The Consumption Origins of Business Cycles: Lessons from Sectoral Dynamics

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Abstract

We measure the impact of household consumption shocks on aggregate fluctuations. These shocks affect household consumption directly, and production and prices indirectly through their impact on aggregate consumption. We show how to identify such shocks using prior knowledge of their differential impact across sectoral variables. Shocks independently affecting household consumption demand have accounted for around 35% of business cycle fluctuations since the mid-1970s, playing a central role in recessions within that period. The inferred household consumption shock series correlates well with measures of changes in consumer confidence and household wealth.

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1 Introduction

Household consumption accounts for more than two-thirds of GDP. Accordingly, cash transfers to households are often used to mitigate recessions. Nevertheless, in canonical business cycle models household consumption decisions play a role mainly as a propagation channel for shocks generated elsewhere. We use cross-sectoral information to show that this is an important omission. We define a consumption shock as one that affects aggregate consumption directly and then propagates to the rest of the economy, for example, because firms may want to reduce their output and hire fewer workers in response to lower consumer demand. One example of such a shock is a deterioration in household expectations or "sentiments", another would be a sudden and exogenous reduction in household wealth. Other sources of a consumption shock with similar effects would include a shock to consumer credit or employment uncertainty. We find that, in combination, such shocks have accounted for close to 35% of output fluctuations since the mid-1970s.

Our findings are made possible by an identification strategy that allows us, with minimal structural assumptions, to use rich cross-sectional information to infer aggregate responses to shocks. We show that, given a large enough panel dataset, one can identify the time-path of an aggregate shock based on information of its differential impact on cross-sectional observations. To efficiently use this identification strategy, we devise a new time series model that can handle large cross-sections of data while allowing for substantial heterogeneity across sectors.

We find that the identified household consumption demand shock generates not only a significant impact on aggregate consumption but, more interestingly, also on GDP. The response of other aggregate variables is furthermore consistent with the typical characteristics of "aggregate demand" shocks: an increase in inflation and the interest rate. At the same time, the impact on corporate credit spreads and measured TFP is small so that the shock is distinct from a productivity or corporate credit shock. Overall, we find that shocks to household consumption have accounted for close to 35% of output fluctuations at business cycle frequencies from 1973 till 2017, and an increasing fraction of output declines in recessions within that period. Those shocks had a maximal impact in the 2008 recession, where they accounted for close to 70% of the shortfall relative to projected output.

We show how we can identify the consumption shock based on information on its marginal effect on a large enough panel of economic variables. Specifically, we propose to capture its effect on sectoral prices and quantities by a sector's degree of specialization in the production of goods and services consumed by households. For example, apparel manufacturing, which mostly caters to households will, all else equal, react more to a household demand shock than software, which also caters to firms and government.

Our identification strategy is designed to be robust to a number of issues. First, we take into account that some industries are more likely to react to all macroeconomic shocks, for example because they produce durables or luxury goods. Second, our procedure can identify consumption shocks correctly even if there is another shock that has a correlated cross-sectoral impact. This is because, as is usual in business-cycle analysis, shocks are assumed to be independent over time. Third we explicitly take into account that our identification assumption can only hold as an approximation. For example, reactions may also depend on inter-sectoral linkages, the differing intensity of various frictions, and other factors. We formally account for this lack of precision in identification assumptions by casting them in terms of uncertain prior distributions within a Bayesian setting, so that our estimates reflect both estimation and identification uncertainty. We are able to obtain reliable estimates in spite of that because of the use of rich cross-sectional data.

In order to be able to make the best use of detailed cross-sectoral data, we need a tractable econometric methodology that allows us to estimate aggregate and idiosyncratic dynamics jointly and efficiently. We accomplish that with the use of a Hierarchical Vector Autoregression (Hi-VAR). Given that framework, we can then use established time-series techniques to identify the dynamic response of the economy to the identified shocks, their relevance in explaining the variance of aggregate variables, and their role in particular historical episodes.

How should we interpret the estimated household consumption shocks? To provide additional interpretation, we compare the consumption shock that we infer to time series not used in estimation. Specifically, we show that fluctuations in our inferred shock line up well with fluctuations in household wealth and with a survey-based measure of consumer sentiment. The correlation with household wealth is especially salient around the Great Recession, as one might expect, whereas the correlation with consumer sentiment holds consistently over time. Together those exercises suggest a role for both shocks to household wealth and household sentiment as central driving forces in business cycles.

More generally, we verify that the consumption shock that we identify can be interpreted as a shock to the Euler equation. We make this case both in a general theoretical framework and through a specific calibrated multi-sector DSGE model. We verify that, in such a model, a discount factor shock, which is one example of a consumption shock, has a larger impact on sectors with high consumption shares. We then use the same model to inspect the effects of a TFP news shock and find that those do not generate the same type of pattern. This distinction implies that, to the extent that we are identifying expectational shocks, those are not the news shocks commonly used in the literature. We find that, in a realistic calibration, the posterior median for the shock recovered by our methodology is highly correlated with a shock to the discount factor.

We extend our methodology to identify several types of shocks simultaneously. While not strictly necessary for identification of the consumption shock, in doing this, we can directly measure the importance of the consumption against alternative sources of fluctuations. For our baseline analysis, we identify six structural shocks explicitly. To guard ourselves against misspecification, we further add three shocks for which we do not impose any identifying restrictions. These three additional shocks do not turn out to be important quantitatively.

Apart from the shock to household consumption, we identify shocks to technological progress, government consumption, monetary policy, corporate financial conditions, and energy cost. To identify the first three shocks, we rely on input or demand intensity shares that can be read directly from input-output tables. Furthermore, we identify corporate credit shocks by exploiting heterogeneity in external financial dependence measures as in Rajan and Zingales (1998), and monetary shocks using sectoral price stickiness data from Nakamura and Steinsson (2008). Together, we find that those six shocks can account for most of the output fluctuations at business cycle frequencies. Consumption (35%) accounts for the largest share. Credit (18%), government consumption (14%), and energy (11%) play a significant if secondary, role. Lastly, monetary and technology shocks play a relatively minor role (5.6% and 7.5%, respectively). In a robustness exercise in Appendix M.6, we further consider an investment shock identified in an analogous manner, but do not find that it changes results in meaningful ways.[1]

Since at least the 1990s, consumption shocks have been recognized as playing a potentially important roles in business cycles (Blanchard (1993), Hall (1993)). Blanchard (1993) and Hall (1993) focus in particular on the role of consumption shocks in the 1990-1991 recession, which we also find to be largely driven by consumption shocks. In the Great Recession, consumption fluctuations in response to the housing bust have been identified as a primary driving force (Mian and Sufi, 2015). We provide evidence that consumption shocks have been an important contributor to business cycles at least since the 1970s.

Our evidence is consistent with prior findings by Cochrane (1994), who finds that shocks to technology, monetary policy, credit conditions, and energy prices cannot explain the bulk of business cycle fluctuations. He then proposes that "consumption shocks" can be important drivers of economic fluctuations. Where we differ is that Cochrane (1994) equates consumption shocks with news shocks, while our consumption shocks have a broader interpretation in some aspects, but also does not identify standard news shocks (which we show in section 2).

The shock that we identify can be interpreted as a shock to an aggregate consumption Euler equation. Such a shock can emerge from fluctuations in aggregate disaster risk (Gourio, 2012) and idiosyncratic income risk (Werning, 2015), among others, so long as they act primarily through the consumption Euler equation, without other direct supply effects. Similar shocks have also been incorporated as an element in estimated DSGE models (Smets and Wouters, 2003). Relative to

[1] The investment shock we identify may not align with the one identified by Justiniano et al. (2010). This is because Justiniano et al. (2011) argues that the investment shock may be partially driven by financial market disturbances, which we separately capture with our financial shock. In the model estimated in Appendix M.6, the two shocks together explain 18.4 % of the variance of output (this is the posterior mean of this fraction).

those economic model-based approaches, we identify consumption shocks using fewer identifying restrictions.

The importance of consumption decisions is reflected in the usefulness of consumer sentiment indices for forecasting (Matsusaka and Sbordone, 1995). The recognition of this fact has given rise to explorations of consumer sentiment indices as a relevant source of information about frictions in expectation formation (see Barsky and Sims (2012) and Bhandari, Borovička, and Ho (2019)). As mentioned above, however, our identification assumption does not capture regular TFP expectations shocks.

In recent years the interconnections between household wealth, consumption, and employment have become the object of a rapidly expanding literature on quantitative models with heterogeneous agents, summarized in Krueger, Mitman, and Perri (2016). Some of those approaches build in feedbacks from consumption decisions to employment through the use of new Keynesian frictions (Kaplan, Moll, and Violante, 2018). Such frictions matter because they allow shocks that primarily affect consumption (or the household Euler equation) to generate co-movement between output, hours, investment, and consumption (Basu and Bundick, 2017). Our results suggest that further work may do well to concentrate on shocks that emerge within the detailed consumption block implied by those models.

At different points in time, economists have been interested in evaluating the relative importance of "demand" vs. "supply shocks". Classic approaches include a priori long-run restrictions (Blanchard and Quah (1989) and Gali (1999)). More recently, Angeletos, Collard, and Dellas (2020) and Bachmann and Zorn (2013) have argued that demand shocks are the dominant driver of output growth fluctuations in the US and Germany, respectively. Apart from relying on an altogether different source of identification, our methodology singles out household consumption as a particularly relevant source of demand shocks. However, it cannot be characterized as the main business cycle shock as it accounts for less than half of GDP fluctuations and, in fact, does not correlate well with the shock identified by Angeletos et al. (2020). More broadly interpreted, our results suggest that business cycles are best explained as stemming from a variety of shocks, with the consumption shock being a particularly important one.

Our methodological innovations are twofold: First, with our tractable time series model, we provide a method to use a *large* number of variables (we use all PCE sectors in the United States at a low level of aggregation) while emphasizing tractability and parsimony. As a result, we can add a large number of relatively "soft" (i.e., non-dogmatic) identification restrictions that add up to precise estimates. Second, we propose a new identification strategy that exploits the large number of variables we can use in our time series model. We show how to create a large number of identification restrictions using insights from general equilibrium models with sectoral heterogeneity. This allows our approach to side-step the identification issues pointed out by Wolf (2020) for VARs with standard aggregate variables and number of sign restrictions.[2] We build on Amir-Ahmadi and Drautzburg (2020), who add sign and magnitude restrictions on selected sectoral responses to identify aggregate shocks processes in a standard VAR framework, and ingeniously show that this can lead to substantially improved identification. De Graeve and Karas (2014) also make the case for using information on the relative magnitude of the responses to shocks to help identify shocks in standard VARs. Compared to these papers, our econometric approach is able to handle much larger panels. We also lever the restrictions for a very different economic question and use very different identification assumptions.

In not imposing "hard" identifying restrictions, our approach also connects more generally to the use of sign restrictions, pioneered by Uhlig (2005), Faust (1998), Canova and Nicolo (2002), and Rubio-Ramirez, Waggoner, and Zha (2010). We also build on papers that propose using Bayesian priors instead of hard identification restrictions (Kociecki (2010) and Baumeister and Hamilton (2015)). [3]

Our approach adds to efforts to find a "general-purpose" methodology for identification that researchers can apply in a broad range of contexts. For example, recently Gabaix and Koijen (2020) proposed to use weighted averages of idiosyncratic shocks as instruments in various settings.[4] It is also related to the extensive literature on Bartik instruments (Bartik (1991)) in applied microeconomics. We share with this approach the insight that differential exposure to aggregate shocks can be a powerful tool for identification (Goldsmith-Pinkham et al. (2018)). The Hierarchical VAR that we use adds to the existing suite of time series models designed to incorporate large panels, including dynamic factor models (Stock and Watson (2005a)), factor augmented VARs (Bernanke et al. (2005), Boivin et al. (2009)), and global VARs (Chudik and Pesaran (2016), Holly and Petrella (2012)).

More broadly, the paper also contributes to the general trend within macroeconomics of using cross-sectional data to inform inference on questions of relevance to macroeconomists (Holly and Petrella (2012), Beraja et al. (2016), Sarto (2018), Chen et al. (2018), and Guren et al. (2019), for example). In the terms laid out by Nakamura and Steinsson (2018), it highlights that the impact elasticities of cross-sectional units to particular aggregate shocks are especially relevant "portable" moments. The use of rich cross-sectional data allows for the use of minimal structural assumptions,

[2] While we use sectoral data to gain more information for identification, another approach to add identification restrictions on aggregate data alone is to use zero restrictions and sign restrictions jointly along the lines of Arias et al. (2018) and Arias et al. (2019).

[3] Also, Schwartzman (2014) and Fulford and Schwartzman (2015) use cross-sectional information to identify shocks. Whereas the first paper uses a structural small open economy model, the second paper leverages the cross-sectional impact of a shock identified from a historical narrative.

[4] The underlying idea is that shocks specific to 'large' idiosyncratic units can have sizeable aggregate effects. In our setting we would call these shocks aggregate shocks (one example is the energy price shock that we model) even if they emanate from one sector.

which are, furthermore, allowed to be uncertain.

The paper proceeds as follows: In Section 2, we define the consumption shock, provide the basic propositions that establish our identification method, and discuss the specifics of how we implement it using the information on sectoral consumption and output. Section 3 provides the details of the Hi-VAR econometric model used to infer aggregate dynamics from the cross-sectional data. Section 4 provides the results, Section 5 interprets the inferred consumption shock in light of information not used in the estimation. Section 6 shows the result of a Monte Carlos study using a multi-sector DSGE model as a data generating process, cross-validates our methodology against the use of external instruments and provides a more detailed analysis of the role of priors and model fit.

2 The Consumption Shock

We now describe in detail how we define the consumption shock, and how we can use cross-sectoral variation to estimate it. We discuss how to apply knowledge of the marginal effects of shocks for identification. Then we show how to explicitly build in uncertainty surrounding those marginal effects, and that these identification restrictions can nonetheless be very powerful given the use of rich panel data. Finally, we present details on how we implement our identification strategy, and provide validation against a multi-sector DSGE model.

2.1 Theory

We start our discussion by defining the shock to household consumption. Armed with this definition, we show how this shock can be backed out from data given knowledge of its impact on a cross section of economic variables. Later we allow for uncertainty in this impact, and show how a large cross section of sectoral variables can guide the estimation. The discussion clarifies that the shock can be interpreted as a composite of various sources of fluctuations that emerge first in the household sector and propagate from there to the rest of the economy primarily through consumption expenditures. Those might include shocks to household credit, to the expectations of households, or to household income risk, so long as those are not a reflection of broader economic shocks. To keep the notation transparent, we start with a simple case where those shocks only affect aggregate consumption. We later extend our discussion to a more general case where consumption shocks can be interpreted as equivalent to a discount rate shock, affecting primarily current consumption decisions, but also potentially other choices made by households.

Consider a log-linearized economic system, in which innovations to different variables can be expressed as:[5]

[5] In Section 3 we show how a system of this form maps into the time series model that we use for inference.

$$c_t - E_{t-1}c_t = \sum_{s\in\mathscr{C}} \frac{\partial c}{\partial \varepsilon_s}\varepsilon_{s,t} + \sum_{s\notin\mathscr{C}} \frac{\partial c}{\partial \varepsilon_s}\varepsilon_{s,t}$$
$$\mathbf{x}_t - E_{t-1}\mathbf{x}_t = \frac{\partial \mathbf{x}}{\partial c}(c_t - E_{t-1}c_t) + \sum_{s\notin\mathscr{C}} \frac{\partial \mathbf{x}}{\partial \varepsilon_{\mathbf{s}}}\varepsilon_{s,t} + \mathbf{w}_t$$

where c_t is the log deviation of aggregate consumption at time t from its steady-state, $\mathbf{x}$ is a vector including log deviations of other variables in the economy, including sectoral prices and quantities, $\varepsilon_{s,t}$ are macroeconomic shocks of interest, $\mathscr{C}$ is the set of consumption shocks, $\mathbf{w}_t$ are shocks specific to each element of $\mathbf{x}_t$. As usual, we assume that aggregate shocks are drawn independently over time and from each other.

The key assumption is that innovations to consumption depend on a set of shocks, $s \in \mathscr{C}$ that do not affect any other variables directly. In this context, the consumption shock is the linear combination of those innovations, $\varepsilon_t^C \equiv \sum_{s\in\mathscr{C}} \frac{\partial c}{\partial \varepsilon_{s,t}}\varepsilon_{s,t}$. A straightforward substitution allows us to express innovations to $\mathbf{x}_t$ as a function of exogenous shocks only:

$$\mathbf{x}_t - E_{t-1}\mathbf{x}_t = \frac{\partial \mathbf{x}}{\partial c}\varepsilon_t^C + \sum_{s\notin\mathscr{C}} \theta_s^x \varepsilon_{s,t} + \mathbf{w}_t \tag{1}$$

where now $\theta_s^x \equiv \frac{\partial \mathbf{x}}{\partial \varepsilon_{\mathbf{s,t}}} + \frac{\partial \mathbf{x}}{\partial c}\frac{\partial c}{\partial \varepsilon_{s,t}}$. It follows that the vector of loadings of the consumption shock on the variables $\mathbf{x_t}$ is given by $\frac{\partial \mathbf{x}}{\partial c}$, which is the key object in our analysis since it encodes the effects of changes in consumption.

Equation 1 can be read as the observation equation of a state-space representation of the economy, with the shocks ε_t^C, $\varepsilon_{s,t}(s \notin \mathscr{C})$ and $\mathbf{w}_t$ representing the unobservable state variables, and innovations to $\mathbf{x}_t$ as the observable data – in our application those are estimated using the VAR dynamics of the raw data. Because, by assumption, the unobservable components ε_t, $\mathbf{w}_t$ are *iid*, the state equation is simply given by the identity

$$s_t = [\varepsilon_t \ \mathbf{w}_t]' \tag{2}$$

where s_t is the state-vector and ε_t is a vector that collects ε_t^C and $\varepsilon_{s,t}(s \notin \mathscr{C})$ so that $\varepsilon_t \sim_{iid} N(0,\mathbf{I})$. Equation (1) can then straightforwardly be rewritten as a linear function of the state vector s_t so that the system is cast in conventional state-space form.

Casting the model as state-space representation allows us to use the information on $\frac{\partial \mathbf{x}}{\partial c}$ to infer the time path of ε_t^C as well as its contribution to the variance of various economic variables using the Kalman filter. In effect, in Proposition 1 below, we show that information on $\frac{\partial \mathbf{x}}{\partial c}$ and the covariance matrix of innovations to $\mathbf{x}_t$ are sufficient to recover the least squares projection of ε_t^C on current and past observables. If ε_t^C are Gaussian, those are also their maximum likelihood estimates. It is a key building block of the estimation algorithm described in detail in Section 3.

Proposition 1 *Consider the state-space representation encoded in the observation equation* (1) *and the state equation* (2)*. The least squares projection of* ε_t^C *based on current and past observables (obtained using the Kalman Filter) only depends on* $\frac{\partial \mathbf{x}}{\partial c}$ *and the covariance matrix of* $\mathbf{x}_t - E_{t-1}\mathbf{x}_t$*, irrespective of initial conditions for the state.*

The proposition states that one can infer the shock based on two pieces of information: The loadings of a set of observed variables on that shock and the covariance of those variables. Importantly, the estimate *does not* depend on the loadings of innovations to $\mathbf{x}_t$ on other shocks. The proposition also holds irrespective of initial conditions for the state because the state is *iid*, so that the Kalman estimates of the state at each point in time is independent of their previous values.[6]

Proposition 1 holds because, by design, the Kalman filter separates the overall movements of an observable variable into movements driven by an unobserved state variable (in our case, the consumption shock multiplied by its effect on each variable, $\partial \mathbf{x}/\partial c \times \varepsilon_t^C$) and a "noise" term. The latter is typically identified with measurement error and defined to be orthogonal to the state variable. In the current context, this noise term includes the effect of other macroeconomic shocks. Those shocks can be treated as noise because, by a standard assumption in macroeconomics, they are orthogonal to the consumption shock.[7]

To fix ideas further, suppose there were two macroeconomic shocks of interest, the consumption shock, defined as above, and a shock to the financial system. A shock to the financial system would affect the non-consumption variables in the economy directly by reducing the supply of credit to non-financial firms. It could also have a substantial impact on consumption through a reduction in the supply of consumer credit. Suppose we had an estimator that erroneously attributed the part of the financial shock that propagates through consumption to the consumption shock, while the part that affects production directly through the credit supplied to non-financial firms remained attributed to a financial shock. Then, the two misidentified shocks would be correlated. It follows that the Kalman filter estimate would not make this erroneous attribution and would identify the shocks correctly. We give a detailed proof in appendix A as well as a Monte Carlo demonstration in the appendix I.

A second result motivates the use of rich cross-sectoral data for shock identification. Specifically, as one might expect, increasing the dimension of the $\mathbf{x}_t$ vector included in the estimation will improve the precision of our estimates.

[6] Moreover, this implies that filtered estimates are identical to smoothed estimates of the state.

[7] This is a significant difference with the approach in Fulford and Schwartzman (2015), that requires restrictions on factor loadings associated with other shocks. The reason such restrictions are not necessary here is because the Kalman filter also exploits the information in the covariance matrix of $\mathbf{x}_t$ and the restriction that macroeconomic shocks are independent of each other.

Proposition 2 *Suppose the state-space system described by equation (1) and (2) satisfies the assumptions in Section 4 of Bai and Ng (2008). Suppose further that the sample size is large in the time dimension ($T \to \infty$, where T is the number of $\mathbf{x}_t$ observations). The estimation error disappears as the dimensionality of $\mathbf{x}_t$ goes to infinity.*

The proof builds on the result, stated by Bai and Ng (2008), that under certain conditions it is possible to consistently estimate the space spanned by the aggregate shocks ε_t. Given this space, one then easily back out the values for ε_t^C by projecting the part of each observation explained by the aggregate shocks on $\frac{\partial \mathbf{x}}{\partial c}$.[8]

The results above make clear that the identification of the consumption shock only requires the effects of various economic variables and the covariance matrix between innovations to different cross-sectional units. It is straightforward to see that the propositions apply more broadly to any shock. In the language of Nakamura and Steinsson (2018), the marginal effects of a shock on a large panel of economic variables, possibly including several sectors or regions, would be the "portable" statistic useful to identify the time-path of the shock and its aggregate effects.

2.1.1 Discount rate shocks

One may legitimately ask whether meaningful economic systems of the form expressed above exist. Aggregate consumption may also depend on other variables in complicated ways, so that isolating a consumption shock may not be realistic. Fortunately, as shown in Werning (2015), in a large class of economic systems, including many with heterogeneous consumers and incomplete markets, the set of variables determined jointly with aggregate consumption is small. In fact, Werning shows that for those models one can write a "generalized" Euler equation which, log-linearized in innovation form, would be expressed as:

$$c_t - E_{t-1}c_t = -\phi\left(r_t - E_{t-1}r_t\right) + E_t c_{t+1} - E_{t-1}c_{t+1} + \varepsilon_t^C + \sum_{s \notin \mathscr{C}} \frac{\partial c}{\partial \varepsilon_s}\varepsilon_{s,t}$$

where r_t is the interest rate faced by households. Now, the consumption shock effectively acts like a shock to the household's discount factor. This implies that not only innovations to consumption, but also to the interest rate faced by households and revisions to future consumption will depend on the consumption shock ε_t^C. In this more general setting, $\mathbf{x}_t$ becomes:

[8] In our empirical implementation there will be additional uncertainty about ε_t^C because we allow for uncertainty about $\frac{\partial \mathbf{x}}{\partial c}$.

$$
\begin{aligned}
\mathbf{x}_t - E_{t-1}\mathbf{x}_t = &\frac{\partial \mathbf{x}}{\partial c}(c_t - E_{t-1}c_t) \\
&+ \frac{\partial \mathbf{x}}{\partial Ec}(E_t c_{t+1} - E_{t-1}c_{t+1}) \\
&+ \frac{\partial \mathbf{x}}{\partial r}(r_t - E_{t-1}r_t) \\
&+ \sum_{s \notin \mathscr{C}} \frac{\partial \mathbf{x}}{\partial \varepsilon_{\mathbf{s}}} \varepsilon_{s,t} + \mathbf{w}_t
\end{aligned}
$$

so that the loading of the consumption shock ε_t^C on $\mathbf{x}_t$ becomes $\frac{\partial \mathbf{x}}{\partial c}\frac{\partial c_t}{\partial \varepsilon_t^C} + \frac{\partial \mathbf{x}}{\partial Ec}\frac{\partial Ec_{t+1}}{\partial \varepsilon_t^C} + \frac{\partial \mathbf{x}}{\partial r}\frac{\partial r_t}{\partial \varepsilon_t^C}$.

