_{ss}) - \frac{C_{ss}}{Y_{ss}}log(C_{ss}) - \frac{G_{ss}}{Y_{ss}}log(G_{ss}) - \frac{I_{ss}}{Y_{ss}}log(I_{ss}) - \frac{R^k_{ss}(1-\tau_{ss})K_{ss}}{Y_{ss}}log(\upsilon_t)$
$Const_\pi$	$(1 - \frac{\beta}{(1+\chi^p beta)} - \chi^p(1+\beta\chi^p))log(\pi_{ss}) - \kappa_p log(mc_{ss}) - \kappa_p log(\eta^p_{ss})$
$Const_{Lam}$	$-log(R_{ss}) + log(\pi_{ss})$
$Const_Q$	$(1-\beta(1-\delta))log(Q_{ss}) - \beta(1-\tau_{ss})R^K_{ss}log(R^K_{ss}) + \beta\tau_{ss}R^K_{ss}log(\tau_{ss})$
$Const_{R^K}$	$log(R^K_{ss}) - log(W_{ss}) - log(L_{ss}) + log(K_{ss})$
$Const_W$	$(1+\kappa_w - \frac{1}{(1+\beta)} - \beta(1+\beta))log(W_{ss}) - \kappa_w \nu log(L_{ss}) - \kappa_w log(U^b_{ss}) + \kappa_w log(\lambda_{ss}) -$
	$-\kappa_w\frac{\tau_{ss}}{(1-\tau_{ss})}log(\tau_{ss}) - \kappa_w log(\eta^w_{ss}) - (\frac{\chi^w}{(1+\beta)} - \frac{(1+\beta\chi_w)}{(1+\beta)} + \frac{\beta}{(1+\beta)})log(\pi_{ss})$
$Const_I$	$-\frac{1}{s}log(Q_{ss}) - \frac{1}{s}log(U^i_{ss})$
$Const_{MC}$	$log(mc_{ss}) - \alpha log(R^K_s s) - (1-\alpha)log(W_{ss}) + log(A_{ss})$
$Const_L$	$(1-\alpha)log(L_{ss}) + \alpha log(K_{ss}) + log(A_{ss}) - log(Y_{ss})$
$Const_C$	$(\frac{\sigma}{1-h} - \frac{\sigma h}{1-h})log(C_{ss}) + log(\lambda_{ss}) - log(U^b_{ss})$
$Const_V$	$\frac{\psi}{1-\psi}log(V_{ss}) - log(R^K_{ss}) + \frac{\tau_{ss}}{(1-\tau_{ss})}log(\tau_{ss})$
$Const_K$	$log(K_{ss}) - log(V_{ss}) - log(\bar{K}_{ss})$
$Const_{\bar{K}}$	$\delta log(\bar{K}_{ss}) - \delta log(U^i_{ss}) - \delta log(I_{ss})$

C Parameters

Calibrated parameters of benchmark DSGE model

Description	Parameter	Value
impatience	β	0.99
CES utility consumption	σ	1
CES utility labor	ϕ	1.4
Level shifter labor	ν	3.19
habits	h	0.8
Capital intensity	α	0.33
Depreciation rate	δ	0.025
Price indexation	χ_p	0.26
Wage indexation	χ_w	0.35
Calvo prices	θ_p	0.9
Calvo wages	θ_w	0.79
Inv. adjustment cost parameter	γ	4
Capital utilization cost param.	ψ	0.34
Steady state tax rate	τ_{ss}	0.32
coeff. on inflation in TR	ϕ_π	1.5
coeff. on B in tax rules	ρ_b	0.1
AR parameter transfer rule	ρ_z	0.34
AR parameter gov. spending	ρ_g	0.97
AR parameter technology	ρ_a	0.35
AR parameter price mark-up	ρ_{η^p}	0.69
AR parameter wage mark-up	ρ_{η^w}	0.42
AR parameter preference	ρ_{u^b}	0.86
AR parameter investment	ρ_{u^i}	0.87
Std.deviation technology	σ_a	0.69
Std.deviation gov. spending	σ_g	0.15
Std.deviation transfers	σ_z	0.91
Std.deviation tax	σ_τ	0.24
Std.deviation interest rate	σ_r	0.25
Std.deviation investment	σ_{u^i}	0.35
Std.deviation preference	σ_{u^b}	0.38
Std.deviation price mark-up	σ_{η^p}	0.065
Std.deviation wage mark-up	σ_{η^w}	0.21

Table 2: Calibrated parameters of the model

where $\kappa_p = [(1-\beta\theta_p)(1-\theta_p)]/[\theta_p(1+\beta\chi_p)]$ and
$\kappa_w = [(1-\beta\theta_w)(1-\theta_w)]/[\theta_w(1+\beta)(1+\frac{1+\eta^w\nu}{\eta^w})]$

D Substantially smaller noise in the monetary policy rule

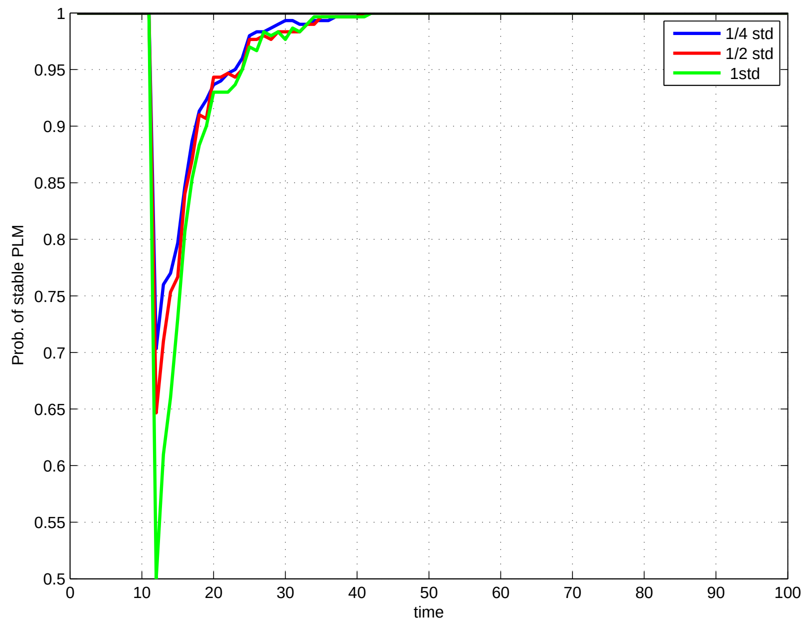

Figure 11: Perceived equilibria in the second experiment when monetary policy error standard deviation is 1/20th of the original value.