The exercise raises the possibility that consumption shocks may affect non-consumption variables through the effect of those shocks on interest rates and expected future consumption. Likewise, richer models may also include effects on labor supply and other household choices. For those and other reasons we allow for "errors" in the sensitivities, by encoding them through non-degenerate prior distributions rather than dogmatic restrictions.

We discuss in detail how we use data on ratios between consumption to gross-output to identify the consumption shocks in section 3.3, after presenting the details of the econometric framework in the coming Section.

3 Estimation: The Hierarchical VAR model

We now describe in detail the full econometric framework used to obtain the results in the paper. The framework allows us to jointly measure innovations to aggregate and sectoral variables, identify aggregate shocks, and estimate impulse response functions, variance, and historical decompositions of the impact of those shocks on different variables. Specifically, we combine a VAR-type time series model (Sims, 1980) for a vector of aggregate variables Y_t with autoregressive models for vectors of sectoral data X_t^j, where t indicates time and i indicates the sector. Aggregate and sectoral data interact in two ways: (i) via structural shocks that affect both types of data and (ii) via direct feedback from (lagged) aggregate data to sectoral data. We describe each of these blocks in turn. We follow up with an in-depth discussion of how the model ought to be interpreted and describe in detail how the model is estimated. We conclude the section with a comparison with other approaches.

3.1 Modeling Aggregate Variables

We model aggregate variables as following a linear vector autoregressive process. A key difference from traditional VARs is that we break the link between forecast errors and structural shocks, thus allowing sectoral data to help identify structural shocks. The aggregate variable vector Y_t (of dimension N by 1) is a function of its past values, structural shocks ε_t, and other shocks w_t:

$$Y_t = \mu + \sum_{l=1}^{L} A_l Y_{t-l} + D\varepsilon_t + w_t \tag{3}$$

where ε_t is of dimension $S \times 1$, and D is an $N \times S$ matrix encoding the impact of the Gaussian structural shocks ε on the aggregate variables, and w_t is a independently and identically distributed $N \times 1$ vector of mean 0, non-structural Gaussian shocks with covariance matrix Ω. We further assume that $\varepsilon_t \sim_{iid} N(0,I)$.[9] As will be clear later, we can allow for $S < N$, $S = N$, or $S > N$, whereas standard VAR analyses require $S \leq N$.

For later discussion, it is useful to note that the one-step ahead forecast error for the aggregate level is given by $D\varepsilon_t + w_t$, whereas a standard VAR model for the aggregate variables would assume that any estimate of a structural shock is a linear combination of the vector of aggregate one-step-ahead forecast errors.[10]

3.2 Modeling Sectoral Variables

There are observations for I disaggregated units (such as industries, regions, or, in our specific application, sectors) with K variables (such as prices and quantities) each. The law of motion for the data from unit i, summarized in the K-dimensional vector X_t^j, is given by:

$$X_t^j = \mu^j + \sum_{l=1}^{L^X} B_l^j X_{t-l}^j + \sum_{l=1}^{L^Y} C_h^j Y_{t-l} + D^j \varepsilon_t + w_t^j \tag{4}$$

where D^j is a $K \times S$ matrix encoding the impact of shocks ε on the idiosyncratic variables (i.e. it collects the coefficients $D_{k,s}^j$ discussed in section 3.3) and the mean zero Gaussian vector w_t^j incorporates the impact of idiosyncratic (or non-structural) shocks on individual units. We denote the covariance matrix of w_t^j by Ω^j. We assume that w_t^j is independent across i and independent from w_t. Our assumptions on the correlation structure of w_t^i allow for correlation in the innovations to different variables within a sector, although not across sectors.

3.3 Identifiying the Consumption Shock

The theoretical discussion in Section 2 makes clear how one can recover the time-path of the consumption shock given knowledge about the impact of that shock on a panel of economic

[9] The distributional assumptions are necessary because we ultimately want to carry out Bayesian inference, for which we need to build a likelihood function.

[10] This is true even if fewer than N shocks are identified, as is common in the literature on sign restrictions in VARs.

variables. In order to implement those procedures we need two objects: (i) A panel of innovations to a large number of variables ($\mathbf{x}_t - E_{t-1}[\mathbf{x}_t]$) and (ii) the "loading" of those innovations on the aggregate shock, $\frac{\partial \mathbf{x}}{\partial c}$ (or $\frac{\partial \mathbf{x}}{\partial c}\frac{\partial c_t}{\partial \varepsilon_t^C} + \frac{\partial \mathbf{x}}{\partial Ec}\frac{\partial Ec_{t+1}}{\partial \varepsilon_t^C} + \frac{\partial \mathbf{x}}{\partial r}\frac{\partial r_t}{\partial \varepsilon_t^C}$ in the case of a shock to the Euler equation).

The problem of obtaining the innovations to different variables is an econometric problem that we tackled in Section 3 above.[11] Precise estimates of the required loadings are hard to obtain. For example, it could require obtaining an instrument for the consumption shock, for which there is currently no clear candidate. Otherwise, one could try to derive those loadings from a structural model, but this would raise the question as to whether the model is correctly specified. To make progress, we use the fact that, in a wide range of models, *relative to overall sensitivity of a sector to aggregate shocks, the marginal effect of a consumption shock on sectoral variables depends on the share of sectoral output that is sold to households*. This is our key identification assumption. As an illustration, we show that this assumption holds for a prototypical multi-sector equilibrium model in Appendix C.

At the same time, this dependence is admittedly imprecise. For that reason, we use Bayesian methods to make this lack of precision explicit and allow it to affect our estimates and statements about our uncertainty surrounding those estimates. In formal terms, we postulate prior distributions for the marginal effects of shocks. The prior means depend on cross-sectional information that we describe below, and the prior variances denote our degree of uncertainty around our assumptions. This use of "soft" prior restrictions for identification was proposed by Baumeister and Hamilton (2015), and contrasts with traditional approaches, which achieve identification by setting hard constraints on the shock process. Relative to that previous work, we can estimate a large scale model tractably by using a Gaussian prior directly for the marginal impact of the structural shocks.

3.3.1 Probing the identification approach

The Cross-Section of Consumption Intensity and Business Cycles We now probe into our key identification assumption, linking the cross-section of consumption intensity of sectors and business cycles.

Table 1 depicts the sectors with the largest and smallest fraction of their output sold to households.[12] The top sector is men's and boys' clothing. It has a ratio of consumption to gross output that is larger than one since a sizeable fraction is imported. Sectors at the top include education (such as elementary and daycare schools), and at the bottom include equipment and machinery (such as cookware and tableware light trucks), and business services (such as employment agency services).

To obtain a rough sense of the relevance of those sectoral differences in predicting business

[11] In practice, the parameters of the model that allows us to filter out innovations are estimated jointly with the shocks.
[12] We use PCE sectors. Details can be found in Appendix B.2.

Rank (out of 187 sectors)	Sector name	C/Y value
Top 1	Men's and boys' clothing	1.30
Top 10	Elementary and secondary schools	1.00
Top 20	Day care and nursery schools	0.98
Top 40	Other video equipment	0.81
Top 60	New domestic autos	0.66
Median (top 94)	Cereals	0.55
Bottom 60	Photographic equipment	0.43
Bottom 40	Other fuels	0.28
Bottom 20	Nonelectric cookware and tableware	0.16
Bottom 10	Employment agency services	0.01
Bottom 1	Used light trucks	0.00

Table 1: Top and bottom sectors, by the ratio of consumption to gross output

cycles, we calculate the difference in 12 month output and consumption growth, and inflation in the top 40 sectors by consumption orientation as compared to the bottom 40. Figure 1 shows the correlation between those differences with year on year output growth at different lags and leads. We also include the correlation between aggregate consumption growth and aggregate output growth and of output with itself for further reference. The figure shows that consumption tends to lead output by a little, but that the difference between sectors in different points in the cross-section leads output by a greater amount. While only a rough test of predictive value, this picture suggests that this particular way of looking at the cross-section of sectors has value as a lens through which to understand output fluctuations.

Our Identification Assumption in an Equilibrium Model We can further verify that our prior assumption is sensible in the context of a fully specified multi-sector extension of the medium-scale New-Keynesian model of Justiniano et al. (2010) (described in detail in Appendix D and further explored in Section 6.1).[13] We examine the impacts of a discount factor shock (which affects household consumption directly through the Euler equation), a monetary shock and a news shock.

Panel (a) of figure 2 shows the relationship between the consumption-gross output ratio implied by the model calibration for each sector and the immediate impact of a shock to the Euler equation on output in each sector. It confirms that in the context of this canonical business cycle framework, the impact of the discount shock on output does increase with the consumption-gross output ratio, albeit imperfectly.

The Euler equation can also be affected by news shocks (see, for example, Schmitt-Grohé and

[13] When quantifying the equilibrium model, we allow for 52 sectors calibrated to match US inter-sectoral linkages. This is just for numerical efficiency. In our empirical application we use the full 187 sectors mentioned earlier.

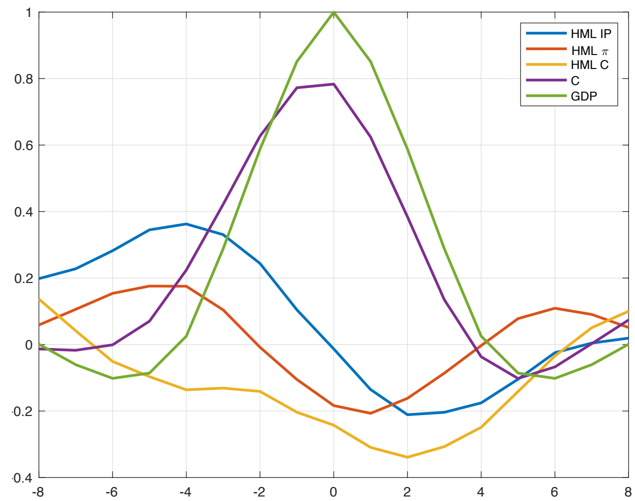


Figure 1: Correlation with lags of GDP

The horizontal axis refers to the quarterly lag of GDP, with negative numbers corresponding to leads. HML IP is the difference between the FRB Industrial Production index for high and low consumption share sectors. HML π and HML C refer to the same difference for inflation and consumption growth among BEA personal consumption expenditure categories.

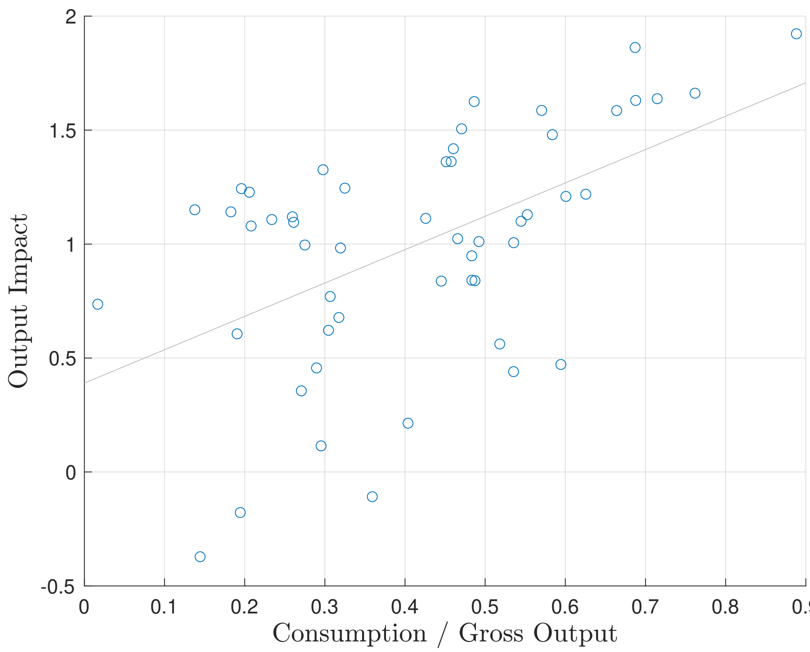
Uribe (2012)) and monetary shocks. However, those shocks affect current production, consumption and prices, through other channels. For example, in the case of news shock, it also boosts investment and production in upstream sectors. Monetary policy shocks will likewise have a more pervasive impact in the economy. In the end, those other effects lead to a very different pattern of impact. In particular, monetary and news shocks that have a *positive* impact on output will have the opposite correlation pattern of an discount rate shock that also has such a positive impact, as shown in panels (b) and (c) of figure 2.[14] Our identification assumption thus does generally not mistake news shocks or monetary shocks for consumption shocks.

3.3.2 Implementing the Identification of Consumption Shocks

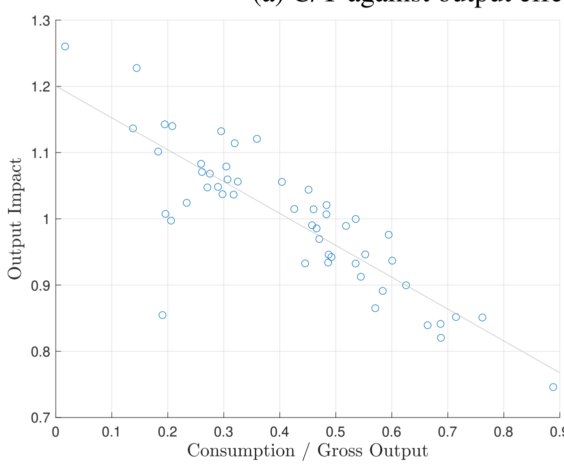
We now describe how we implement the identification of the Consumption Shocks in practice. We denote the marginal effect on impact of a shock s to a variable k in sector j by $D^j_{k,s}$. To set the prior mean for $D^j_{k,s}$, we decompose the prior mean as follows:[15]

[14] We model a news shock in the evolution of Total Factor Productivity. Details can be found in Appendix D.

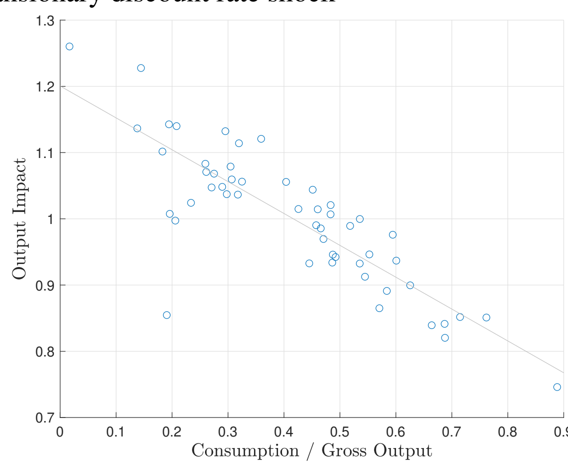
[15] We work with the squared prior mean here (i) to focus on pinning down the magnitudes of the prior mean in this step (economic theory will then help us pin down the sign of the prior mean) and (ii) because we find it easier to work at the level of contributions to the overall variance since equation (6) gives us information about those contributions.



(a) C/Y against output effect of expansionary discount rate shock



(b) C/Y against output effect of expansionary monetary shock



(c) C/Y against output effect of expansionary tfp news shock

Figure 2: Sectoral Consumption/Gross output ratios plotted against the sectoral output effect on impact of different shocks in calibrated multi-sector DSGE model (see Appendix C)

$$(E\left[D_{k,s}^{j}\right])^{2}=\gamma_{k}^{j}\beta_{k,s}\alpha_{k,s}^{j} \tag{5}$$

$$\left(E\left[D_{k}^{i}\right]\right)^{2}=\sum_{s=1}^{S}\left(E\left[D_{k,s}^{i}\right]\right)^{2} \tag{6}$$

where

1. $\alpha_{k,s}^{j}$ is a measure of the *relative* impact of shock s on variable k *for sector i as compared to other sectors*. We encode in this component the notion that, the more a sector sells of its output directly to households, the more sensitive it is to household consumption shocks.[16]

2. $\beta_{k,s}$ is a measure of the *overall* impact of shock s on variable k *across all sectors*. We use an 'ignorance prior,' and set this variable to $1/S$, where S is the number of structural shocks that we will allow for in our implementation.

3. γ_{k}^{j}: a measure of the overall sectoral sensitivity of variable k in sector i to all shocks. For example, this variable encodes the notion that consumption of durable goods is overall more sensitive to all shocks than the consumption of nondurables.

Given $\alpha_{k,s}^{j}$ and $\beta_{k,s}$, we can back out γ_{k}^{j} if we have values for $\sum_{s=1}^{S}(E\left[D_{k,s}^{j}\right])^{2}$. Those correspond to the portion of the variance of individual variables explained by the aggregate shocks. They do not depend on the identification of specific shocks and can be estimated with conventional factor-analytic tools. In particular, we estimate those by estimating the model described in Section 3 below in a training sample with agnostic priors.[17] This procedure above thus allows us to set a magnitude for the prior mean $E\left[D_{k,s}^{j}\right]$. We use *a priori* information on the sectoral impact of shocks to set the sign.

As an example of what this procedure achieves, consider a scenario where we only have two shocks (named shock 1 and 2), and one observable per sector. Also, to cut down on notation define $\tilde{D}_{k,s}^{i}\equiv[E(D_{k,s}^{i})]^{2}$ and $\tilde{\alpha}_{k,s}^{i}\equiv\beta_{k,s}\alpha_{k,s}^{i}$. Let's focus on one specific sector, sector a. There are three equations in three unknowns for that sector. We also drop the subscript k since there is only one

[16]To keep units consistent, we normalize our indicator variable, the fraction of output a sector sells directly to households, to be between 0 and 1. If there are missing values for the indicator variables for some sectors, we assume that the indicators for those sectors take on the average value of the relevant indicator. We normalize indicator variables for other shocks (which we discuss in Table 2) in the same way.

[17]The prior we use for the agnostic estimation is the same as for our actual estimation except that we use priors with large variances on the impact of the structural shocks and the residual covariances. For the choice of the training sample, our default is the full sample - our approach can, therefore, be interpreted as an empirical Bayes approach. When doing this we also use a very loose prior on the covariance matrix of the non-structural shocks

variable.

$$\tilde{D}_1^a = \gamma^a \tilde{\alpha}_1^a \tag{7}$$

$$\tilde{D}_2^a = \gamma^a \tilde{\alpha}_2^a \tag{8}$$

$$\tilde{D}^a = \tilde{D}_1^a + \tilde{D}_2^a \tag{9}$$

Adding the first two equations gives

$$\tilde{D}^a = \gamma^a \tilde{\alpha}_1^a + \gamma^a \tilde{\alpha}_2^a$$

and thus

$$\gamma^a = \frac{\tilde{D}^a}{\tilde{\alpha}_1^a + \tilde{\alpha}_2^a}$$

which in turns allows us to solve for the squared shock loadings:

$$\tilde{D}_1^a = \frac{\tilde{D}^a}{\tilde{\alpha}_1^a + \tilde{\alpha}_2^a}\tilde{\alpha}_1^a$$

Our procedure thus produces (squared) prior means that weight different shocks according to both the relevant indicator α and the overall impact of a shock on a variable, β. We pin down the sign of the prior mean by referring to predictions coming from economic theory.

By choosing normal priors, we do not necessarily force the sign restrictions to hold with certainty. Because of the normality assumption, the posterior mean might have a different sign from the prior mean. This probability depends on the prior variance. In our baseline estimates, we set it such that the prior standard deviation is $1/2 \times abs\left(E\left[D_{k,s}^j\right]\right)$, ensuring a wide band of uncertainty around our prior assumptions while stating that we expect the sign to hold with high probability. Appendix G provides descriptive statistics on the cross-sectoral distribution of prior on the impact of the consumption shock.

Lastly, we also need to set the prior mean for the impact of shocks on aggregate variables. We intentionally choose to not impose substantial prior information about the aggregate effects because we want the sectoral variables to inform our results on the effects of aggregate shocks. For the consumption shock, we use the minimal assumption that it tends to increase consumption while remaining agnostic on its impact on other aggregate variables. To be precise, we set the prior mean impact of the consumption shock on consumption innovations to be consistent with the overall variance of those innovations driven by aggregate shocks (which we obtain from our identification-agnostic estimation on a training sample). We further set the prior impact of the consumption shock on other aggregate variables to have mean zero, and a standard deviation of 0.25 (we use the same prior for the impact of the other structural shocks in our model on consumption). When implementing our empirical strategy, we will further constrain our approach by simultaneously identifying various shocks. The identification approach for those other shocks follows a similar structure as the identification of the consumption shock, as we discuss below.

3.4 Further implementation Details

We now explain how we set priors on other parameters and estimate the model.

3.4.1 Priors on D^j and D

We described the prior distributions for the impact coefficients D and D^j for the consumption shock in detail above. We use a similar procedure to add priors to the other five shocks: technology, credit supply, government consumption, monetary, and energy cost. Table 2 describes the aggregate and sectoral indicators used to construct the prior means for the impact matrices. The impacts on aggregate shocks are set similarly to the consumption shock, with the technology shock having a priori a positive impact on TFP, credit shocks having a positive impact on spreads, government shocks on government expenditures, monetary shocks on interest rates, energy shocks on the price of energy.

We describe the direction of impact on sectoral variables and the ranking for which variables are most affected in table 3. Those follow basic economic theory: the household consumption shock has a larger positive output and price impact on sectors with higher consumption to gross-output ratio and a smaller impact on consumption. The technology shock has a more positive quantity impact and more negative price impact in sectors with high R&D expenditures. Credit shocks reduce quantities and increase prices in sectors with high external dependence. Government shocks increase prices and output (and reduce consumption) in sectors with high government consumption. Monetary shocks increase prices by less and increase output and consumption by more in sectors where prices are stickier.[18] Energy shocks increase prices and reduce quantities by more in sectors that are more intensive in energy inputs.

In Appendix B.2 describes the data used to set those priors in detail. In Appendix C, we develop

[18] One may wonder whether the same would not be true of all demand shocks. The point, however, is that other demand shocks may have other stronger biases - for example, a shock to the discount factor would have a strong bias towards consumption intensive sectors that a monetary shock would not necessarily have. We check that intuition in the same structural model used in Sections 3.3.2 and 6.1. In particular, we examine the correlation pattern between the probability of prices staying in place and the cross-sectoral impact of the monetary shock on consumption, prices and output. As expected, the correlation between the sectoral effect of a contractionary monetary policy shock and sectoral price stickiness (as measured by the probability of a price remaining in place between periods) is 0.74 (recall that overall prices drop in response to the monetary shock), whereas for an contractionary consumption shock the correlation the same correlation is much weaker, at 0.19. The correlations with the impact in consumption are the same, but with flipped signs. There is also a notable, although less sharp, difference in the correlation between price stickiness and sectoral output impact, at -0.27 for the monetary policy shock but -0.13 for the consumptino shock.

a tractable two-period model that allows us to derive those relationships. We also introduce three additional structural shocks (elements of ε) to allow for the possibility that we do not explicitly model some important source of fluctuation. We use loose Gaussian priors centered at 0 for the corresponding elements of D and D^i. As we will see later, when we present a variance decomposition, these three additional shocks are not important drivers of the aggregate variables in our model.

	Positive aggregate impact	Index for sector-specific $\alpha^j_{k,s}$ in $E[D^j_{k,s}]$
Household	Household consumption	Household consumption / Gross output
Technology	TFP (Fernald, 2014)	R&D expenditures / Gross output
Credit	Baa-Treasury credit spread	External finance dependence
Government	Government consumption	Government consumption / Gross output
Monetary	Fed funds rate	Average price duration
Energy	Energy price index	Cost of energy inputs / Gross output

Table 2: Assumptions used for prior means for impact coefficients for different shocks. See Appendix C for detailed motivation

	PCE price	PCE quantity	Industrial Production
Household	$+\uparrow$	$+\downarrow$	$+\uparrow$
Technology	$-\downarrow$	$+\uparrow$	$+\uparrow$
Credit	$+\uparrow$	$-\downarrow$	$-\downarrow$
Government	$+\uparrow$	$-\downarrow$	$+\uparrow$
Monetary	$+\downarrow$	$+\uparrow$	$+\uparrow$
Energy	$+\uparrow$	$-\downarrow$	$-\downarrow$

Table 3: Signs and ranking of impact $\alpha^j_{k,s}$: $\uparrow$ implies that impact increases with sector-specific index for α^j_k in table 2 and $\downarrow$ that it decreases.

3.4.2 Priors on other parameters

For the intercepts μ^j at the sectoral and aggregate level, we use Gaussian priors with mean zero and large variances. For the A_l matrices (the VAR coefficients at the aggregate level), we use a Minnesota-type prior (Koop and Korobilis (2010)).[19] We do this because we have a relatively large number of observables at the aggregate level, so some prior shrinkage is useful. At the sectoral level, we have fewer variables (per sector), so we simply use priors for B^i_l and C^i_h centered at 0 with a standard deviation of 0.5.

To use the Gibbs sampler, we use inverse-Wishart priors for the covariance matrices Ω and Ω^j of the reduced form shocks at the aggregate and sectoral levels. As is well known, this imposes

[19] We use their benchmark choice of hyperparameters.

some restrictions on what prior beliefs we can impose on our model. One is that the variances are bounded away from 0 (really not much of a problem in our case), while the main problem is that there is no genuinely uninformative prior (as we increase the variance, we also have to at some point increase the prior mean since variances are bounded below by 0). To set this prior, we will use the results from an estimation of our model with a training sample and an agnostic prior, as also outlined in Table 4.[20]

Prior on Ω To set the prior for Ω, we use results from our agnostic prior estimation. We set the prior mean to the estimated posterior mean of Ω and use as degrees of freedom the size of our overall sample. Table 4 below summarizes the priors on the different parameters.

Prior on Ω^j For Ω^j, we follow the same strategy as for its aggregate counterpart Ω, except that we use a smaller number of degrees of freedom (there is less need for shrinkage as the number of variables per sector is smaller than the number of aggregate variables).

3.4.3 Gibbs Sampler

As mentioned before, we exploit the Gibbs sampler throughout by imposing independent Normal-inverse Wishart priors.

Drawing ε_t Given All Other Parameters We assume Gaussian innovations for tractability. If we use a variant of equations (3) and (4), it is straightforward to see that, conditional on A_l, B_l, C_l, Σ, D, and D^j, ε_t can be drawn via exploiting the Kalman filter (simply put all known quantities on the left-hand-side: all that remains on the right-hand side are the ε terms, w^j and w), based on Carter and Kohn (1994). To make this step more numerically efficient, we follow Durbin and Koopman (2012) and collapse the large vector of observables into a vector with the same dimension as the structural shocks. As discussed by Durbin and Koopman (2012), this can be done without loss of information.

Drawing Other Parameters Given ε Since we condition on ε at this stage, drawing all other parameters amounts to drawing from Gaussian and inverse Wishart posteriors. One helpful insight here is that, conditional on ε, all other blocks can be run in parallel. This means that our approach can be scaled up easily. This is especially useful for extensions where a researcher might want to depart from the Normal-inverse Wishart prior.

[20] As mentioned earlier, the prior we use for the agnostic estimation is the same as for our actual estimation except that we use priors with large variances on the impact of the structural shocks and the residual covariances. For the choice of the training sample, our default is the full sample - our approach can, therefore, be interpreted as an empirical Bayes approach.

Table 4: Summary of prior distributions

Parameters	Prior Density	Prior Parameters
μ, A_l	Normal	Minnesota prior as in Koop and Korobilis (2010)
Ω	Inverse Wishart	Mean: training sample Degrees of freedom: sample size
D, constrained elements	Normal	Mean and standard deviation: system of equations
D, unconstrained elements	Normal	Mean 0, standard deviation 0.25
μ^i,B_l^i, C_h^i	Normal	Mean 0, standard deviation 0.5 (each element)
Ω_i	Inverse Wishart	Mean: training sample, degrees of freedom = 15
D^i	Normal	Mean and standard deviation: system of equations

3.5 Interpretation and Comparison With Other Approaches

We now present a more detailed discussion of how to interpret the model, and a formal discussion of the relevant identification issues. This interpretation will also allow us to compare the model with other existing approaches.

To interpret the econometric model, it is useful to rewrite our model as follows: First, define the vector of all observables

$$Z_t = [Y_t{}' \; X_t^{1\prime} \; X_t^{2\prime} \; \dots \; X_t^{I\prime}]'$$

We can then recast our model in the following way:

$$Z_t = \mu^Z + \sum_{l=1}^{max(L^X,L^Y,L)} B_l^Z Z_{t-l} + \underbrace{D^Z \varepsilon_t + w_t^Z}_{u_t^Z} \tag{10}$$

where w_t^Z is a vector that stacks the non-structural shocks according to the ordering of observables in Z_t.

Our assumptions imply that the residuals u_t^Z are orthogonal to all variables in Z_{t-1}. This model can be mapped back into the system described in Section 2 - the conditional mean of the variables $(E_{t-1}\mathbf{x}_t)$ is given by $\mu^Z + \sum_{l=1}^{max(L^X,L^Y,L)} B_l^Z Z_{t-l}$, so that $u_t = \mathbf{x}_t - E_{t-1}\mathbf{x}_t$, and the impact of the consumption shock and other structural shocks is determined by $D^Z \varepsilon_t$, where each column of D^Z is equal to the effect of one shock on all variables. In particular, the column of D^Z corresponding to a consumption shock is equal to $\frac{\partial \mathbf{x}}{\partial c}$ (or the corresponding expression from Section 2.1.1).

3.5.1 Identification of model parameters

We now characterize the identification of model parameters and aggregate shocks. Because of the structure of the one-step-ahead forecast error, we can identify μ^Z as well as the coefficient matrices B_l^Z, as is usually the case in VAR analyses. Focusing on u_t^Z, we can see that it follows a factor structure, where the common factors are the *iid* structural shocks ε_t, so that standard results on identification in factor models apply (Bai and Ng (2008)). While we cannot identify the effects of individual structural shocks without additional assumption, we can identify the overall effect of *all* structural shocks.

To identify ε_t, we need identification restrictions akin to those used in the structural VAR literature. To see this, define

$$u_t = D\varepsilon_t + w_t, \tag{11}$$

$$u_t^j = D^j \varepsilon_t + w_t^j \ \forall i. \tag{12}$$

For any conformable orthogonal matrix Q, we can construct alternative models that feature the same first and second moments and thus the same Gaussian likelihood:

$$u_t = \underbrace{DQ^{-1}}_{\tilde{D}} \underbrace{Q\varepsilon_t}_{\tilde{\varepsilon}_t} + w_t \tag{13}$$

$$u_t^j = \underbrace{D^j Q^{-1}}_{\tilde{D}^j} \underbrace{Q\varepsilon_t}_{\tilde{\varepsilon}_t} + w_t^j \ \forall i. \tag{14}$$

It follows that, even though the overall impacts $D\varepsilon_t$ and $D^i\varepsilon_t$ $\forall i$ are identified, the impact of each shock (captured by the matrices D and D^i) is not, so that additional restrictions are necessary to pin those down. The priors on D and D^j described in section 2 influence the posterior of Q but, unless they are degenerate, do not pin it down entirely - even asymptotically, there will be some posterior uncertainty about D and D^i (and hence the implicit posterior of Q). However, as shown by recent work, even such "set" identification can be very informative if applied to enough variables (Amir-Ahmadi and Drautzburg, 2020) and having many (even weak) restrictions (Amir-Ahmadi and Uhlig, 2015). This insight is especially relevant for our setting, where we have many sectors and thus a substantial amount of prior information we can exploit.

It is also important to point out that this identification discussion does not mean that all prior distributions of D and D^j are equally consistent with the data. What is true is that for any given value of D and D^j, there is a set of other values that have the same likelihood, but some sets are more likely than others. Different Gaussian prior distributions for D and D^j of the type we use will put different weights on D and D^j matrices belonging to different sets. They can, therefore, be assessed via their marginal likelihoods obtained by integrating across all their possible values. The outcome is that, while the priors are essential for identification, the priors for D and D^j will generally not be equal to the posterior distributions of those matrices.

3.5.2 Restrictions on B_l^Z and comparisons with other approaches

Without further restrictions on B_l^Z, equation 10 implies a very large number of parameters to be estimated. Our approach differs from Dynamic Factor Models (DFM) and factor augmentmented VARs (FAVAR) approaches through the restrictions that it imposes.

In particular, our model imposes restrictions on the matrices B_l by assuming that one sector's variables cannot directly respond to any other sector's lagged variables. Nevertheless, we do allow idiosyncratic units to respond to their own lags and to lagged values of aggregates. The restrictions on B_l^Z are, therefore, weaker restriction than the one adopted in DFM and FAVAR approaches, which assume that idiosyncratic units are only a function of unobservable factors and idiosyncratic error terms. A further way in which the Hierarchical-VAR relaxes assumptions typically made in DFMs and FAVAR's is by allowing explicitly for correlation across the variables within each idiosyncratic block driven by the idiosyncratic shocks.[21]

As compared to standard dynamic factor models Stock and Watson (2005a) and FAVARs (Bernanke et al. (2005)), the Hi-VAR features a factor structure with two types of factors: static, unobserved, factors, identified by the structural shocks, as discussed above, and dynamic, observed, factors, identified with observed lagged aggregate variables, which appear in equation 4 as well as in the aggregate dynamics (equation 3).

Having the unobserved factors be *iid* shocks allows for a greater number of latent factors. In contrast, DFMs and FAVARs often select a small number of factors, which follow persistent VAR processes. Those descriptions of the data are mutually consistent, since multiple *iid* shocks could drive each of the "VAR factors" (our counterpart of these VAR factors would be the observable aggregate variables that influence sectoral dynamics).[22] Furthermore, our approach allows for a more parsimonious identification procedure in comparison with DFMs and FAVARs, where identification of structural shocks requires imposing identifying assumptions for both the unobserved factors and the structural shocks.

3.5.3 Separation of link between one-step-ahead forecast errors and structural shocks

Our econometric model breaks the close link between one-step-ahead forecast errors and structural shocks implied by standard VARs. This distinction is useful for two reasons: (i) this allows sectoral data and aggregate data to *jointly* identify structural shocks and (ii) it does not necessarily force

[21] As Stock and Watson (2016) discuss, likelihood-based approaches to factor model estimation typically assume that idiosyncratic shocks are uncorrelated across series. The primary approach that allows for such correlation are non-parametric approximate factor models estimated with frequentist methods.

[22] Typically, however, in standard dynamic factor models, the number of *iid* shocks driving the 'VAR factors' is imposed to be the same as the number of 'VAR factors.' Notice that, for applications where no structural shocks are identified, one can assume as many iid shocks as 'VAR factors' without loss of generality under Gaussianity and linearity.

structural shocks to explain large fractions of the variances of our observables if the data do not call for structural shocks to be important.[23] To safeguard ourselves against overestimating the contribution of w_t to aggregate variation, we suggest adding shocks to the vector of structural innovations ε_t with loose priors that do not use any identification information. The additional 'structural' shocks will soak up any explanatory power that the model would otherwise falsely attribute to w_t. In our application, we add three of those shocks. We should note that others have imposed a factor structure on residuals of time series models. Altonji and Ham (1990), Clark and Shin (1998), Stock and Watson (2005b), and Gorodnichenko (2005) follow the same route to estimate common shocks in time series models with many observables.[24] In particular, we share with Stock and Watson (2005b) the assumption that non-structural shocks cannot contemporaneously affect variables in other blocks of the model. Gorodnichenko (2005) interprets w_t as shocks that can arise in equilibrium models due to "expectations errors, measurement errors, heterogeneous information sets (e.g., consumers and the central banker can have different information sets), myopia and other forms of irrational behavior." Gorodnichenko (2005) also describes an equilibrium model with imperfect information that has such a factor structure in residuals.
Another modeling approach that touches on issues similar to ours is Global VAR (Chudik and Pesaran (2016)). Those do not break the link between aggregate shocks and one-step-ahead forecast errors at the aggregate level and require a priori restrictions on how shocks propagate between idiosyncratic variables.

3.5.4 Further links with VAR literature

What sets our approach apart from the previous literature on structural VARs is that (i) because of our model structure, we can use substantially larger datasets than standard VAR applications can, (ii) we can identify several shocks simultaneously, rather than one or two.[25] Finally, as can be seen from equation 10, our model is a restricted VAR using many variables. As such, there is a natural connection to the literature that uses shrinkage priors for such VARs (Banbura et al. (2010)). Instead of using shrinkage priors (such as the Minnesota prior) in a VAR for all of our variables, we instead impose restrictions implied by the grouping of variables into sectoral and aggregate variables.[26].

[23] Our model does not preclude structural shocks from being the main drivers of business cycles a priori: the estimated variances of the non-structural shocks could be very small.

[24] Cesa-Bianchi and Ferrero (2020) use this assumption in the context of a panel VAR for sectors of the US economy. Their work focuses on identifying shocks via restrictions on aggregate variables after exploiting this factor structure.

[25] By estimating the responses to structural shocks directly, we do not need to post-process reduced-form VAR estimates to obtain the structural representation that allows us to compute the effects of structural shocks. Eliminating this additional step is useful because the algorithms used to deliver the impulse responses after estimating a reduced-form model can be numerically time-consuming because not all proposed candidate parameter vectors of the structural VAR satisfy the identification restrictions as in Rubio-Ramirez et al. (2010) or because the imposed restrictions are overidentifying as in Amir-Ahmadi and Drautzburg (2020).

[26] We do still use a Minnesota-type prior for the aggregate variables in our VAR.

One key innovation relative to many prior studies is that, as discussed above we apply a Gaussian prior directly to the effects of the structural shocks on aggregate and sectoral data.[27] This procedure allows us to use more prior information on the magnitudes of these effects compared to what would be feasible in the standard sign restriction approach.[28] By exploiting our specific model structure, we can efficiently estimate very large scale models. Also, because we directly estimate a structural VAR, our approach can handle set-identified, exactly identified, and over-identified environments. The difference between those alternatives depends on the priors on the parameters governing the contemporaneous impact of structural shocks.

Importantly, our approach is computationally very efficient because, as we will show below, it relies solely on standard steps in Gibbs samplers (drawing from Normal and inverse-Wishart priors as described in Koop and Korobilis (2010) as well as using Gibbs sampling for linear and Gaussian state-space models as in Carter and Kohn (1994)). The hierarchical structure of our model implies that those procedures are amenable to parallelization.[29] This implies that our approach can be very efficient even in applications that have a much larger scale than our application in this paper.

4 Estimation Results

We now describe the main results. To obtain those, we used eight aggregate US time series (in year-over-year growth rates where applicable): (i) real GDP growth (denoted gdp later), (ii) CPI inflation (denoted π), (iii) the effective Federal Funds rate (denoted i), (iv) growth rate in real government spending (denoted g), (v) real PCE consumption growth (denoted c), (vi) Moody's Seasoned Baa Corporate Bond Yield Relative to the Yield on a 10-Year Treasury of Constant Maturity (denoted $spread$), (vii) Fernald's utility adjusted Total Factor Productivity (TFP) (Fernald (2014), denoted tfp) , (viii) and energy inflation based on the relevant producer price index (denoted $energy$). We use data from the first quarter of 1961 to the last quarter of 2017. The data are described in detail in Appendix B.1.

For the sectoral data, we use three variables for each sector, where available: (i) the year on year

[27] We can do this because we directly estimate the impact of structural shocks rather than first estimate a reduced-form model and then infer the structural model afterward, as is common in the VAR literature. By directly estimating a structural representation, we follow in the footsteps of, for example, Baumeister and Hamilton (2015) and Sims and Zha (1998), who estimate structural VARs. Baumeister and Hamilton (2018) and Baumeister and Hamilton (2019) are closest to our approach because they also use the information on the contemporaneous impact of the structural shocks to inform their priors.

[28] In the standard approach to impose sign restrictions, as outlined in Rubio-Ramirez et al. (2010), inequality restrictions are imposed on impulse responses in conjunction with a uniform (Haar) prior on the rotation matrices that map reduced form parameters to initial impulse responses. We could incorporate strict inequality restrictions in our framework by incorporating a Metropolis step into our algorithm.

[29] This parallelization argument does not hold, for example, in large scale VARs. And while certain aspects of Gibbs samplers for factor models might also be amenable to parallelization, these models do not directly emphasize the dynamics of all variables in the sector transparently.

growth rate of sectoral PCE (ii) the year on year sectoral inflation as measured by the associated price index and (iii) the year on year growth in Industrial Production as made available by the Federal Reserve Board. The latter is not available for all sectors, so we only use it where available. The sectoral data are described in Appendix B.2.

In terms of specification, we use six lags of the left-hand-side variables as a conservative choice throughout to insure that we capture the dynamics of our observables at both aggregate and sectoral levels. For the lagged aggregate variables in the sectoral equations (where they enter as additional variables) we use two lags for parsimony.[30]

4.1 Impulse Response Functions

We now show the impulse response functions obtained from the model estimation. Figure 3 shows the median and various percentiles of the impulse responses to a one-standard-deviation shock for the household consumption shock.[31] The results conform to the expected response to a generic aggregate demand shock. There is an increase in inflation, output, and nominal interest rates. Energy costs also increase, which again is consistent with an increase in demand for energy. At the same time, 90 percent posterior bands of the responses of TFP and credit spreads contain 0, implying that the consumption shock is not, first and foremost, a response to technology changes or financing conditions.

[30] Adding more lags for the 8 aggregate variables would significantly increase the number of parameters at the sectoral level. For example, adding two more lags for sectoral inflation and consumption *alone* would add 5984 parameters (187 sectors $\times$ 2 sectoral variables $\times$ 8 aggregate variables $\times$ 2 lags). On top of that, we would need to add 32 parameters for each sector with IP data. Introducing such a large number of parameters would require alternative (shrinkage) priors for the parameters on aggregate variables in the sectoral equations, but there is limited guidance available in the literature on how to set up such a shrinkage prior for exogenous variables.

[31] The impulse responses to other shocks are in Appendix L. These other responses are broadly in line with previous responses obtained for these shocks in the literature.

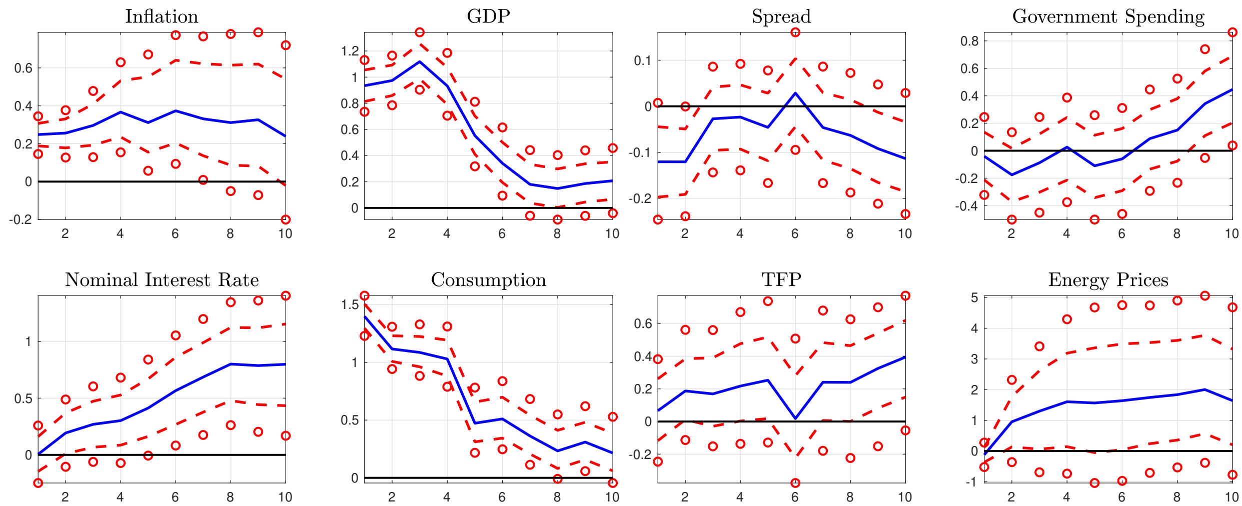


Figure 3: Responses to Household Demand Shock. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis shows time in quarters.

We also examine how incorporating the sectoral data helps with identification. Specifically, figure 4 shows that, relative to a specification where the shock is identified only from its impact on aggregate consumption, the impulse response functions for the household demand shock becomes much more tightly estimated once we incorporate priors on the sectoral responses. It is those tighter posteriors that make clear the impact of those shocks on inflation and interest rates.[32]

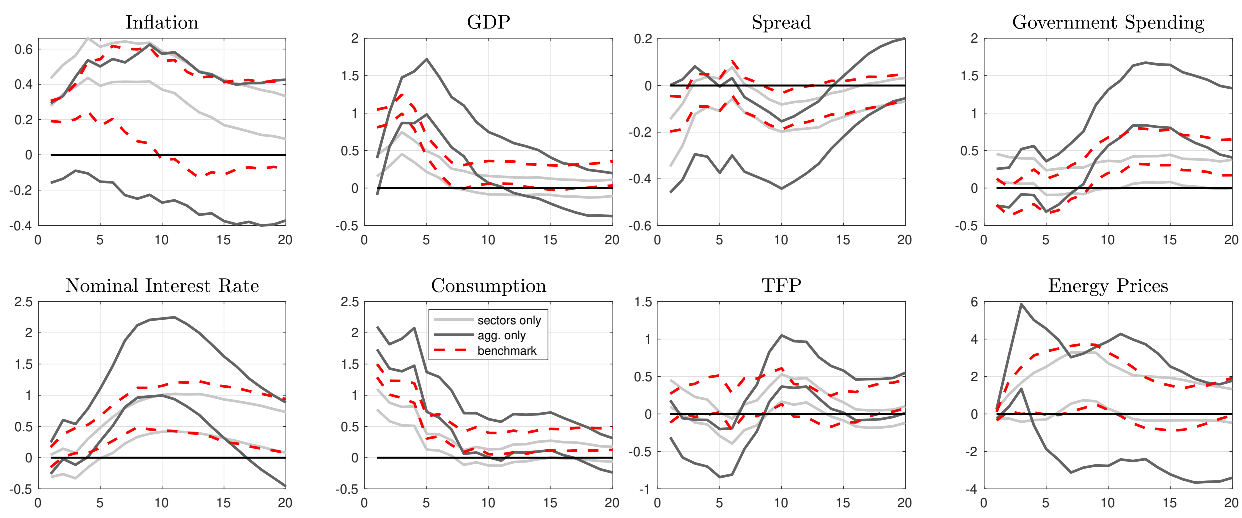


Figure 4: Responses to Household Demand Shocks: Comparison of Identification Schemes. Error Bands are 16th and 84th Percentile Posterior Bands.

[32] To obtain the impulse responses based only on sectoral or aggregate information, we choose very loose priors for D and D^i, respectively, and re-estimate our model.

4.2 The Sources of Business Cycles

In this section, we examine how the different identified structural shocks explain business cycles. We do this in two ways: through a variance decomposition, describing the fraction of business cycle variance explained by the various shocks, and through a historical decomposition, which shows the contribution of each shock to various cyclical downturns.

The results for the variance decomposition are presented in table 5 below. To obtain the numbers in the table, we decompose for each variable the fraction of the overall forecast error variance at business cycle frequencies into different components.[33] The numbers refer to average variances for forecast errors 6 to 32 quarters ahead. For all variables the elements of ε_t account for more than 85% of overall variance, with the remaining explained by the residuals w_t.[34] The table shows that household consumption shocks play a prominent role not only in explaining nominal interest rates and inflation (as one would expect), but also GDP, consumption, and energy prices. The other shock with a prominent role is to corporate credit, accounting for a large part of the variance of GDP and consumption. If we count household consumption, government consumption, and monetary policy shocks as "demand" shocks and energy and technology as "supply" shocks, we find that demand shocks account for substantially more of GDP variation at business cycle frequencies than supply shocks.

Our results suggest that consumption shock plays a prominent but not dominant role. In particular, our results do not support the view that there is a single "main" business cycle shock accounting alone for most of output fluctuations.[35]

Table A-5 in Appendix M.2 shows how the decomposition for the household shock would appear if one only used the prior on aggregate variables to identify the model. The aggregate-only identification implies that the household consumption shock explains a smaller portion (20%) of GDP variation compared to the baseline estimate (34%). The 90% error band for the aggregate-only identification is also twice as large then the one implied by the model estimated using sectoral data (depicted in Table A-2 in the appendix). This means that the average variance share estimated using sectoral identification falls within the error band when only aggregate restrictions are used.

[33] We focus here on the posterior mean. The 5th and 95th percentiles for the household consumption shock can be found in Appendix M.

[34] It follows that our model performs similarly to the Dynamic Factor Model in Stock and Watson (2016). For example, they find that the 8 factors explain 83% of the four quarter ahead variance of GDP and 67% of the variance in inflation, whereas in our Table 5 we find that our model explains close to 91% of the business cycle variance of output and 92% of inflation.

[35] In particular, if we regress the main business cycle shock from Angeletos et al. (2020) on the various shocks we identify, we find that this main shock has poor correlation with the consumption shock and can be better understood as a combination of various shocks. The results of this exercise are in Table M.8 in Appendix M.8. Our results are, therefore, in line with the observation in Dieppe et al. (2021), that, in the presence of multiple shocks explaining a substantial part of the variance of endogenous variables, a variance maximizing estimator may in fact generate a linear combination of multiple shocks.

However, the mean value (20%) found in the aggregate-only estimate is outside of the error band when sectoral restrictions are included.

	tech	credit	household	gov	energy	monetary	total
Inflation	4.0	7.0	13.9	33.6	21.9	4.4	92.2
GDP	6.5	16.0	33.9	12.8	10.6	3.7	90.7
Nominal Interest Rate	4.6	6.6	22.9	31.2	10.5	4.8	87.6
Consumption	5.0	9.6	42.6	16.3	9.9	3.5	93.3
Spread	13.8	33.0	9.8	14.6	10.1	3.2	92.1
Government Spending	3.9	6.9	26.1	24.3	9.0	6.2	84.9
TFP	18.4	5.4	10.9	22.8	11.0	5.0	93.4
Energy Prices	4.3	5.4	8.3	11.4	53.9	3.6	92.2

Table 5: Mean of variance decomposition across business cycle frequencies and posterior draws. 'Total' referes to the fraction of variance explained by all elements of ε_t.

The variance decomposition provides a view of the average importance of different shocks in driving different variables. Alternatively, one might ask how relevant the various shocks were in particular recession episodes. This question allows for the possibility that recessions are qualitatively different from expansions, and that they may have been caused by different shocks. To answer this question, we use a historical decomposition.

Table 6 provides the results of such a decomposition for the recession episodes fully included in the sample. The first column shows the peak-to-trough changes in the *level* of (log) real GDP for the various recessions, and the second column shows the expected change in GDP in the absence of any shocks after the recession peak. It is typically positive, reflecting, among other, things, that the estimated growth rate of real GDP is positive. The subsequent columns show the difference between this baseline behavior and the one that would result if the economy was only hit by each inferred sequence of shocks (we provide point estimates based on posterior means). Thus, for example, in the 1980 recession, output dropped by 2.2% when it was expected to grow by 0.7%. Out of that 2.9% short-fall, the shock to household consumption accounted for 0.9%, or about a third, with the credit shock accounting for a slightly smaller part. Both shocks appear to have large impacts in most subsequent recessions, with household consumption having an increasingly large role. By the 2007-09 recession, household consumption accounts for more than two-thirds of the difference between projected and realized output growth and the credit shock for half as much.

The monetary shock is not an important driver of recessions. Note that this does not mean that the monetary shock is not important for economic fluctuations more generally: Table 5 shows that the monetary shock has substantial impact on the variance of the aggregate observables at business cycle frequencies (broadly similar to, for example, the variance decomposition in Smets and Wouters (2007)). For the Volcker disinflation, a possible interpretation of our results is that the

"Volcker shock" propagated into the economy primarily by depressing household consumption and by changing credit conditions, with nominal frictions leading to infrequent price changes playing a minor role.

	data	no shocks	tech	credit	household	gov	energy	monetary
80	-2.2	0.7	-0.1	-0.7	-0.9	-0.1	-0.1	-0.0
81-82	-2.5	3.5	0.6	-2.3	-3.2	-0.1	0.3	-0.1
90-91	-1.4	1.5	0.3	-0.8	-1.6	-0.1	-0.1	-0.0
2001	0.4	2.4	-0.5	-0.1	-1.3	-0.0	0.2	0.0
2007-2009	-4.1	4.5	-0.2	-3.8	-6.3	-0.2	0.3	-0.1

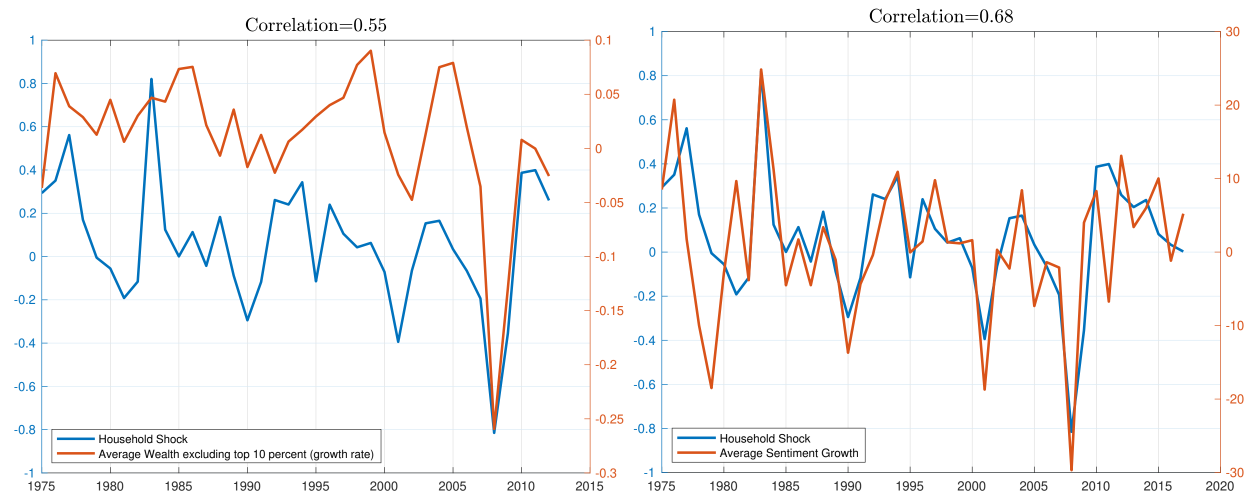
Table 6: Counterfactual Recessions. Contributions of various shocks to peak to trough change in the level of GDP relative to No Shock Forecast.

5 Interpretation: Sentiments and Wealth

How should we interpret the consumption shock? As the derivation in section 2 makes clear, the shock may be a combination of various shocks that affect households first, and sectoral output and prices in response to household spending decisions.

We provide some insight into the interpretation of our estimated shock by examining the behavior of the inferred consumption shock series in comparison to data not used in its estimation. This exercise provides external validation for our findings since those series were not used in the estimation at all.

One potential source of consumption shocks are fluctuations in housing wealth. This was strongly highlighted in empirical, theoretical and quantitative work by Mian et al. (2013), Kaplan et al. (2016) and Berger et al. (2018). Figure 5a compares the time-series for the household consumption shock inferred using our methodology to the growth rate of average wealth of households in the bottom 90% of the wealth distribution, obtained from Saez and Zucman (2016). We aggregate our shock to an annual frequency since the wealth measure is only available at that frequency. The two series correlate well, especially from the late 1990s onward, and very prominently so around the 2007-09 recession.



(a) Household wealth fluctuations and the household consumption shock

(b) Consumer sentiment and the Household Consumption Shock

Figure 5: Comparison of Household Consumption shock with Wealth and Sentiment Changes.

At the same time, the correlation is smaller earlier in the sample, indicating that the source of household consumption fluctuations may have been different over that period. Figure 5b compares the shock to a measure of changes in consumer sentiment derived from the Michigan Survey. The two series track each other very closely for the entire sample, including the early part.[36] Our findings are thus not inconsistent with the statement that fluctuations in sentiments are important determinants of economic fluctuations, as argued for example by Farmer (2013) or Chahrour and Jurado (2018).[37]

[36]This tracking becomes apparent in the figure, which plots annual averages. Annual averages are used here to aid comparison to the wealth measure in the left panel, which is ony available at an annual frequency. The correlation for the original quarterly series is not much lower at 0.57.

[37]In Appendix L.1 we examine whether a sentiment shock identified using consumer sentiment as an instrument has similar properties to the consumption shock that we identify. We find that this is true in some instances, but not all. In particular, it does result in an increase in output, interest rates, and consumption. However, it does not have a positive effect on inflation. We believe the reason for this discrepancy is that the sentiment shock identified in this way also incorporates perceptions about supply-side fundamentals. This is supported by the fact that the shock identified in this manner is associated with an increase in TFP, a decline in energy prices, and a decline in spreads, none of which are present in the same way in our identified shock. The shock identified through this method exhibits similar properties to the one we have estimated in some instances, but not all. In particular, it does result in an increase in output, interest rates, and consumption. However, it does not have a positive effect on inflation. We believe the reason for this discrepancy is that the sentiment shock identified in this way also incorporates perceptions about supply-side fundamentals. This is supported by the fact that the shock identified in this manner is associated with an increase in TFP, a decline in energy prices, and a decline in spreads, none of which are present in the same way in our identified shock. This suggests that consumer sentiment is a useful, but not perfect, indicator of the demand-side forces at play in the economy.

6 Further Validation and Analysis

In this section, we conduct some further analysis to validate our approach in various ways. We start with a validation of our approach by using a multi-sector New Keynesian model as a data-generating process to benchmark our approach, followed by comparing estimated impulse-response functions for the monetary policy shock, estimated using our method, to IRFs measured with the use of external instruments established in the literature. Next, we evaluate the importance of prior information on the impact of shocks, and the extent to which it is modified by data. Finally, we perform an analysis of the model fit, by assessing the extent to which the structural restrictions in the Hi-VAR constrain the interplay between sectoral and aggregate variables.

6.1 Validation of our Identification Assumption in an Equilibrium Model

We now show that our identification scheme is well suited to estimate the evolution of a discount rate shock in a multi-sector New Keynesian dynamic equilibrium model calibrated to US data. The model, which we describe in detail in Appendix D, is a generalization of Justiniano, Primiceri, and Tambalotti (2010) to allow for multiple sectors and sectoral linkages. The calibration builds on Justiniano et al. (2010) and Carvalho, Lee, and Park (2021). We furthermore use information from sectoral linkages and consumer shares obtained from the input-output tables made available by the BEA and on sector-specific price stickiness from Nakamura and Steinsson (2008). We calibrate the volatility of the discount-rate shock in our benchmark so that it explain the same fraction of one-quarter ahead variance as the household demand shock in our estimates. Tables A-1 and A-2 in the Appendix D.7 lists the calibrated parameters together with their sources.

The model features 5 aggregate shocks -technology, consumption demand (a shock to the discount factor), monetary policy, government spending and investment. We simulate one data set of length 256 quarters and then estimate our Hi-VAR model on this dataset using various approaches to identification (i.e. setting the prior on D^Z).[38] The variables we use in our estimation are aggregate inflation, output, the nominal short-term interest rate, consumption, government spending, TFP, and investment.[39] The lag lengths and the setting of the aggregate response to aggregate shocks (D) are set as in the empirical application. Where the various specifications differ is in the setting of D^i, the sectoral responses to aggregate shocks. Here, we examine two strategies to set the prior mean of D^i (the standard deviation is set as in the empirical application):

1. Center D^i at the true impact responses obtained from the equilibrium model. The prior

[38] While the theoretical concept of identification in econometrics is a population concept, we focus on a more stringent test of our approach using a standard sample size in macroeconomics. Increasing the sample size does not substantially alter our quantitative findings, as we discuss in Appendix F.

[39] All variables are measured in deviations from the model-implied trend.

standard deviations are the same as in our empirical setting (and as such in the specification below).[40]

2. Set the prior means as in our empirical application. For the technology shock this is not possible as we do not have R&D in our equilibrium model, so we set the weight α equal across sectors.

To assess how well our approach fares, we focus on the estimated consumption shock. We simulate one sample from the equilibrium model and then estimate the Hi-VAR using the two priors described above. To assess fit, we concentrate on the posterior distribution of the correlation coefficient between the actual consumption shock and the median estimated shock.[41]

Panel (a) in Figure 6 shows, in a scatterplot, the distribution of the actual discount rate shock plotted against the posterior median estimate for the two experiements. When using the actual sectoral impact of the discount rate shock (in red), the posterior median and the actual shock comove tightly. The comovement between actual and posterior median (in blue) becomes less tight but remains substantial when the prior means based on C/Y ratios are used. Also, there is no indication of a systematic bias in either estimate, with both scatterplots clustering around the 45 degree line.

The two lower panels show the posterior distribution of the correlation coefficients. Panel (b) shows that the correlation coefficient for the prior using the true IRFs is tightly concentrated around 0.96, so that, given accurate knowledge of the sectoral impact of the discount rate shock, cross-sectional data can help one estimate its time-path with high accuracy.

Panel (b) shows the corresponding distribution for the correlation coefficient for the case in which the econometrist is unwilling to take a stance on the true underlying model uses sectoral variation in C/Y to center sectoral prior impact. As one might expect, the correlation coefficient is further away from 1, with mode somewhat above 0.7, and considerably more disperse. This dispersion reflects the uncertainty and approximate nature inherent in our identification assumptions.

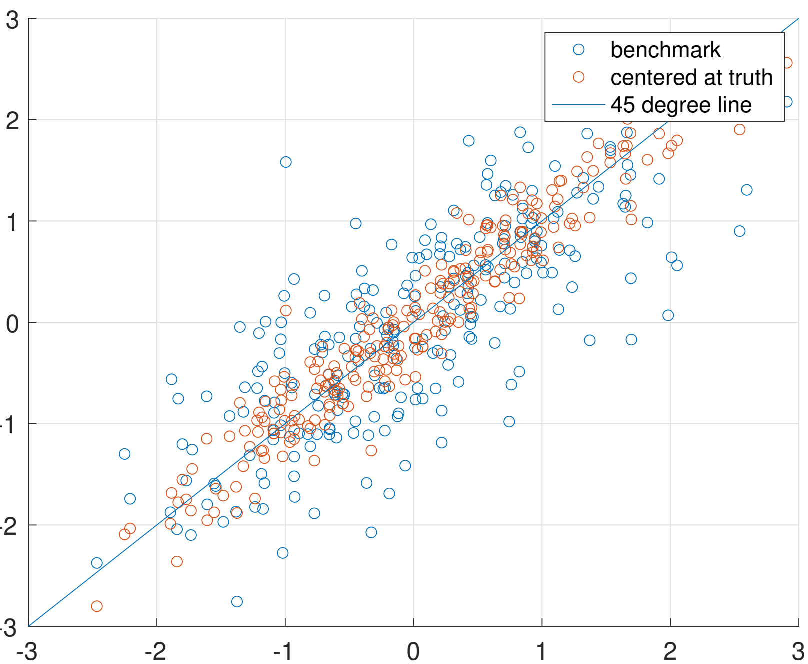
The role of confounding sources of variation Within our methodology, prior uncertainty induces set-identification so that points within the identified set are given probabilities implied by the prior distributions. We illustrate this point by manipulating other sources of estimation uncertainty.

[40]Note that in this specification there is still estimation uncertainty as the correct response is not dogmatically imposed, but just used to set the prior. We view this specification as representing an upper bound of what can be achieved with our approach.

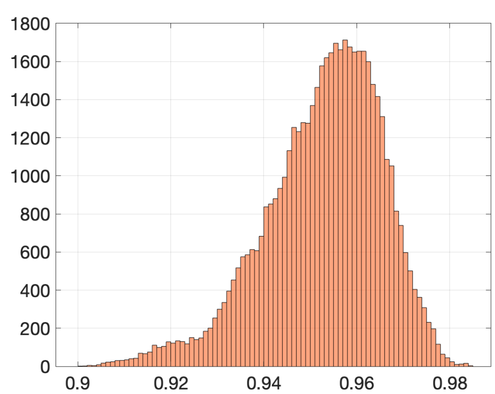
[41]This is just the coefficient from the following OLS regression for each posterior draw i

$$\tilde{\varepsilon}^C_{t,i} = \beta^i \varepsilon^C_t + u^i_t \tag{15}$$

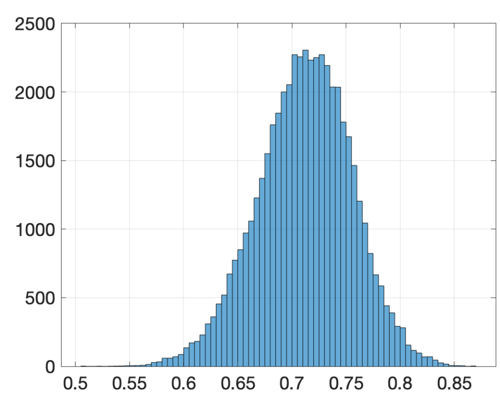
where $\tilde{\varepsilon}^C_{t,i}$ is the i-th draw of the consumption shock at time t, whereas ε^C_t is the true consumption shock at time t. Running this regression for each draw i, we are left with as many regression coefficients as we have posterior draws. We standardize both the estimated shock (separately for each draw i) and the true shock to have unit standard deviation so that β^i can also be interpreted as a correlation coefficient.



(a) Estimated (median) vs true shock.

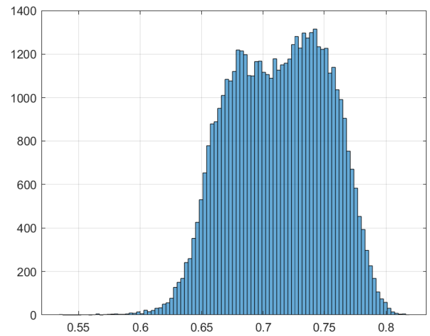


(b) Distribution of β^i, identification centered on true sectoral impact

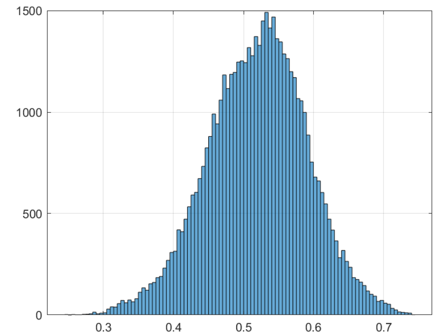


(c) Distribution of β^i, benchmark identification (based on C/Y ratios)

Figure 6: Posterior distribution of correlation between true discount rate shock and estimates based on data generated from multi-sector New Keynesian model



(a) Distribution of β^i, smaller volatility of sectoral shocks



(b) Distribution of β^i, smaller volatility of consumption shock, 50 % reduction.

Figure 7: Robustness checks, Monte Carlo

The first source of estimation uncertainty that we manipulate are the idiosyncratic shocks affecting sectoral outcomes (thus approximating a "population" outcome). In particular, we reduce the standard deviation of those shocks by a factor of 100. As we can see in Figure 7a, the results are very similar to our benchmark, meaning that the differences between our benchmark specification and whether the true shock or the specification where we center our prior at the truth will not vanish as we increase the sample size. Instead, the binding constraint is the specification uncertainty inherent in our identification restrictions.

The second source of estimation uncertainty are the confounding effects of other aggregate shocks. We show that those become more relevant as the discount rate shock accounts for a smaller fraction of overall output variation. In particular, if we reduce the volatility of the consumption shock by 50 percent from the benchmark value the posterior mode for β^i falls to 0.55, which is still sizeable.

6.2 Comparison with IV-based Identification

Our approach uses sectoral data to provide additional information about the various shocks of interest. Another source of information that has been used repeatedly in empirical macroeconomics are instruments for aggregate shocks (see, for example, Mertens and Ravn (2013) and Stock and Watson (2018)). We use the Romer & Romer monetary shock (Romer and Romer (2004)) as updated by Wieland and Yang (2019). We drop sectoral information and instead incorporate information coming from the instrument along the lines of Caldara and Herbst (2019). We denote the monetary shock by ε_t^m, the observed Romer & Romer shock by m_t, and add the following equation to our aggregate block (which we do not change otherwise):

$$m_t = \mu^m + \beta^m \varepsilon_t^m + u_t^m \tag{16}$$

where u_t^m is a mean zero Gaussian shock. We use loose priors on all parameters in this equation. The priors on μ^m and β^m are centered on 0 and 1, respectively. In Figure 8, we compare the median estimated response of all aggregate variables obtained from using the instrument to the estimated response using the sector-based identification approach.

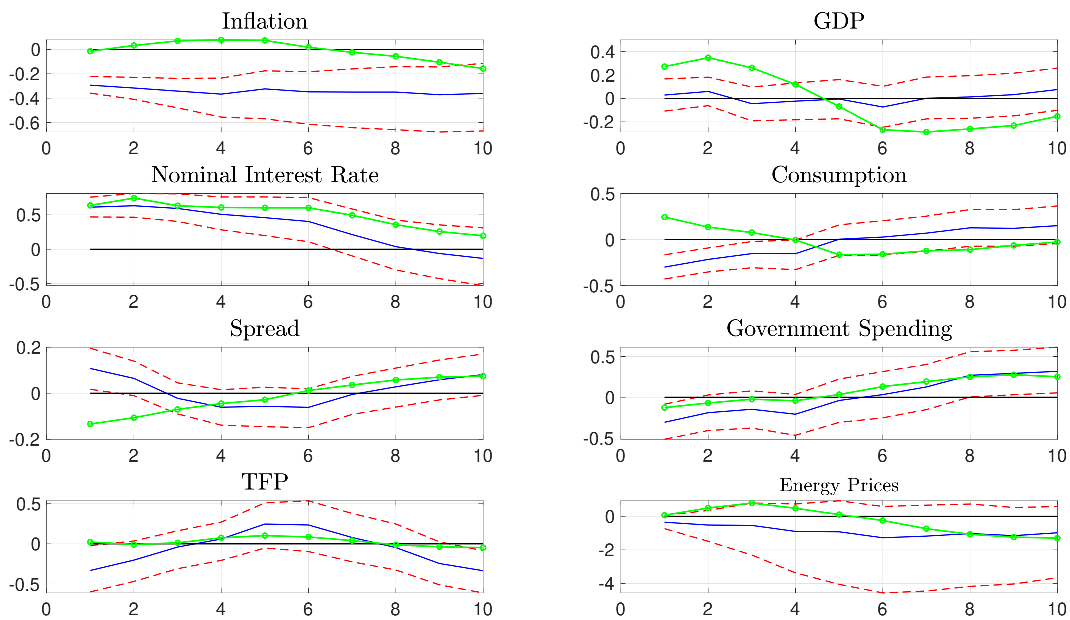


Figure 8: Benchmark Responses (Median, 5th and 95th Percentiles) to a One-Standard Deviation Monetary Shock and Median Response based on Romer & Romer Shock (green).

The impact response of the nominal interest rate is very similar under the two identification approaches. The responses were the approaches differ the most (inflation and GDP) are those were our approach arguably yields more credible responses (a decline in inflation and no significant increase in GDP growth on impact). It is by now well known that analyses based on Romer & Romer-type shocks can lead to such incredible responses (see Bu et al. (2020)). For the other variables, the instrument-based responses are mostly within our 90 percent posterior bands.

6.3 The Importance of Prior Information

While standard asymptotic results imply that most parameters in our analysis, such as VAR coefficients and the variance of innovations, are well identified by the data, this is not the case for the impact matrix D^Z. In particular, the posterior distribution of D^Z is influenced by the priors even asymptotically. This influence confirms that the priors are necessary for the identification of the structural shocks.

There is no amount of data that can be completely informative about the impact of each individual shock, D^Z. However, standard results from factor analysis imply that one can identify the part of the covariance of innovations that is accounted for by aggregate shocks.[42] That part is equal to $D^Z D^{Z\prime}$, since the covariance of macroeconomic shocks ε_t is itself equal to the identity matrix.[43], and in general converges in large samples to a known matrix, ϕ. In Appendix H we show that, given $\lim_{T\to\infty} D^Z D^{Z\prime} = \phi$ the asymptotic posterior distribution of D^Z satisfies

$$P(D^Z | D^Z D^{Z\prime} = \phi) \propto \mathbb{1}\left(D^Z D^{Z\prime} = \phi\right) p(D^Z) \tag{17}$$

where $\mathbb{1}$ is the indicator function. That is, asymptotically, only the parts of the prior distribution that are consistent with ϕ are retained. Since there are multiple values of D^Z for which $D^Z D^{Z\prime} = \phi$, the posterior distribution for D^Z remains non-degenerate in large samples. At the same time, it is constrained by the data. This is reminiscent of results for standard VARs in Baumeister and Hamilton (2015) (see their Proposition 2).

Note that expression 17 describes the joint distribution of D^Z, which is itself a matrix. The dependence of the distribution on $D^Z D^{Z\prime}$ induces dependence between the elements of this matrix: We may take the a priori stance that a certain level of impact for the consumption shock on a certain variable is probable, but it will only remain so if it is compatible with the level of impact for other shocks that are themselves also probable.

To assess the importance of the data relative to our prior distributions, we plot the prior median of the impact of a given shock on the variables in a sector against the posterior median (i.e., the prior and posterior medians of the relevant entries of D^Z). We do this to (i) check that our prior information is not completely overruled by the data (in which case we should go back to the drawing board) and (ii) that our analysis indeed adds information relative to the prior so that the data is indeed helpful to identify the effects of shocks. We focus here on the consumption shock - the figures for the other shocks look similar. Figure 9 shows a scatter plot of the prior vs. the posterior medians across sectors for our three sectoral variables as well as the identity function (all dots would be on this line if the data were not informative at all).[44] The data is informative in that it shifts the median impact of the shock across sectors.

[42] We check this result numerically in the Monte Carlo exercise described in Appendix I.

[43] Specifically, the vector of aggregate shocks at any time t is given by ε_t and the part of innovations accounted for those shocks is given by $D^Z\varepsilon_t$, so that $E\left[D^Z \varepsilon_t \varepsilon_t' D^{Z\prime}\right] = D^Z E\left[\varepsilon_t \varepsilon_t'\right] D^{Z\prime} = D^Z D^{Z\prime}$

[44] Note that IP data is not available for all sectors.

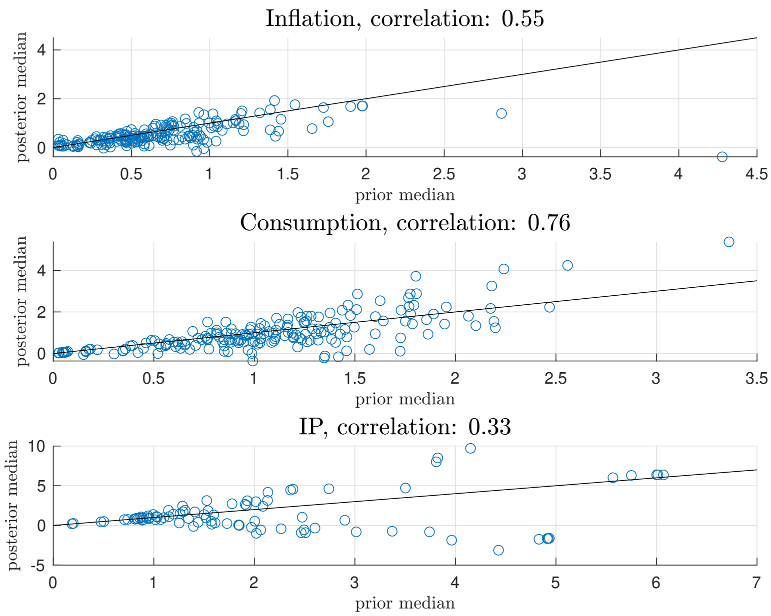
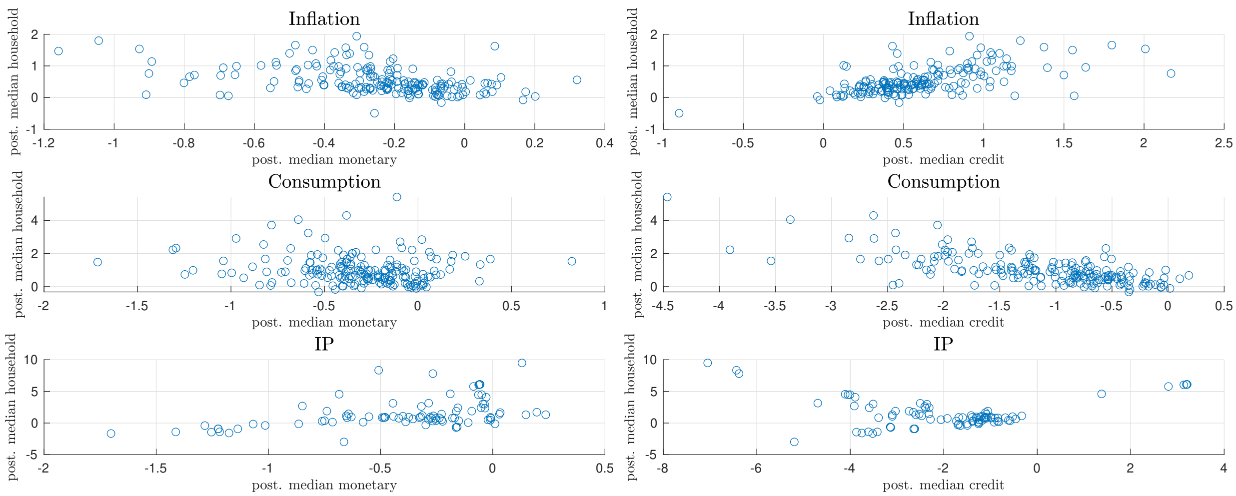


Figure 9: Prior vs Posterior Impact of Household Shock Across Sectors.

Given the uncertainty about the marginal effect of the consumption shock on different prices and quantities, our estimate is more credible if the marginal effect of other shocks does not look too similar to that of the consumption shock. The two prime candidates for shocks that could have similar estimated effects as the household shock are the credit shock and the monetary shock. Therefore we produce a scatter plot of the posterior medians for the (impact) consumption shock response across sectors against the posterior median of the (impact) monetary and credit shock responses across sectors. Figures 10a and 10b shows these scatter plots. As can be seen from those scatter plots, the impacts of shocks across sectors are not strongly correlated, highlighting that we are identifying a shock that is very different from monetary and credit disturbances.



(a) Household vs. Monetary Shock (b) Household vs. Credit Shock

Figure 10: Comparison of Impact Responses Across Sectors.

6.4 Model Fit and the Interplay Between Sectoral and Aggregate Data

Our model is restrictive because correlations between sectors or between sectors and aggregate variables have to come through either the structural shocks ε_t or lagged aggregate variables. These restrictions could lead to misspecification, casting doubt on our identification strategy. To address this possible concern, we first compute the correlations between aggregate consumption growth and consumption growth at the sectoral level that appear in our dataset as well as the corresponding correlations for aggregate and sectoral inflation. We then draw 1000 parameter values from the posterior, simulate data of the same length as our dataset for each set of parameters (after discarding 1000 burn-in observations), and compute the same correlations for our simulated data. This exercise gives us the posterior distribution of the correlations we are interested in. We are thus carrying out a posterior predictive check as advocated for by Rubin (1984) and further discussed by Gelman et al. (2013) and Geweke (2005), for example. The top two panels in Figure 11 plot the correlations from the data (black) as well as the median (red), and the 5th and 95th percentiles (blue) of the posterior distribution. We sort the correlations from the actual data by size (starting with the largest correlation) to make the figure easier to interpret. We order the sectors in the same order for the simulated data. As can be seen from figure 11, our model can replicate the correlation patterns between aggregate and sectoral data.

An inquisitive reader might ask for a more stringent test, namely a check of the correlation of variables *across* sectors rather than between any sector and the corresponding aggregate variable. We show the results for this posterior predictive check in the bottom two panels of Figure 11. The figure looks noisier just because there are many more data points (pairwise correlations between

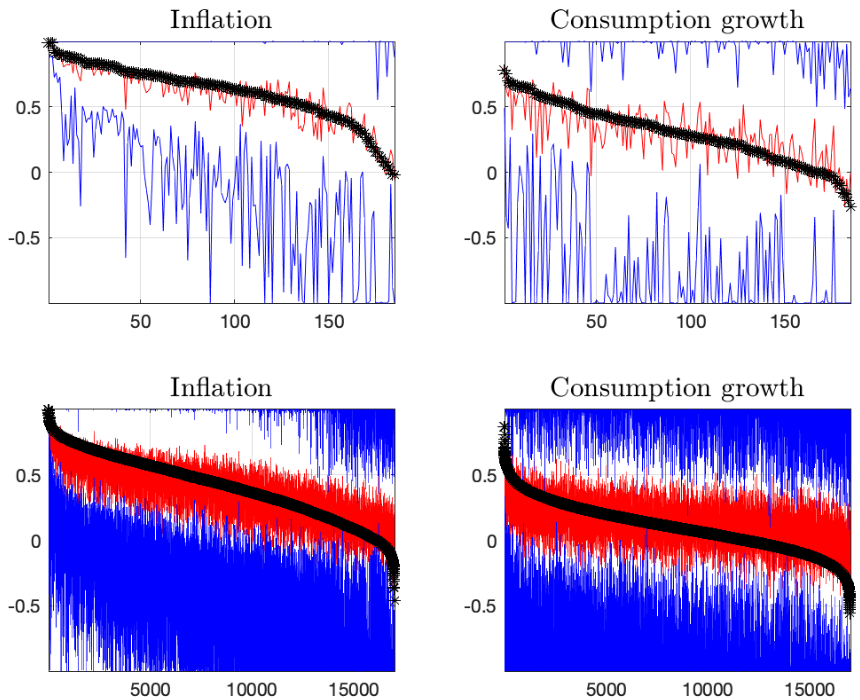


Figure 11: Posterior Predictive Check, Model-Implied Correlations vs Data. Top Two Panels: Correlation with Aggregate Inflation and Consumption Growth, sector on x-axis. Bottom Two Panels: Correlation Across Sectors, sector pairs on x-axis. Red line: Posterior Median, Blue Lines: 5th and 95th Posterior Percentiles. Data in Black.

the 187 sectors in our sample), but the main pattern remains, our model can replicate the broad correlation patterns. Our model misses at the very tail ends of the spectrum of correlations (more so for inflation than for consumption growth), but given that our model is tightly parameterized and parsimonious, we think of these results as very encouraging.

6.5 Further Analysis

In the Appendix, we give further results for our model. In particular, we show analytically in Appendix J why a researcher would generally want to use the most disaggregated data possible (like we do), and we characterize in detail the asymptotic behavior of the impact of economic shocks on aggregate and sectoral variables in Appendix H. More details and results for the Monte Carlo exercise can be found in I. Finally, in Appendix M we show that the role of the household consumption shock is robust to various changes in the specification, from having a larger prior variance on the aggregate impact of this shock to dropping the Great Recession from the sample as well as using fewer lags in our model. Appendix M also contains results for a specification where we separately identify an investment shock.

7 Conclusion

We propose an approach to use rich cross-sectional data in order to measure business cycle shocks and their aggregate impacts. The approach relies on using a priori information on the differential impact of the shock on different sectors, casting that information as a Bayesian prior to properly account for any uncertainty surrounding it, and relying on a rich set of cross-sectional data to "average out" identification errors at the level of individual sectors.

We use this method to measure shocks to aggregate consumption, defined as shocks that affect sectoral output and prices through their impact on aggregate consumption but not otherwise. We find that such shocks account for approximately 34% of output fluctuations at business cycle frequencies, and a large part of output losses during recessions.

The results highlight the value of detailed work in understanding the sources of aggregate consumption dynamics, and suggests that policies that stabilize consumption can have a significant business cycle stabilization effect.

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For Online Publication

Appendix For "The Consumption Origins of Business Cycles: Lessons from Sectoral Dynamics"

A Proof of Propositions

A.1 Proof of Proposition 1

Here we give a proof of Proposition 1. For notational convenience, we cast this proof in terms of the time series model developed in Section 3 for our application, but translating it into the notation used in section 2 is straightforward. In particular, we let $u_t = \mathbf{x}_t - E_{t-1}\mathbf{x}_t$, and let ε_t be a vector collecting all structural shocks, and w_t is the idiosyncratic noise. D collects the effects of all structural shocks. In particular, its first column corresponds to $\frac{\partial \mathbf{x}}{\partial c}$. Consider a version of our model without dynamics (to focus our attention on the identification of shocks)[1]:

$$u_t = D\varepsilon_t + w_t \tag{A-1}$$

$$\varepsilon_t = \varepsilon_t \tag{A-2}$$

u_t are the stacked forecast errors at the aggregate level and sectoral level stacked into one vector. The second equation/identity is added to turn our model into a state space model. Because all shocks are Gaussian, we can apply the Kalman filter to calculate filtered estimates of our structural shocks ε_t. Note that because our state ε_t does not feature any dynamics, the application of the Kalman filter does not require specifying initial condition. Likewise, filtered estimates will generally equal smoothed estimates, so there is no need to have a separate treatment for smoothed estimates below.[2] We assume the equations above are the true data-generating process. Without loss of generality, we assume that the shock whose responses are not misspecified is the first element of ε_t. The Kalman

[1] All VAR-type parameters are identified in our setting, so this is without loss of generality.

[2] Because our state is *iid*, the initial distribution of the state does also not matter for smoothed/filtered estimates of the state. To see this, consider a generic linear Gaussian state space system with observables $\bar{y}_t$ and state $\bar{x}_t$:

$$\bar{y}_t = \bar{A}\bar{x}_t + \bar{u}_t \tag{A-3}$$

$$\bar{x}_t = \bar{C}\bar{x}_{t-1} + \bar{w}_t \tag{A-4}$$

where $\bar{u}_t \sim_{i.i.d.} N(0, \bar{B})$ and $\bar{w}_t \sim_{iid} N(0, \bar{D})$. The one-step ahead conditional expectation and conditional variance of the state are then given by

$$E_{t-1}\bar{x}_t = \bar{C}E_{t-1}\bar{x}_{t-1} \tag{A-5}$$

$$Var_{t-1}\bar{x}_t = \bar{C}Var_{t-1}\bar{x}_{t-1}\bar{C}' + \bar{D} \tag{A-6}$$

In our application, $\bar{C} = \mathbf{0}$ (the state is *iid*), and hence the one step ahead expectation and variance do not feature any temporal dependence. This then also means that $E_t\bar{x}_t$ and $Var_t\bar{x}_t$ do not depend on $E_{t-1}\bar{x}_{t-1}$ and $Var_{t-1}\bar{x}_{t-1}$.

filter returns a least squares estimate of $E_t \varepsilon_t = \varepsilon_{t|t}$:

$$\varepsilon_{t|t} = \beta u_t$$

The matrix of coefficients β is given by the standard formula linking the covariance matrix of the right hand side variable u_t with the covariance of the right-hand-side variable with the left-hand side variable, the vector of structural shocks ε_t:

$$\beta = E(\varepsilon_t u_t')[E(u_t u_t')]^{-1}$$

The second term on the right hand side, $E(u_t u_t')$, can be identified from the data as the second moment matrix of the observables. As such, it does not depend on whether or not D is correctly specified as long as our choice of D is consistent with the overall variability of the data. Where identification matters is in the first term on the right-hand side:

$$E(\varepsilon_t u_t') = D'$$

Let's now assume that we have a misspecified version of the model where, instead of using the true impact matrix D, we use a matrix $\tilde{D}$ such that the first column of D and $\tilde{D}$ coincide. Therefore, the response to the first element of ε is correctly identified, whereas the others are not. This means that the first row of D' and $\tilde{D}'$ coincide. This in turn, means that the first row of $D'[E(u_t u_t')]^{-1}$ equals the first row of $\tilde{D}'[E(u_t u_t')]^{-1}$ and thus that the first element of the estimated shock series is independent of whether D or $\tilde{D}$ is used to form the estimate.

In terms of the notation in the proposition statement, it follows that information on the covariance matrix of $\mathbf{x}_t - E_{t-1}\mathbf{x}_t$ (equal to $[E(u_t u_t')]^{-1}$) and the vector of effects $\frac{\partial \mathbf{x}}{\partial c}$ (equal to the first column of $\tilde{D}$ and of D) are sufficient for the identification of ε_t^C (the first element of ε_t)

A.2 Proof of Proposition 2

As in the proof of Proposition 1, we use the notation in Section 3. In particular, Let ε_t be the vector of all macroeconomic shocks $\varepsilon_{s,t}$. Let D be a matrix where each row corresponds to an element of $\mathbf{x}_t$ and each column to one of the shocks $\varepsilon_{s,t}$ so that each element has the effect of $\varepsilon_{s,t}$ on $\mathbf{x}_t$. Without loss of generality, we assume that $\varepsilon_{1,t} = \varepsilon_t^C$, in which case the first column of D is equal to $\partial \mathbf{x}_t / \partial \varepsilon_t^C$. Also, let $u_t \equiv \mathbf{x}_t - E_{t-1}\mathbf{x}_t$. Finally, let N denote the dimensionality of $\mathbf{x}_t$ or, equivalently, u_t. To prove the proposition, it is sufficient to construct an estimator for ε_t^C and show that it converges asymptotically to its true value as $N \to \infty$.

Step 1 - Obtain estimates of the space spanned by the macroeconomic shocks ε_t: Result

A.2(a) in Bai and Ng (2008) states that, given the assumptions in Section 4, as $N \to \infty$, one can estimate a $\widehat{\varepsilon}_t$ such that $\sqrt{N}(\widehat{\varepsilon}_t - H\varepsilon_t) \to N(0, \Xi_t)$ where Ξ_t is a matrix defined in their paper and H is a rotation matrix. The estimation error therefore concentrates around zero as $N \to \infty$. In other words, using factor-analytic methods one can consistently estimate the space spanned by ε_t.

Step 2 - Obtain $\widehat{D} \equiv DH'$

Recall that $u_t = D\varepsilon_t + w_t = DH'\widehat{\varepsilon}_t + w_t$, where we use the fact that, for rotation matrices, $H' = H^{-1}$. As $N \to \infty$, $\widehat{\varepsilon}_t$ is measured without error. Since w_t is orthogonal to $\widehat{\varepsilon}_t$, we can recover $\widehat{D} \equiv DH'$ by regressing u_t on $\widehat{\varepsilon}_t$

Step 3 - Estimate ε_t^C: Given that D has a column matching $\partial \mathbf{x}_t / \partial \varepsilon_t^C$, one can find a rotation matrix $\tilde{H}$ such that (i) $\tilde{D} = \widehat{D}\tilde{H}'$ and (ii) $\tilde{D}$ has its first column equal to $\partial \mathbf{x}_t / \partial \varepsilon_t^C$. Such a matrix exists, since $\tilde{H} = H'$ would satisfy the condition. In general, however, there may be multiple such matrices. We take $\tilde{H}$ to be any matrix of that set.

Let $\bar{u}_t = D\varepsilon_t = \widehat{D}\widehat{\varepsilon}_t = \tilde{D}\tilde{H}\widehat{\varepsilon}_t$ denote the part of u_t explained by ε_t. Note that with $N \to \infty$, one can construct $\bar{u}_t$ given Steps 1 and 2 above. Consider now a projection of $\bar{u}_t$ on $\tilde{D}$. The projection coefficients satisfy

$$\tilde{\varepsilon}_t = (\tilde{D}'\tilde{D})^{-1}\tilde{D}'\bar{u}_t$$

Note that $\tilde{D}'\tilde{D} = \tilde{D}'\tilde{H}'\tilde{H}\tilde{D} = \widehat{D}'\widehat{D} = \widehat{D}'HH'\widehat{D}' = D'D$, so that $(\tilde{D}'\tilde{D})^{-1} = (D'D)^{-1}$ irrespective of H or $\tilde{H}$. Moreover, given that we chose $\tilde{H}$ to ensure that the first column of $\tilde{D}$ is equal to $\partial \mathbf{x}_t / \partial \varepsilon_t^C$, the first row of $\tilde{D}'\bar{u}_t$ will also be the same for all H and for all $\tilde{H}$ satisfying that restriction. In particular, that will be true for $\tilde{H} = H'$, so that $\tilde{D} = D$. It follows that $\tilde{\varepsilon}_{1,t} = \varepsilon_{1,t} = \varepsilon_t^C$.

B Data

B.1 Aggregate Data

See figure 3 for a depiction of the aggregate time-series. The sources and definitions are given below. Growth refers to year over year changes of quarterly data.

- Real GDP growth: Real Gross Domestic Product, Billions of Chained 2012 Dollars Series (FRED Series GDPC1) Quarterly, Seasonally Adjusted Annual Rate.

- CPI inflation: FRED Series CPIAUCSL, Consumer Price Index for All Urban Consumers: All Items. Quarterly, seasonally adjusted.

- The effective Federal Funds rate: FRED Series FEDFUNDS, Quarterly, not seasonally adjusted, Percent

- Growth rate in real government spending: FRED Series GCEC1, Quarterly, seasonally adjusted, Billions of chained 2009 Dollars.

- Real PCE consumption growth:FRED Series PCECC96, Quarterly, sea- sonally adjusted, Billions of chained 2009 Dollars.

- Moody's Seasoned BAA Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity: FRED Series BAA10YM, Quarterly, not seasonally adjusted.

- Fernald's utility adjusted TFP (Fernald (2014)): Percent Change (natural log difference);

- Inflation based on the relevant producer price index: Producer Prices Index: Economic Activities: Total Energy for the United States, FRED Series PIEAEN01USQ661N.

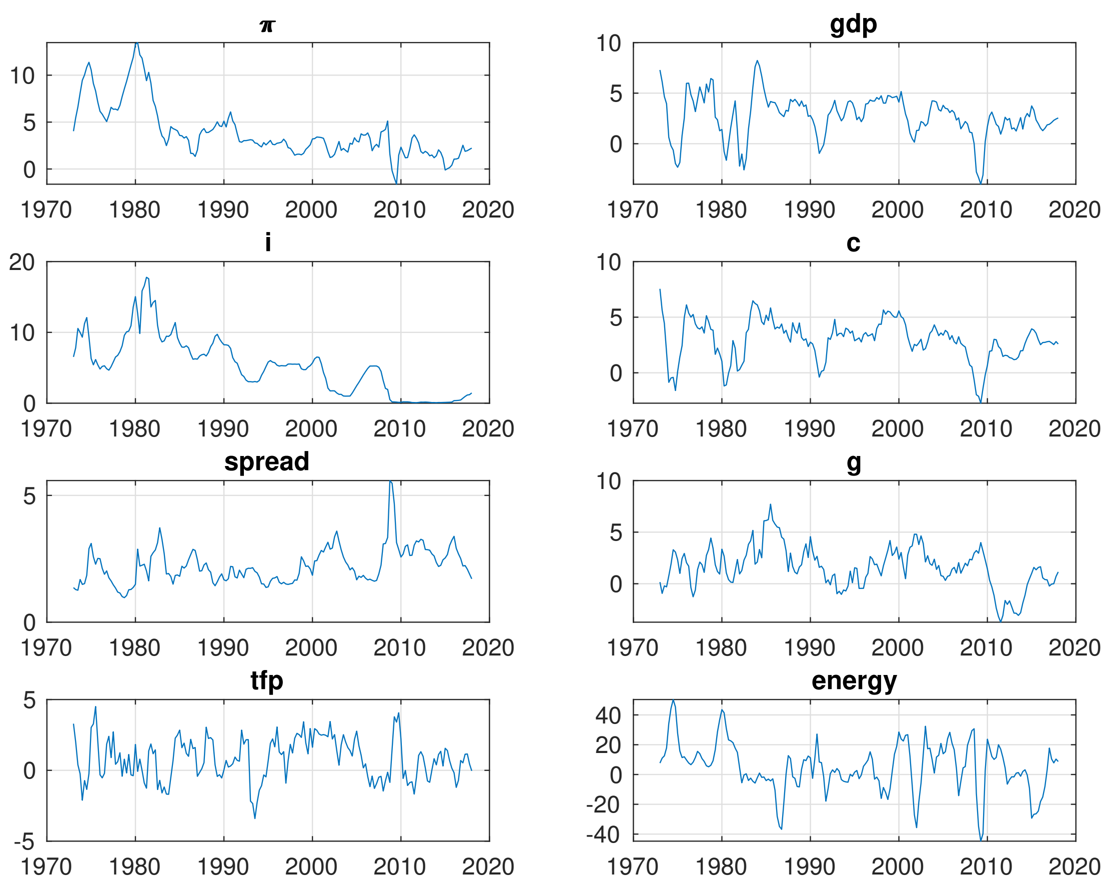

Figure A-1: Aggregate Data

B.2 Sectoral Level Data

We use PCE sectors throughout. For Industrial Production, the data originally was classified by 4-digit 2007 NAICS and was converted to PCE using the 2007 PCE Bridge Table published by the BEA.

- PCE Price Index (PCEPI): BEA Table 2.4.4U. Price Indexes for Personal Consumption Expenditures by Type of Product. See figure A-2 upper panel for a depiction of the data series.
- PCE Quantity Index (PCEQI): BEA Table 2.4.3U. Real Personal Consumption Expenditures by Type of Product, Quantity Indexes. See figure A-2 middle panel for a depiction of the data series.
- Industrial production index: This is the Fed Board of Governor's IP data. One can access the IP data release here: https://www.federalreserve.gov/releases/G17/. See figure A-2 lower panel for a depiction of the data series.
- R&D intensity: The ratio of RD expenditure to total revenue (sales). Provided by the NSF. The most recent data from the NSF, 2014, is used when available for that industry.
- External financing: Using capital expenditure and cash flow by firm and year from Compustat for 1979 to 2015, we can construct the external financing ratio as in Rajan and Zingales (1998), as one minus the ratio between cash flow to capital expenditure. Then matching each firm to its industry, we take the median capital expenditure value across firms for each industry and year. Then, we take the median again across years to obtain a single value for each industry.
- Household Consumption Share: We calculate the Household share as the proportion of output that goes to Personal Consumption Expenditures from the BEA IO Use Table.
- Government Consumption Share: We calculate the government share as the total output sold to all federal, state, and local government categories listed in the Use Table, divided by total industry output.
- Energy exposure: We take the ratio of intermediate inputs from energy sectors to total intermediate inputs using the BEA Use Table. Energy sectors are defined as electrical power generation, oil and gas extraction, natu- ral gas distribution, and petroleum and coal manufacturing.
- Price stickiness: The median price adjustment duration from Nakumura Steinsson (2008) across PCE categories. To capture the frequency of price changes within in industry, we take

the price adjustment durations estimated by Nakamura and Steinsson (2008). The estimates are provided at the Entry Line Item (ELI) level. By using the ELI/PCE crosswalk provided by the BLS, we can transfer these ELI level duration values to the PCE classification. For each PCE category, we assign the average of the duration values for the set of ELIs with which the PCE category is matched.

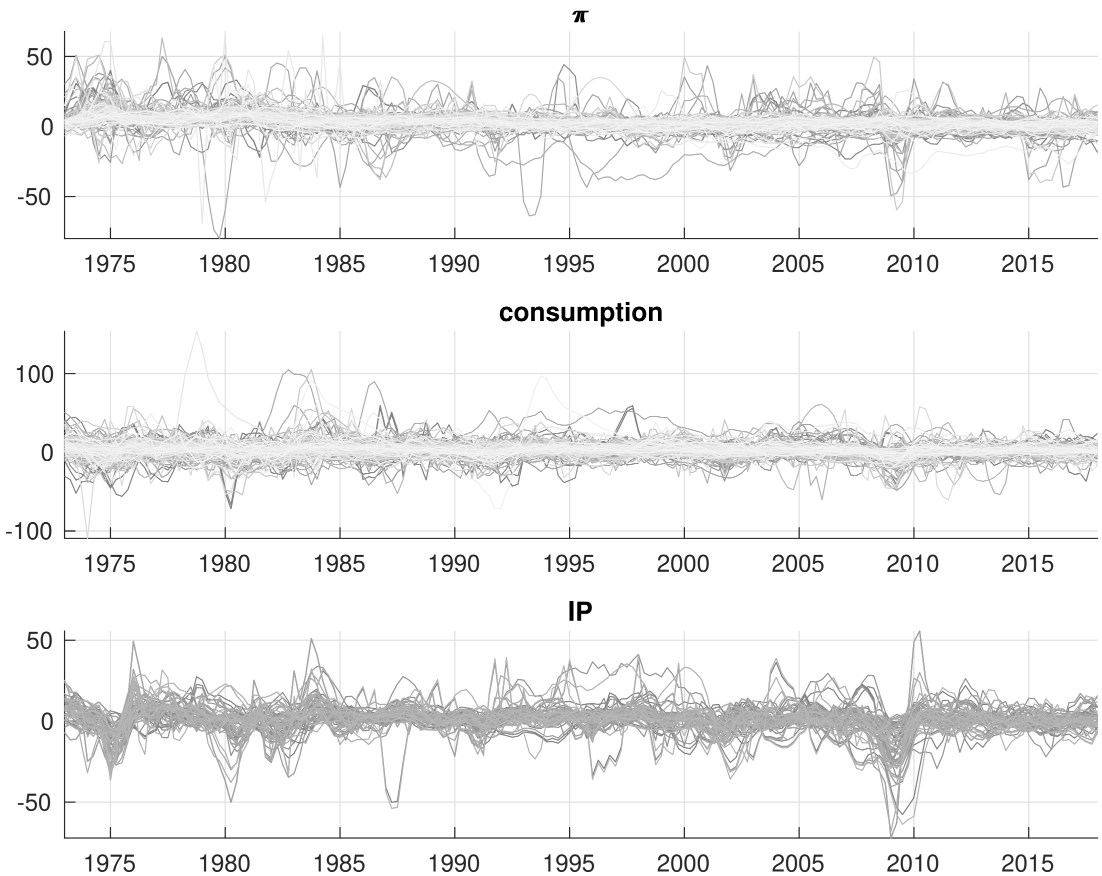

Figure A-2: Sectoral Data

C A Tractable Multi-Sector Model with Nominal Rigidites

We now lay out a tractable, multi-sector model with nominal rigidities to motivate the shock identification scheme. Nominal rigidities allow for a non-trivial "aggregate demand" channel. Since our main focus is in the cross-sectional differences between industries, rather than their individual dynamics, we lay out a static multi-sector economy. This is appropriate for our empirical analysis since we use identifying restrictions (via our priors) on the impact of shocks rather than on the dynamic responses to those. The model shares many elements with the framework developed in

Pasten et al. (2018), while also allowing for nominal wage stickiness and for several aggregate shocks.

C.1 Households

There are J sectors, indexed $i \in \{1,...,J\}$. There is a representative household with Cobb-Douglas preferences over the various goods, with share-parameter α_j for a good of industry i.

$$U = \prod_j C_j^{\alpha_j},$$

where $\sum_j \alpha_j = 1$. The household chooses its the amount it consumes of good i, C_j, to maximize its utility subject to the budget constraint

$$\sum_j P_j C_j + T = WL + \Pi + \sum_j r_j \bar{K}_j,$$

where T is a lump-sum tax levied by the government to finance its consumption, W is the wage rate, Π are profits rebated from firms, $\bar{K}_j$ is the stock of capital specific to sector i owned by the household, with r_j the corresponding rental rate, and $L < 1$ is employment to be determined in equilibrium.

Finally, households supply one unit of labor inelastically, but nominal wages are rigid so that labor is rationed.

Given those constraints, optimal household consumption choice satisfies:

$$P_j C_j = \alpha_j^C PC$$

for $P^C \equiv \prod_j \left(\frac{P_j}{\alpha_j}\right)^{\alpha_j}$ and $C \equiv \prod_j (C_j)^{\alpha_j}$.

C.2 Fiscal Authority

The fiscal authority minimizes the cost of consuming an exogenously given aggregate government consumption G,

$$\min \sum_j P_j G_j$$

$$s.t.: \prod_j (G_j)^{\alpha_j^G} = G,$$

where G is exogenously determined and α_j^G are expenditure shares. The optimality condition for the government is:

$$G_j = \alpha_j^G \frac{P_G}{P_j} G$$

where

$$P_G = \prod_j \left(\frac{G_j}{\alpha_j^G} \right)^{\alpha_j^G}.$$

C.3 Firms

Within each sector there is a continuum of varieties of intermediate products indexed $v \in [0,1]$. Those varieties are purchased by final goods producers that bundle them into the I goods according to a CES aggregator:

$$Y_j = \left[\int_0^1 Y_j(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}}$$

The demand for final good producer in sector i for intermediate input of variety v is

$$Y_j(v) = \left(\frac{P_j(v)}{P_j} \right)^{-\theta} Y_j$$

where

$$P_j = \left[\int P_j(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}}$$

For each variety, production takes place with a Cobb-Douglas production function:

$$Y_j(v) = e^{\varepsilon_j} \prod_j \left(X_{j'j}(v) \right)^{\gamma_{j'j}} \times \left(L_j(v) \right)^{\lambda_j} \left(K_j(v) \right)^{\chi},$$

where $X_{j'j}(v)$ is the quantity of final goods materials produced in sector j used as materials in sector i for variety v, $L_j(v)$ is labor, $K_j(v)$ is sector-specific capital, and ε_j is a sector-specific exogenous productivity shock. The share parameter for good j used in sector i is $\gamma_{j'j}$. We assume that $\sum_j \gamma_{j'j} + \lambda_j + \chi = 1$, so that firms in the industry face constant returns to scale.

Producers of varieties are monopolists. Firms differ on the information set available to them regarding prices and the demand for their intermediate input. Letting $\mathbf{s}$ denote the state of the economy, they take the wage rate, final goods prices, and household demand as given and choose their inputs to maximize expected profits.

$$\max_{M_{j'j}} E\left[P_j(v)Y_j(v,\mathbf{s}) - \sum_j P_j(\mathbf{s})X_{j'j}(v,\mathbf{s}) - w(\mathbf{s})L_j(v,\mathbf{s}) - r_j(\mathbf{s})K_j(v,\mathbf{s})|\mathscr{I}_j(v)\right]$$

$$s.t. : Y_j(v,\mathbf{s}) = \left(\frac{P_j(v)}{P_j(\mathbf{s})}\right)^{-\theta} Y_j(\mathbf{s})$$

$$Y_j(v,\mathbf{s}) = e^{\varepsilon_j}\prod_j \left(X_{j'j}(v,\mathbf{s})\right)^{\gamma_{j'j}} \left(L_j(v,\mathbf{s})\right)^{\lambda_j} \left(K_j(v,\mathbf{s})\right)^{\chi}$$

where $\mathscr{I}_j(v)$ is the information set for variety v in sector i. For a fraction ϕ_j of variety producers in sector i ($v \in [0,\phi_j]$) the information set does not includes the realized vector of shocks $\mathbf{s}$. For the remainder, the information set does includes it. Yet, firms commit to producing as much as necessary to satisfy demand at the prices that they choose.

Given cost-minimization, marginal cost is

$$\mathrm{mc}_j(\mathbf{s}) = e^{-\varepsilon_j}\prod_j \left(\frac{P_j(\mathbf{s})}{\gamma_{j'j}}\right)^{\gamma_{j'j}} \left(\frac{w(\mathbf{s})}{\lambda_j}\right)^{\lambda_j} \left(\frac{\mathbf{r}(\mathbf{s})}{\chi}\right)^{\chi}$$

Firms with full information set prices to

$$P_j(v,\mathbf{s}) = \frac{\theta}{\theta-1}\mathrm{mc}_j(s)$$

Firms without full information set prices to

$$P_j(v) = \frac{\theta}{\theta-1}E\left[\frac{P_j(\mathbf{s})^{\theta}Y_j(\mathbf{s})}{E\left[P_j(\mathbf{s})^{\theta}Y_j(\mathbf{s})\right]}mc_j(\mathbf{s})\right]$$

We thus have that the price index for sector i is

$$P_j(\mathbf{s}) = \left[\phi_j\left(\frac{\theta}{\theta-1}E\left[\frac{P_j(\mathbf{s})^{\theta}Y_j(\mathbf{s})}{E\left[P_j(\mathbf{s})^{\theta}Y_j(\mathbf{s})\right]}mc_j(\mathbf{s})\right]\right)^{1-\theta} + (1-\phi_j)\left(\frac{\theta}{\theta-1}\mathrm{mc}_j(\mathbf{s})\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Given that all firms in a sector have the same marginal cost, we can write the average markup as

$$\mu_j = \frac{P_j(\mathbf{s})}{mc_j(\mathbf{s})} = \left[\phi_j \frac{\theta}{\theta - 1} E\left[\frac{P_j(\mathbf{s})^{\theta} Y_j(\mathbf{s})}{E\left[P_j(\mathbf{s})^{\theta} Y_j(\mathbf{s})\right]} mc_j(\mathbf{s}) \right]^{1-\theta} \left(\frac{1}{mc_j(\mathbf{s})} \right)^{1-\theta} + (1 - \phi_j) \left(\frac{\theta}{\theta - 1} \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

C.4 Market Clearing

Market clearing for each sector i, requires that all output is used either as materials, for household consumption or for government consumption:

$$Y_j = \sum_j X_{jj'} + C_j + G_j$$

Also, there is a fixed stock of capital $\bar{K}_j$ for each sector. Market clearing in capital markets thus requires that the demand for capital in sector i equals supply:

$$K_j = \bar{K}_j$$

The resource constraint in the labor market is

$$\sum_j L_j \leq 1$$

With sticky wages the inequality need not hold. We assume that wages are stuck at a level high enough that it doesn't bind. Labor rationing thus implies that

$$L = \sum_j L_j$$

C.5 Shocks

As in Woodford (2003), we assume exogenous processes for nominal aggregates. In particular, we assume that nominal private consumption and nominal government consumption are set exogenously. Specifically, we assume that

$$P^C C = M^C M^Y$$
$$P^G G = M^G M^Y$$

so that nominal private and government consumptions can be affected either by an exogenous component which is specific to each type of final expenditure M^C or M^G, or by a common component

M^Y.

Finally, we also allow for industry level productivity shocks ε_j. We assume that $\varepsilon_j = \sum_{r=1}^{R} \lambda_{ir}\varepsilon_r + \hat{\varepsilon}_j$, where ε_r are aggregate shocks, F_j captures the sensitivity of various sectors to that shock, and $\hat{\varepsilon}_j$ is a sector-specific shock. In our application, we will allow ε_r to incorporate shocks to technology and financial shocks.

C.6 Log-linearized system

Up to a first-order approximation the economy is described by the following system of equations (small letters indicate log deviations from steady-state):

$$p^C + c = m^C + m^Y$$

$$p^G + g = m^G + m^Y \tag{A-7}$$

$$w = 0 \tag{A-8}$$

$$g_j - g = p^G - p_j \ \forall i \tag{A-9}$$

$$c_j - c = p^C - p_j \ \forall i \tag{A-10}$$

$$y_j = \varepsilon_j + \sum_j \gamma_{j'j} x_{j'j} + \lambda_j l_j + \chi k_j \ \forall i \tag{A-11}$$

$$w + l_j = p_j + y_j - \mu_j \ \forall i \tag{A-12}$$

$$p_j + x_{j'j} = p_j + y_j - \mu_j \ \forall i, j \tag{A-13}$$

$$r_j + k_j = p_j + y_j - \mu_j \ \forall i \tag{A-14}$$

$$k_j = \bar{k}_j \tag{A-15}$$

$$\mu_j = -\phi_j \left(\sum_j \gamma_{j'j} p_j + \lambda_j w + \chi r_j - \varepsilon_j \right) \tag{A-16}$$

$$y_j = \sum_j \frac{X_{jj'}}{Y_j} x_{jj'} + \frac{C_j}{Y_j} c_j + \frac{G_j}{Y_j} g_j \tag{A-17}$$

The system can be reduced to:

$$p_j - (1-\chi)\mu_j = -\varepsilon_j + \sum_j \gamma_{j'j} p_j + \chi\left(p_j + y_j - \bar{k}_j\right)$$

$$p_j + y_j = \sum_j \gamma_{jj'} \frac{Y_j}{Y_j}(y_j + p_j - \mu_j) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y)$$

$$\mu_j = -\frac{\phi_j}{1-\phi_j\chi}\left(-\varepsilon_j + \sum_j \gamma_{j'j} p_j + \chi\left(p_j + y_j - \bar{k}_j\right)\right)$$

Or, eliminating μ_j,

$$p_j = \frac{1-\phi_j}{1-\chi}\left(-\varepsilon_j + \sum_j \gamma_{j'j} p_j + \chi\left(y_j - \bar{k}_j\right)\right)$$

$$p_j + y_j = \sum_j \gamma_{jj'} \frac{Y_j}{Y_j}(y_j + \frac{1}{1-\phi_j} p_j) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y)$$

The system can be rewritten as

$$p_j = \frac{1-\phi_j}{1-\chi}\chi\left[\left(1-\chi\Phi_j\right)\left[\sum_j f_{jj'}(y_j + \frac{1}{1-\phi_j} p_j) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y)\right] + \Phi_j\left(\varepsilon_j + \chi\bar{k}_j\right)\right]$$
$$-\Phi_j\left(\varepsilon_j + \chi\bar{k}_j\right) + \Phi_j \sum_j b_{j'j} p_j$$

$$y_j = \left(1-\chi\Phi_j\right)\left[\sum_j f_{jj'}(y_j + \frac{1}{1-\phi_j} p_j) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y)\right] + \Phi_j\left(\varepsilon_j + \chi\bar{k}_j\right) - \Phi_j \sum_j b_{j'j} p_j$$

with $f_{jj'} = \gamma_{jj'}\frac{Y_j}{Y_j}$ capturing forward links and $b_{j'j} = \gamma_{j'j}$ capturing backward links

After log-linearizing and rearranging, the model can be reduced to:

$$p_j = \frac{1-\phi_j}{1-\chi}\left(-\varepsilon_j + \sum_j \gamma_{j'j} p_j + \chi\left(y_j - \bar{k}_j\right)\right)$$
$$p_j + y_j = \sum_j \gamma_{jj'} \frac{Y_j}{Y_j}\left(y_j + \frac{1}{1-\phi_j} p_j\right) + \frac{C_j}{Y_j}(m^C + m^Y) + \frac{G_j}{Y_j}(m^G + m^Y)$$

where small caps letters denote log deviations from a reference level. The first set of equations are "sectoral supply" equations, relating marginal production cost to prices. The second set of equations are "sectoral demand" equations, relating nominal expenditures to sectoral prices. The last set of equations link nominal consumption expenditures and exogenous demand shocks.

The system has the form

$$Z = AZ + b = A^N Z + \sum_{n=0}^{N-1} A^n b$$

with Z including prices and quantities in all sectors, b including the direct impact of all exogenous shocks, and A including the indirect impact of shocks through linkages.

Lemma 1 characterizes the direct and indirect impacts of the shocks on prices, output and consumption:

Lemma 1 *The direct impact of shocks is given by* $b = \left[\mathbf{p}^{Direct}, \mathbf{y}^{Direct}, \mathbf{c}^{Direct}\right]^T$, *where*

$$p_j^{Direct} = \Phi_j \chi \left[\frac{C_j}{Y_j} m^C + \frac{G_j}{Y_j} m^G + m^Y\right] - \Phi_j\left(\varepsilon_j + \chi \bar{k}_j\right) \tag{A-18}$$
$$y_j^{Direct} = \left(1 - \Phi_j \chi\right)\left[\frac{C_j}{Y_j} m^C + \frac{G_j}{Y_j} m^G + m^Y\right] + \Phi_j\left(\varepsilon_j + \chi \bar{k}_j\right) \tag{A-19}$$
$$c_j^{Direct} = \left(1 - \Phi_j \chi \frac{C_j}{Y_j}\right) m^C + (1 - \Phi_j \chi) m^Y - \Phi_j \chi \frac{G_j}{Y_j} m^G + \Phi_j\left(\varepsilon_j + \chi \bar{k}_j\right) \tag{A-20}$$

and

$$\Phi_j \equiv \frac{1-\phi_j}{\chi(1-\phi_j) + 1 - \chi}$$

is inversely related to ϕ_j. *Indirect effects are* $AZ = \left[\mathbf{p}^{Indirect}, \mathbf{y}^{Indirect}, \mathbf{c}^{Indirect}\right]^T$, *where*

$$p_j^{Indirect} = \Phi_j \sum_j \left(\chi \frac{f_{jj'}}{1-\phi_j} + b_{j'j} \right) p_j + \chi \Phi_j \sum_j f_{jj'} y_j \tag{A-21}$$

$$y_j^{Indirect} = \left(1 - \chi \Phi_j\right) \sum_j f_{jj'} y_j + \sum_j \left[\frac{1-\chi\Phi_j}{1-\phi_j} f_{jj'} - \Phi_j b_{j'j} \right] p_j \tag{A-22}$$

$$c_j^{Indirect} = -p_j^{Indirect} \tag{A-23}$$

where $f_{jj'} = \gamma_{jj'} \frac{Y_j}{Y_j}$ *capture forward linkages and* $b_{j'j} = \gamma_{j'j}$ *captures backward linkages.*

Lemma 1 implies that the direct impact of a consumption shock m^C on prices increases in $\Phi_j \chi \frac{C_j}{Y_j}$

D Dynamic Model

In what follows we present a dynamic model with multiple sectors, sticky nominal prices and sticky nominal wages. The exposition largely follows Justiniano et al. (2010), with some simplifications (we omit markup shocks) and extensions where needed.

D.1 Final good producers

There are J sectors (indexed $j \in [1,...,J]$). In each of these sectors there are perfectly competitive firms producing final goods Y_t^j combining a continuum of intermediate goods $\{Y_t(i)\}_r$, $i \in [0,1]$, according to the technology

$$Y_t^j = \left[\int_0^1 Y_t^j(i)^{\frac{\varepsilon^p - 1}{\varepsilon^p}} di \right]^{\frac{\varepsilon^p}{\varepsilon^p - 1}}$$

From profit maximization and zero profit conditions we have that

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon^p} Y_t^j$$

where P_t is the price of final good j and satisfies

$$P_t = \left[\int_0^1 P_t(i)^{\frac{1}{1-\varepsilon^p}} di \right]^{1-\varepsilon^p}$$

D.2 Intermediate good producers

A monopolist produces the intermediate good i in sector j according to the production function

$$Y_t^j(i) = \max\left\{\left(\frac{K_t^j(i)}{(1-\gamma^j)\omega^j}\right)^{(1-\gamma^j)\omega^j}\left(\frac{A_t^j L_t^j(i)}{(1-\gamma^j)(1-\omega^j)}\right)^{(1-\gamma^j)(1-\omega^j)}\prod_{j'}\left(\frac{M_t^{j'j}(i)}{\gamma^{j'j}}\right)^{\gamma^{j'j}} - F^j, 0\right\}$$

where $K_t^j(i)$, $L_t^j(i)$ denote the amounts of capital and labor emploiyed by firm i in sector j, $M_t^{j'j}(i)$ is the amount of materials produced in sector j' used by firm i in sector j and F^j is a fixed cost of production, chosen so that profits are zero in stead state. A_t^j respresents exogenous technological progress in sector j. We assume that it consists of a combination of aggregate and sector specific components:

$$A_t^j = A_t \widehat{A}_t^j$$

where

$$\ln A_t = \rho^A \ln A_{t-1} + \varepsilon_t^A$$

where ε_t^A is *iid* with standard deviation σ^A

Furthermore,

$$\ln \widehat{A}_t^j = (1-\rho^{A^j})\ln \widehat{A}^j + \rho^{A^j}\ln \widehat{A}_{t-1}^j + \varepsilon_t^{A,j}$$

where $\varepsilon_t^{A,j}$ has, likewise, standard deviation σ^{A^j} Every period in each sector j, a fraction ξ^{pj} of intermediate firms cannot choose its price optimally, and as in Smets and Wouters (2003), they reset it according to the indexation rule

$$P_t(i) = P_{t-1}(i)\left(\Pi_{t-1}^j\right)^{\iota^p}\Pi^{1-\iota^p},$$

where $\pi_t^j = \frac{P_t^j}{P_{t-1}^j}$ is gross sector j inflation and π is its steady state. The remaining fraction of firms chooses its price $P_t(i)$ optimally, by maximizing the present discounted value of future profits

$$E_t\left\{\sum_{s=0}^{\infty}\left(\xi^{pj}\right)^s\frac{\beta^s\Lambda_{t+s}}{\Lambda_t}\left[P_t(i)\left(\Pi_{t,t+s}^j\right)Y_{t+s}(i) - W_{t+s}^j L_{t+s}(i) - R_{t+s}^{k,j}K_{t+s}(i) - \sum_{j'}P_{t+s}^{j'}(i)M_{t+s}^{j'}(i)\right]\right\}$$

where

$$
\Pi^j_{t,s} \equiv \prod_{k=1}^{s} \left(\Pi^j_{t+k-1}\right)^{\iota^p} \Pi^{(1-\iota^p)k} \text{ for } s \geq 1
$$
$$
\Pi^j_{t,t} = 1
$$

and

$$
Y_{t+s}(i) = \left(\frac{P_{t+s}(i)}{P_{t+s}}\right)^{-\varepsilon^p} Y^j_{t+s}
$$

subject to the demand function and to cost minimization. In this objective, Λ_t is the marginal utility of nominal income for the representative household that owns the firm, while W_t and $r^{k,j}_t$ are the nominal wage and the rental rate of capital specific to sector j.

Cost minimization by firms implies that

$$
\frac{K^j_t(i)}{L^j_t(i)} = \frac{W^j_t}{R^{k,j}_t}\frac{\omega^j}{1-\omega^j}
$$

and

$$
\frac{M^{j'j}_t(i)}{L^j_t(i)} = \frac{W^j_t}{P^{j'}_t}\frac{\gamma^{j'j}}{(1-\gamma^j)(1-\omega^j)},
$$

so that nominal marginal cost in sector j is common to all firms and given by

$$
MC^j_t = \left(R^{k,j}_t\right)^{(1-\gamma^j)\omega^j} \left(\frac{W^j_t}{A^j_t}\right)^{(1-\gamma^j)\left(1-\omega^j\right)} \prod_{j'} \left(P^{j'}_t\right)^{\gamma^{j'j}}.
$$

Substituting back input choices, and ignoring the fixed costs, yields employment in each variety as a function of sectoral output and the price of the variety,

$$
L^j_t(i) = (1-\gamma^j)(1-\omega^j)\frac{MC^j_t}{W^j_t}\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon^p} Y^j_t.
$$

Integrating both sides yields sectoral employment:

$$
L^j_t = (1-\gamma^j)(1-\omega^j)\frac{MC^j_t}{W^j_t} P^{\varepsilon^p}_t Y^j_t \int P_t(i)^{-\varepsilon^p} di.
$$

From the intermediate input demand function,

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon^p} Y_t^j.$$

Given that, with our production function, average variable costs and marginal costs coincide, the objective function for firms setting prices optimally can be rewritten as

$$\max_{P_t(i)} E_t \left[\sum_{s=0}^{\infty} \left(\xi^{pj}\right)^s \frac{\beta^s \Lambda_{t+s}}{\Lambda_t} \left[\left(P_t^j(i)\Pi_{t,t+s}^j - MC_t\right) Y_{t+s}(i)\right]\right]$$
$$s.t. : Y_{t+s}^j(i) = \left(\frac{P_t(i)\Pi_{t,t+s}^j}{P_{t+s}}\right)^{-\varepsilon^p} Y_{t+s}^j$$

The first order condition can then be written as

$$\tilde{P}_t^j = \frac{\varepsilon^p}{\varepsilon^p - 1} \sum_{s=0}^{\infty} \frac{E_t \left\{\left(\beta \xi^{pj}\right)^s \Lambda_{t+s} \tilde{Y}_{t+s}^j MC_{t+s}^j\right\}}{\sum_{s=0}^{\infty} E_t \left\{\left(\beta \xi^{pj}\right)^s \Lambda_{t+s} \tilde{Y}_{t+s}^j \Pi_{t,t+s}^j\right\}}$$

where $\tilde{P}_t^j$ is the optimally chosen price for all firms i choosing their prices in period t (so that $P_t^j(i) = \tilde{P}_t^j$), and $\tilde{Y}_{t+s}$ is the demand they face in $t+s$.

Alternatively,

$$\frac{\tilde{P}_t^j}{P_t} = \frac{\varepsilon^p}{\varepsilon^p - 1} \sum_{s=0}^{\infty} \frac{E_t \left\{\left(\beta \xi^{pj}\right)^s \Lambda_{t+s} P_{t+s} \left(\tilde{Y}_{t+s}^j\right) \frac{MC_{t+s}^j}{P_t^j}\right\}}{\sum_{s=0}^{\infty} E_t \left\{\left(\beta \xi^{pj}\right)^s \Lambda_{t+s} P_{t+s} \left(\tilde{Y}_{t+s}^j\right) \left(\Pi_{t,t+s}^j / \Pi_{t,t+s}\right)\right\}}$$

where

$$\Pi_{t,s} \equiv \prod_{k=1}^{s} \Pi_{t+k} \text{ for } s \geq 1$$
$$\Pi_{t,t} = 1$$

D.3 Employment Agencies

Workers have monopoly power over their labor supply. There is a competitive employment agency which combines specialized household labor into a homogeneous labor input sold to firms in sector j according to

$$L_t^j = \left[\int L_t^j(h)^{\frac{\varepsilon^w - 1}{\varepsilon^w}} dh \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}}.$$

Profit maximization implies that

$$L_t^j(h) = \left(\frac{W_t^j(h)}{W_t^j} \right)^{-\varepsilon^w} L_t^j,$$

and the wage paid by firms for homogeneous labor input is

$$W_t^j = \left[\int_0^1 W_t^j(h)^{1-\varepsilon^w} dh \right]^{\frac{1}{1-\varepsilon^w}}$$

D.4 Households

Each household (h) has labor which is specific to some sector j and utility function given by

$$U_t = \sum_s E_t \beta^s b_{t+s} \left[\ln\left[X_{t+s}(h)\right] - \sum_j \frac{\varphi^j}{1+\nu} L_t^j(h)^{1+\nu} \right],$$

where

$$X_{t+s}(h) = \prod_j \left(C_{t+s}^j(h) - \eta C_{t+s-1}^j \right)^{\alpha_t^j},$$

and where $C_{t+s}^j(i)$, $L_t(i)$ and $X_{t+s}(i)$ are household choices and X_{t+s} and C_{t+s}^j are equilibrium objects that the household takes as given. The formulation corresponds to allowing for habits to consumption of particular goods.

To allow for sector-specific demand shocks, we allow consumption shares, α_t^j to be time-varying. Specifically[3]

$$\ln \alpha_t^j = (1-\rho^\alpha)\alpha^j + \rho^\alpha \ln \alpha_{t-1}^j + \varepsilon_t^{\alpha,j}$$

where ε_t^α is a random normal variable with standard deviation σ^{α^j}. The time-varying parameter b_t is a shock to the discount factor, affecting both the marginal utility of consumption and the marginal disutility of labor. This intertemporal preference shock follows the stochastic process

$$\Delta \log b_t = \rho^b \Delta \log b_{t-1} + \varepsilon_{b,t}$$

where Δ is the time-difference operator and $\varepsilon_{b,t}$ is an *iid* random normal variable with mean zero

[3] While this formulation constrains share parameters to be positive, it does not constrain them to add up to 1. Allowing for this degree of freedom is necessary to give the ability to match the full set of sector-specific variables.

and standard deviation σ^b. There are state contingent securities ensuring that in equilibrium consumption and asset holdings are the same for all households. As a result, the household's flow budget constraint is

$$\sum_j P_t^j C_t^j + \sum_{j,j'} P_t^{j'} I_t^{j'j} + T_t + B_t \leq R_{t-1} B_{t-1} + Q_t(j) + \Pi_t + W_t^j(j) L_t(j) + \sum_j R_t^{k,j} K_{t-1}^j,$$

where $I_t^{j'j}$ is investment in good j' to form capital in sector j, T_t is lump-sum taxes, B_t is holdings of government bonds, R_t is the gross nominal interest rate, $Q_t(j)$ is the net cas flow from household's j portfolio of state contingent securities, and Π_t is the per-capital profit accruing to households from ownership of the firms.

Consumption Given interest rates on riskless debt R_t, the problem induces the Euler equation:

$$\Lambda_t = \beta R_t E_t \Lambda_{t+1},$$

where $P_t = \prod_j \left(\frac{P_t^j}{\alpha_t^j} \right)^{\alpha_t^j}$ is the consumption price index and $\Lambda_t \equiv \frac{b_t}{P_t X_t}$ is the "nominal" marginal utility of consumption. Given that we get the intra-temporal allocation across industries:

$$C_t^j(h) = \alpha_t^j \frac{P_t}{P_t^j} X_t(h) + \eta C_{t-1}^j.$$

The model features a representative household, so that in equilibrium, $C_t^j = C_t(h)$.

Capital accumulation Households own capital specific to each sector j and rent them to firms at the rate $R_t^{k,j}$. The physical capital accumulation equation is

$$K_t^j = (1-\delta) K_{t-1}^j + \left(1 - S\left(\frac{I_t^j}{I_{t-1}^j} \right) \right) I_t^j,$$

where δ is the depreciation rate and is the investment in sector j. The function S captures the presence of adjustment costs in investment, as in Christiano, Eichenbaum, and Evans (2005). In steady state, $S = S' = 0$ and $S'' > 0$.

Production of investment goods in sector j require using goods produced by other sectors according to the production function

$$I_t^j = B_t^j \prod_{j'} \left(\frac{I_t^{j'j}}{\gamma_I^{j'j}} \right)^{\gamma_I^{j'j}}$$

where $I_t^{j'j}$ is the quantity of goods produced in sector j' used for investment in sector j. The production function for investment in each sector is scaled by an investment-specific productivity shock B_t^j. Like the labor-augmenting productivity shock A_t^j, B_t^j has both aggregate and an idiosyncratic components:

$$B_t^j = B_t \widehat{B}_t^j$$

where

$$\ln B_t = \rho^B \ln B_{t-1} + \varepsilon_t^B$$

and

$$\ln \widehat{B}_t^j = \rho^B \ln \widehat{B}_{t-1}^j + \varepsilon_t^{Bj}$$

where ε_t^B and ε_t^{Bj} are *iid* normal variables with zero mean and variance σ^B and σ^{Bj}, respectively. We assume that they have a common persistence parameter ρ^B.

The optimal choice of physical capital stock for sector j satisfies the optimality conditions:

$$\begin{aligned}
\chi_t^j &= \beta E_t \left[R_{t+1}^{k,j} \Lambda_{t+1} + (1-\delta) \chi_{t+1}^j \right], \\
P_t^{j'} \Lambda_t &= \gamma_I^{j'j} \frac{I_t^j}{I_t^{j'j}} \left[\chi_t^j \left[1 - S\left(\frac{I_t^j}{I_{t-1}^j} \right) - S'\left(\frac{I_t^j}{I_{t-1}^j} \right) \frac{I_t^j}{I_{t-1}^j} \right] + \beta S'\left(\frac{I_{t+1}^j}{I_t^j} \right) \left(\frac{I_{t+1}^j}{I_t^j} \right)^2 \chi_{t+1} \right],
\end{aligned}$$

where χ_t is the multiplier on the capital accumulation equation. Defining Tobin's q for sector j as $Q_t^j = \frac{\chi_t^j}{P_t^{I,j} \Lambda_t} = \frac{P_t \chi_t^j}{P_t^{I,j} b_t [X_t(h)]^{-\sigma}}$, where $P_t^{I,j} = \prod \left(P_t^{j'} \right)^{\gamma^{j'j}}$, the relative marginal value of installed capital with respect to consumption, we can also write

$$\begin{aligned}
Q_t^j &= \beta E_t \left[\frac{R_{t+1}^{k,j} \Lambda_{t+1}}{P_t^{I,j} \Lambda_t} + \frac{P_{t+1}^{I,j} \Lambda_{t+1}}{P_t^{I,j} \Lambda_t} (1-\delta) Q_{t+1}^j \right], \\
1 &= \left[Q_t^j \left[1 - S\left(\frac{I_t^j}{I_{t-1}^j} \right) - S'\left(\frac{I_t^j}{I_{t-1}^j} \right) \frac{I_t^j}{I_{t-1}^j} \right] + \beta \frac{\Lambda_{t+1} P_{t+1}^{I,j}}{\Lambda_t P_t^{I,j}} S'\left(\frac{I_{t+1}^j}{I_t^j} \right) \left(\frac{I_{t+1}^j}{I_t^j} \right)^2 Q_{t+1}^j \right].
\end{aligned}$$

Wage setting Every period a fraction ξ^w of households cannot freely set its wage, but follows the indexation rule

$$W_t^j(j) = W_{t-1}^j(j)\left(\pi_{t-1}e^{z_{t-1}}\right)^{\iota^w}(\pi)^{1-\iota^w}.$$

The remaining fraction of households chooses instead an optimal wage $W_t(j)$ by maximizing

$$E_t\left\{\sum_{s=0}^{\infty}\xi^{ws}\beta^s\left[-b_{t+s}\varphi^j\frac{L_{t+s}^j(h)^{1+\nu}}{1+\nu}+\Lambda_{t+s}\Pi_{t,t+s}^w W_t^j(h)L_{t+s}^j(h)\right]\right\},$$

where

$$\Pi_{t,t+s}^w = \prod_{\nu=1}^{s}\left(\Pi_{t+\nu-1}e^{z_{t+\nu-1}}\right)^{\iota^w}(\Pi)^{\nu(1-\iota^w)} \text{ if } s \geq 1$$
$$\Pi_{t,t}^w = 1$$

subject to the labor demand function of the employment agencies.

The F.O.C. for a wage chosen by household h to work in industry j is to maximize

$$E_t\left\{\sum_{s=0}^{\infty}\xi^{ws}\beta^s\left[-b_{t+s}\varphi\frac{L_{t+s}^j(h)^{\nu}}{1+\nu}+\Lambda_{t+s}\Pi_{t,t+s}^w W_t^j(h)L_{t+s}^j(h)\right]\right\},$$

subject to the demand of the employment agency,

$$L_t^j(h) = \left(\frac{W_t^j(h)}{W_t^j}\right)^{-\varepsilon^w} L_t^j,$$

The F.O.C. is

$$\begin{aligned}
&E_t\left\{\sum_{s=0}^{\infty}\xi^{ws}\beta^s\left[b_{t+s}\varphi\left[\left(\frac{\Pi_{t,t+s}^w W_t^j(h)}{W_{t+s}^j}\right)^{-\varepsilon^w} L_{t+s}^j\right]^{1+\nu}\frac{1}{W_t^j(h)}\right]\right\}\\
&\quad = E_t\left\{\sum_{s=0}^{\infty}\xi^{ws}\beta^s\left[\Lambda_{t+s}\Pi_{t,t+s}^w\left[\left(\frac{\Pi_{t,t+s}^w W_t^j(h)}{W_{t+s}^j}\right)^{-\varepsilon^w} L_{t+s}^j\right]\right]\right\},
\end{aligned}$$

which can be rewritten as

$$\left(\tilde{W}_t^j\right)^{1+\nu\varepsilon^w} = \frac{\varepsilon^w}{\varepsilon^w - 1} \frac{E_t\left\{\sum_{s=0}^{\infty} \xi^{ws}\beta^s \left[b_{t+s}\varphi^j \left[\left(\frac{\Pi^w_{t,t+s}}{W^j_{t+s}}\right)^{-\varepsilon^w} L^j_{t+s}\right]^{1+\nu}\right]\right\}}{E_t\left\{\sum_{s=0}^{\infty} \xi^{ws}\beta^s \Lambda_{t+s}\Pi^w_{t,t+s}\left(\frac{\Pi^w_{t,t+s}}{W^j_{t+s}}\right)^{-\varepsilon^w} L^j_{t+s}\right\}}$$

D.5 The government

A monetary policy authority sets the nominal interest rate following a feedback rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho^R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_X}\right]^{1-\rho^R} \eta_{mp,t},$$

where R is the steady-state of the gross nominal interest rate. As in Smets and Wouters (2003), interest rates responds to deviations of inflation from its steady state, as well as to the level and growth rate of the GDP ($Y_t = \sum \gamma^j \frac{P_t^j}{P_t} Y_t^j$). The monetary policy rule is also perturbed by a monetary policy shock $\eta_{mp,t}$, is *iid* $N(0, \sigma^2_{mp})$.

Fiscal policy is fully Ricardian. The government finances its budget deficit by issuing short term bonds. Public spending is determined exogenously as a time varying fraction of output:

$$G_t = \left(1 - \frac{1}{\zeta_t}\right) Y_t$$

where the government spending shock ζ_t follows the stochastic process

$$\log \zeta_t = (1-\rho^G)\zeta + \rho^G \log \zeta_{t-1} + \varepsilon_t^G.$$

where ε_t^G is *iid* normal random variable with standard deviation σ^G.

Public spending is a Cobb-Douglas aggregate of spending in different sectors. The government chooses sector-specific spending to minimize the cost of G_t:

$$\left\{G_t^j\right\}_j = \arg\min \sum_j P_t^j G_t^j$$

$$s.t.: \prod \left(G_t^j\right)^{\alpha_G^j} = G_t$$

so that

$$G_t^j = \alpha_G^j \frac{P_t^G}{P_t^j} G_t$$

where $P_t^G = \prod \left(\frac{P_t^j}{\alpha_j^G} \right)^{\alpha_j^G}$

D.6 Market clearing

The aggregate resource constraint for each sector j is

$$C_t^j + \sum_{j'} I_t^{jj'} + \sum_{j'} M_t^{jj'} + G_t^j = Y_t^j$$

D.7 Model Solution and Calibration

To solve the model we first write it in terms of stationary variables (detrended the permanent part of TFP for real output variables and by the price level for nominal variables), log-linearize it and find the rational expectations equilibrium using Dynare.

The calibration builds onJustiniano et al. (2010) and Carvalho et al. (2021). We furthermore use information from sectoral linkages and consumer shares obtained from the input-output tables made available by the BEA and on sector-specific price stickiness from Nakamura and Steinsson (2008). Tables A-1 and A-2 list the calibrated parameters together with their sources.

Parameter	Description	Value	Source
N	Number of Sectors	52	
ζ	1/steady-state government share of output	2.70	G/Y = 37%
δ	Capital depreciation	0.05	Justiniano et al. (2010)
β	Discount Factor	1.00	Justiniano et al. (2010)
ν	Inverse Frisch elasticity of labor supply	3.79	Justiniano et al. (2010)
η	Consumption habit parameter	0.78	Justiniano et al. (2010)
ε^w	Elasticity of substition for employment	1.87	Justiniano et al. (2010)
ε^p	Elasticity of substition for goods	1.81	Justiniano et al. (2010)
ξ^w	Calvo parameter (wages)	0.70	Justiniano et al. (2010)
ι^p	Indexation coefficient for prices	0.24	Justiniano et al. (2010)
ι^w	Indexation coefficient for wages	0.11	Justiniano et al. (2010)
I''	Investment adjustment cost parameter	2.85	Justiniano et al. (2010)
ϕ_x	Taylor rule, coefficient on output	0.24	Justiniano et al. (2010)
ϕ_π	Taylor rule, coefficient on inflation	2.09	Justiniano et al. (2010)
Π	steady-state inflation rate	0.03	Justiniano et al. (2010)
ρ^R	Taylor rule, smoothing parameter	0.82	Justiniano et al. (2010)
ρ^A	Persistence aggregate TFP	0.99	Carvalho et al. (2019)
ρ^{A^j}	Persistence sectoral TFP shock	0.93	Carvalho et al. (2019), average persistence for sectoral demand shock
ρ^G	Persistence government spending shock	0.99	Justiniano et al. (2010)
ρ^b	Persistence intertemporal preference shock	0.94	Carvalho et al. (2019), average persistence for sectoral demand shock
ρ^B	Persistence investment-specific TFP	0.72	Justiniano et al. (2010)
ρ^α	Persistence sectoral Demand shock	0.94	Carvalho et al. (2019), average persistence for sectoral demand shock
σ^η	Volatility to monetary shock	0.001	Carvalho et al. (2019), adjusted for iid monetary shocks
σ^A	Volatility, aggregate TFP	0.003	Carvalho et al. (2019)
σ^{A^j}	Volatility, sectoral TFP	0.003	Carvalho et al. (2003), average across sectors
σ^G	Volatility, government shock	0.00	Justiniano et al. (2010)
σ^B	Volatility, investment-specific TFP	0.06	Justiniano et al. (2010)
σ^{B^j}	Volatility, sectoral investment productivity	0.06	Same proportion to aggregate as A shock
σ^b	Volatility, preference shock	0.15	see text
σ^{α^j}	Volatility, consumption share	0.019	Carvalho et al. (2003), average across sectors

Table A-1: Calibration of Aggregate Parameters

Parameter	Description	Source
α^j	steady-state consumption share	BEA use tables
α_G^j	government consumption share	BEA use tables
φ_j	Disutility of labor of type j parameter	calibrated so steady-state wage is the same for all sectors
ω_j	Capital Share (by sector)	BEA use tables
γ_j	Materials Share (by sector)	BEA use tables
$\gamma_I^{j'j}$	Share of sector j' in sector j investment	Capital flow table
ξ_j^p	Calvo parameter (prices)	Nakamura and Steinsson (2008)

Table A-2: Sectoral Parameters

E Selected Impulse Responses for the Simulation-based experiment

Our model has variables for 182 sectors as well as 8 aggregate variables. This leads us to focus on the estimated shock series as a low dimensional check in the main text. Nonetheless, we want to give readers a sense of the estimated impulse responses. Below we plot the responses of GDP and consumption. The true impulse responses of those variables in the DSGE model are very similar. As expected, the estimated impulse responses in our model are then very similar across these two aggregate variables. As Figure A-3 shows, we are able to replicate the patterns of the true impulse responses.

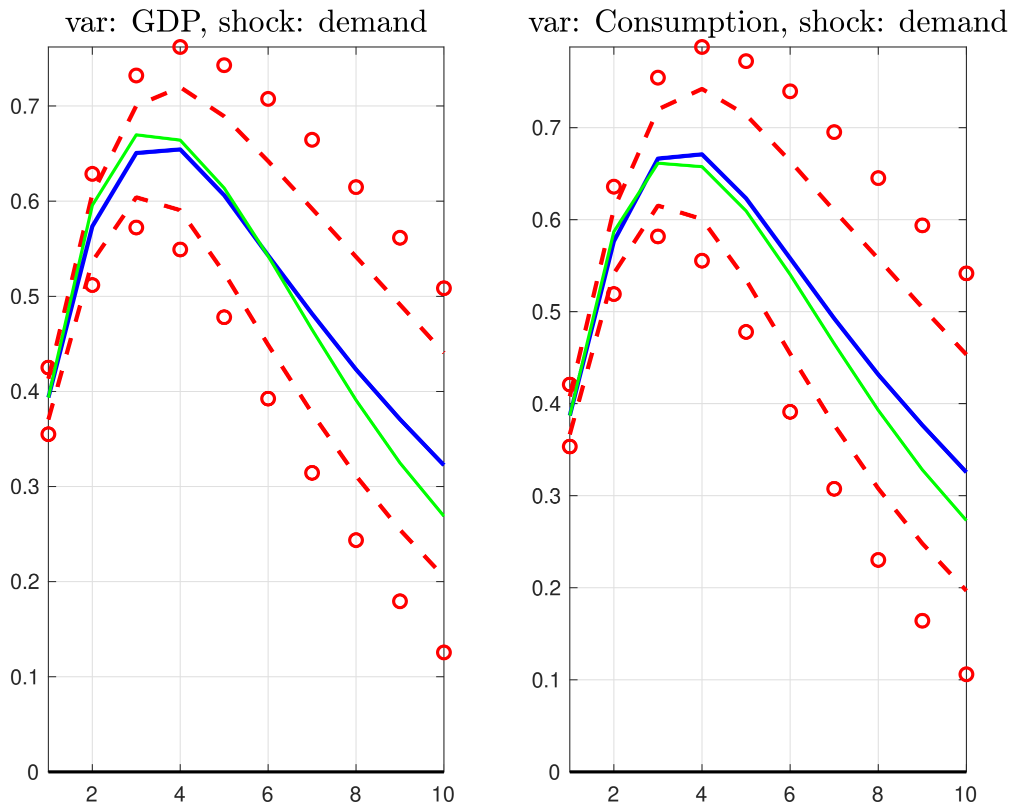

Figure A-3: Responses to Household Demand Shock for consumption and GDP in Monte Carlo exercise. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis shows time in quarters. DSGE-model based IRF in green (normalized to coincide with the median estimated IRF on impact).

F Results with $T = 1,000$

We simulate 1,000 observations from our benchmark DSGE model. As can be seen from Figure A-4, the results are similar to the results in the main text. This confirms that with a macro standard sample size we already achieve what is possible with our specific identification assumptions (as we discuss in the main text, if a researcher had more detailed information on the sectoral responses, that researcher could improve on our benchmark approach using sectoral differences in C/Y ratios, but that is practically infeasible).

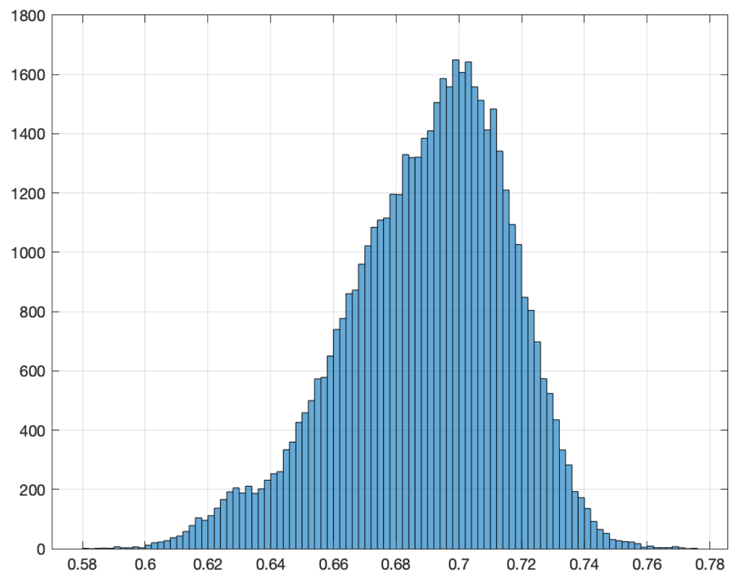

Figure A-4: Posterior of β^i, DGP with 1,000 observations.

G The Prior for the Household Shock

Table A-3 shows the percentiles (across sectors) of the prior mean of the relevant entries of D^i for the household shock. We focus on sectoral inflation and consumption since those variables are available for all sectors. The prior means completely characterize the Gaussian priors since we set the prior standard deviation equal to a fixed fraction of the absolute value of the prior mean.

Variable	5th Percentile	Median	95th Percentile
Inflation	0.07	0.60	1.48
Consumption	0.13	1.03	2.07

Table A-3: Prior on the Impact of the Household Shock.

H Asymptotic Posterior Distribution of D^Z

We can make some progress towards characterizing the asymptotic behavior of the marginal posterior of D. Our prior $p(D^Z, \theta)$ is absolutely continuous with respect to the likelihood function $\mathscr{L}(D^Z, \theta | Z)$ where Z is the array of all observations on Z_t and θ is the vector of all parameters except D^Z.[4] VAR and factor model identification arguments imply that under standard regularity conditions (including linearity and Gaussian innovations) all parameters except D^Z are identified - even with infinite data we can only identify $D^Z D^{Z'}$. All other parameters converge to a unique limiting value θ^* such that the asymptotic posterior $p^*(D^Z, \theta|Z)$ (with conditional distribution $p^*(D^Z|Z, \theta)$ and marginal distribution $p^*(D^Z|Z)$) is given by

$$p^*(D^Z, \theta^*|Z) = p^*(D^Z|Z, \theta = \theta^*) = p^*(D^Z|Z)$$

This equivalence between joint, conditional, and marginal asymptotic posterior is due to the fact that asymptotically the marginal posterior for θ will be degenerate and only have mass at θ^*. Let's define the limit of $D^Z D^{Z'}$ as the sample size T grows large:

$$\lim_{T \to \infty} D^Z D^{Z'} = \phi$$

where this limit should be understood to mean that asymptotically the joint posterior $p(D^Z, \theta|Z)$ will be equal to 0 except when $\theta = \theta^*$ and $D^Z D^{Z'} = \phi$. Then the asymptotic marginal posterior of D^Z (denoted by $p^*(D^Z|Z)$) is the prior restricted to those values of D^Z consistent with ϕ:

$$p^*(D^Z|Z) = p(D^Z|D^Z D^{Z'} = \phi)$$

Applying Bayes' rule to the conditional prior yields:

$$p(D^Z|D^Z D^{Z'} = \phi) = \frac{p(D^Z D^{Z'} = \phi|D^Z)p(D^Z)}{p(D^Z D^{Z'} = \phi)}$$

The first term in the numerator $p(D^Z D^{Z'} = \phi|D^Z)$ can be interpreted as an indicator function because it will only be non-zero when a value for D^Z is consistent with $D^Z D^{Z'} = \phi$. The second term in the numerator is just the prior $p(D^Z)$. The term in the denominator is a normalizing constant that will be independent of D^Z for all values of D^Z such that $D^Z D^{Z'} = \phi$.

I Validating our approach: A Monte Carlo experiment with a Hi-VAR DGP

This section describes the results of an experiment that is meant to highlight the amount of additional information that sectoral information brings to bear on identifying structural shocks of interest.

[4] Since our priors on blocks of parameters are either Gaussian or inverse Wishart this assumption is satisfied in our model.

We simulate one dataset[5] of 170 observations (roughly the size of our actual sample) and discuss results for two sets of priors. We assume there are 4 aggregate variables, 180 sectors (in line with the number of sectors in our actual sample), and 2 observables per sector. All lag lengths (in both the data-generating process and the estimated model) are set to 1 for simplicity. The aggregate VAR coefficients in the data-generating process are set so that all variables are stationary, but persistent. The VAR coefficient matrices for each sector are drawn at random subject to the constraint that dynamics are stationary. We set the values of Ω, Ω^i, and the loadings on the two structural shocks for all variables in such a way that the structural shocks explain a small fraction of the variance at the sectoral level, as depicted in Figure A-5. These fractions are substantially smaller than what we find with our posterior estimates, both at the aggregate and sectoral level, so we are tying our hands with this conservative choice - we are consciously making this exercise hard for our approach. Furthermore, to mimic our empirical setting, we allow the loading on the structural shocks to be correlated within sectors across variables and across sectors.[6] The priors for the shock loadings are centered at the true value. The variance is set in the same fashion as in the empirical analysis of the main text.

We now ask two related questions: (i) How well does the posterior median of the structural shock series line up with the true value? and (ii) Is the estimation uncertainty small enough to draw meaningful conclusions from such an estimation?

We first set the prior means of the effects equal to their true value, and their standard deviations as in the empirical analysis, to be half the absolute values of the prior means.

Figure A-6 plots the true shock series, the posterior medians as well as 98 percent posterior bands centered at the median. We see that the posterior median capture the true evolution of the shock very well (the correlations are 0.93 for both shocks) and the posterior uncertainty surrounding the estimates are small. Why is the posterior uncertainty small? While each piece of identification information we use is not very informative, with a large number of sectors, the set of identification restrictions implicit in our priors is actually informative. This is reminiscent of results in standard dynamic factor models, where the model can become exactly identified even when using standard sign restrictions when the number of sign restrictions grows to infinity (Amir-Ahmadi and Uhlig (2015)). On top of that we get additional identification strength from using information on magnitudes, as highlighted by Amir-Ahmadi and Drautzburg (2020).

As depicted in Figure A-6, we can identify the structural shocks with great accuracy. In the main text we discuss that knowledge of loadings of *other shocks* is not necessary to identify the loadings of one specific shock. To highlight this feature, we now re-estimate our model with the same simulated

[5] We show that even with one datatset the evidence in favor of using sectoral information is so strong that we don't need to simulate a larger number of samples.

[6] We draw all these sectoral coefficients jointly from a multivariate Gaussian distribution with correlation coefficient 0.5.

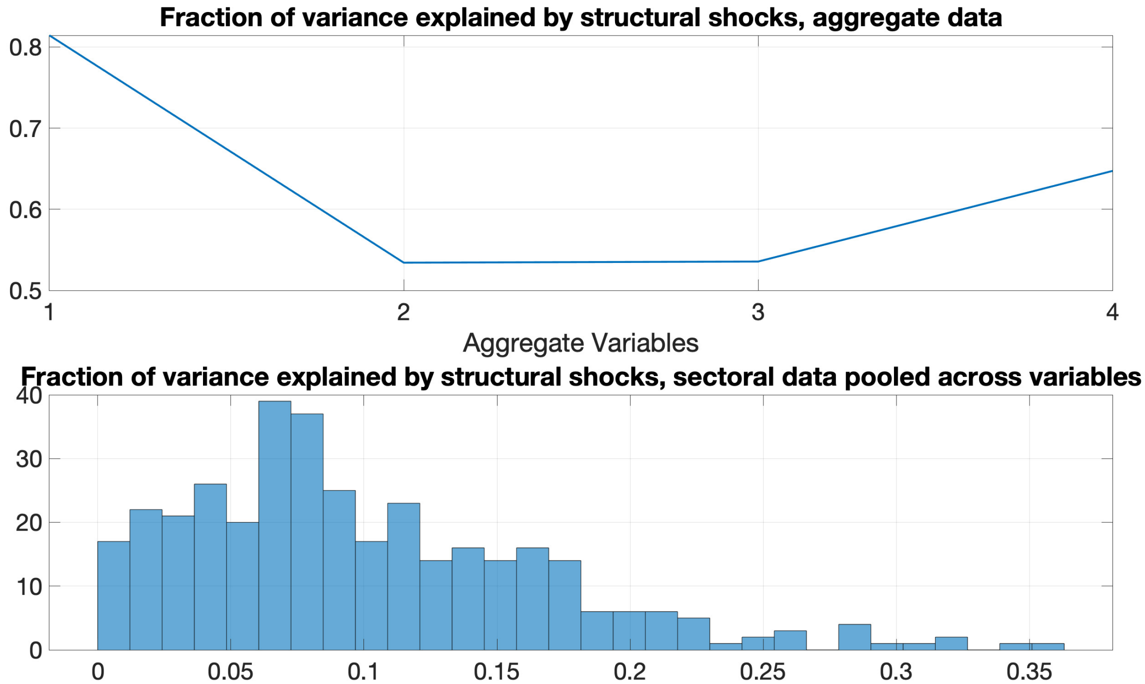

Figure A-5: Fraction of variance explained by structural shocks in our simulation exercise.

data, but setting the prior on all shock loadings of the second shock to a Gaussian distribution with mean 0 and standard deviation 0.25. Figure A-7 shows the results. Two results stand out: first, the first shock is still estimated precisely (the correlation of the posterior median with the true shock series is now 0.77), whereas the estimated second shock series does not match the truth at first sight. However, a further look reveals that the correlation between the posterior median and the true series is actually high in absolute value (-0.89). What happens? Our model correctly estimates the space spanned by the two shocks (i.e. the overall effect of the two shocks). But without any identification information on the second shock (in particular on the sign of the effects of this shock), the algorithm cannot pin down the shock exactly, but only the space spanned by this second shock. In this run of the posterior sampler, it concentrated on the part of the posterior distribution where the sign of the effects and the actual shocks is flipped relative to the true values. [7]

[7] We run the posterior sampler for only 20,000 draws, half of which are discarded, in this simulation exercise. Even with this small amount of draws we can already see that our algorithm performs well. Such a small number of draws is generally not enough to fully capture severe multi-modality of the posterior distribution. In our empirical analyses we use 150,000 draws.

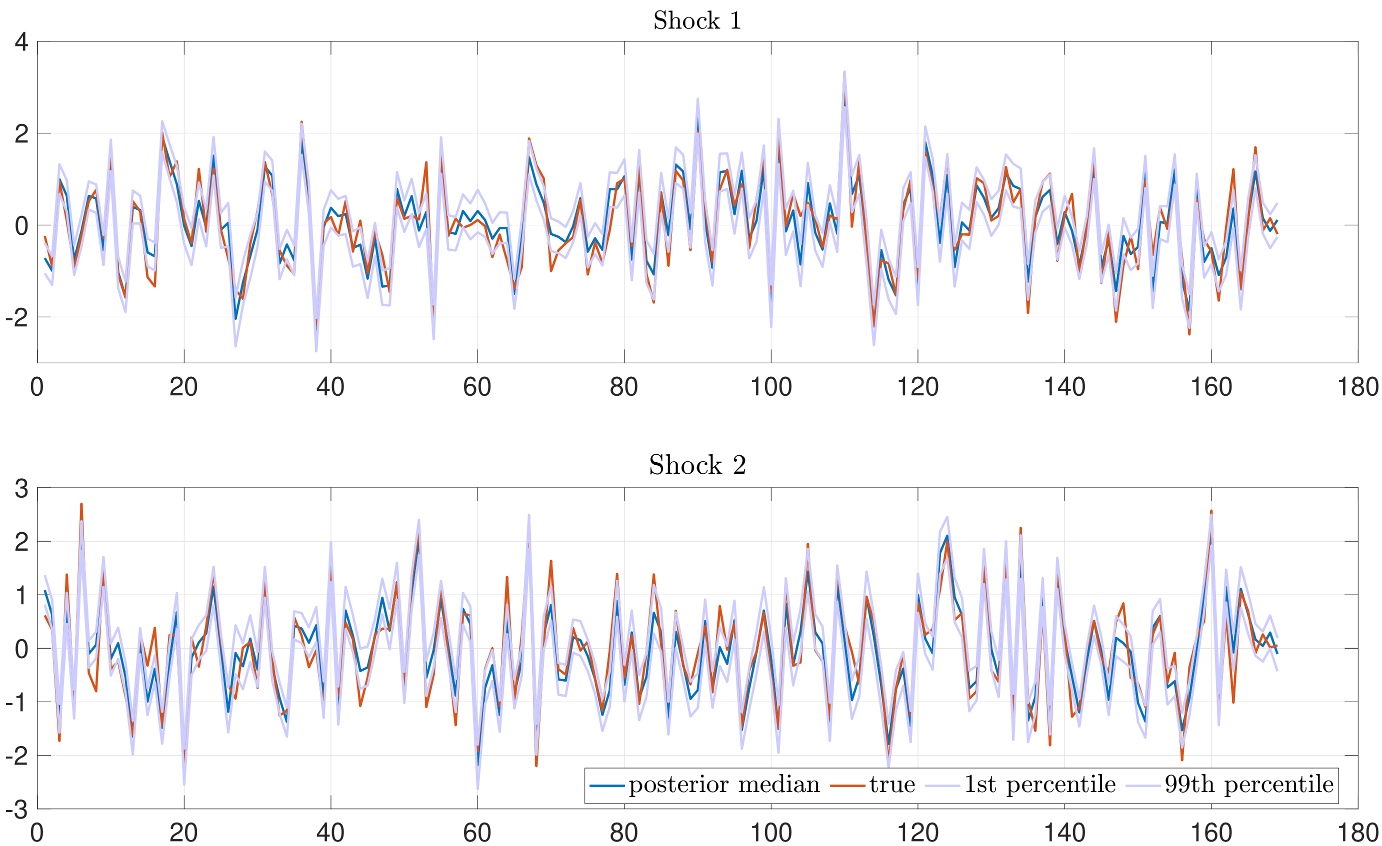

Figure A-6: Estimated and true shocks, Monte Carlo Exercise.Prior centered at true values for both shocks.

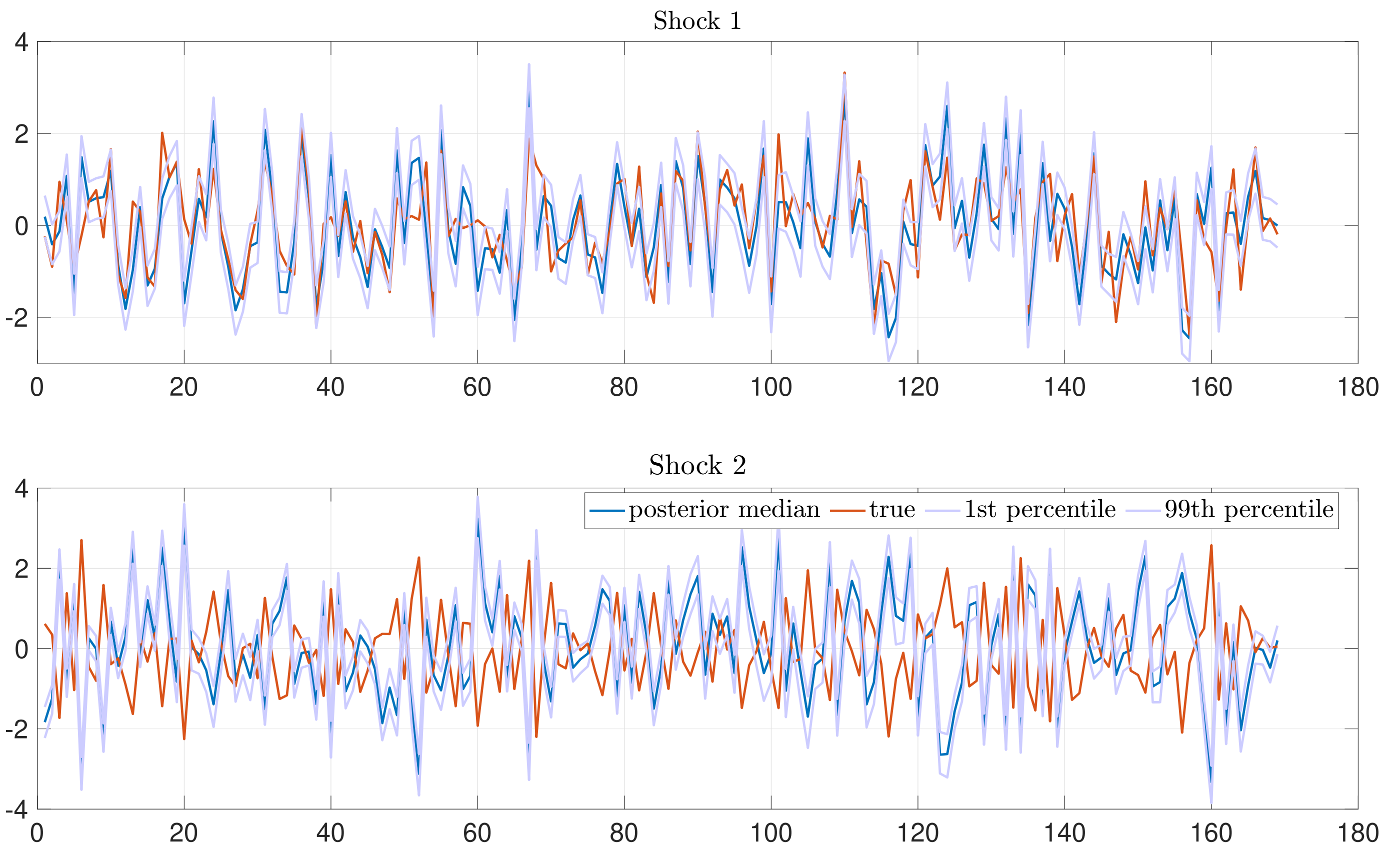

Figure A-7: Estimated and true shocks. Uninformative prior on effects of second shock.

J Why don't we use more aggregated sectoral data?

Sectoral data are available at various levels of aggregation. We choose to use data that is as disaggregated as possible. To justify this choice, we will study a very simple example. Consider an economy consisting of two equally sized sectors (we could easily generalize this argument to more sectors, but this extension would not add anything to our argument). We disregard aggregate variables here because they are not important for the argument. We also consider one observable per sector. So the state space system we study is

$$u_t^1 = \varepsilon_t + w_t^1 \tag{A-24}$$

$$u_t^2 = \varepsilon_t + w_t^2 \tag{A-25}$$

$$\varepsilon_t = \varepsilon_t \tag{A-26}$$

where $w_t^1 \sim (N(0,\Sigma^1)$ and $w_t^2 \sim (N(0,\Sigma^2)$ are two independent Gaussian processes, and, as before, $\varepsilon_t \sim (N(0,1)$. For simplicity, we have normalized D to 1 in this example in both sectors. Alternatively, we could study a system where we aggregate the two sectors (we use equal weights here because we have assumed for simplicity that the sectors have equal size):

$$\overline{u}_t = \varepsilon_t + \overline{w}_t \tag{A-27}$$

$$\varepsilon_t = \varepsilon_t \tag{A-28}$$

Here we have $\overline{w}_t = \frac{1}{2}(w_t^1 + w_t^2)$ and thus $\overline{w}_t \sim N(0, \frac{1}{4}(\Sigma^1 + \Sigma^2))$. First note that we abstract in this example from two aspects that would make a researcher want to use more disaggregated data:

1. We don't model any dynamics in the sector. It is well known in the time series literature that aggregating VAR processes generally leads to VARMA processes for the aggregated variables. To at the very least be able to approximate these VARMA dynamics in our framework we would need to incorporate more lags of observables into the sectoral equations when using more aggregated data.

2. We focus here on the case of one aggregate shock. If there is more than one shock and different sectors have heterogeneous exposures to the different shocks then averaging over this heterogeneous exposure can lead to a substantial loss of information.

Coming back to our example, we can ask which of the two systems leads to a more precise estimate of the shock ε_t. We focus here on the variance of the estimation error for ε_t [8]. While it is easy to derive the formulas for the variance in closed form in our simple examples, we can already illustrate

[8] To be precise, we study $var(\varepsilon_t|I_t)$ where I_t is the information set including time t observations

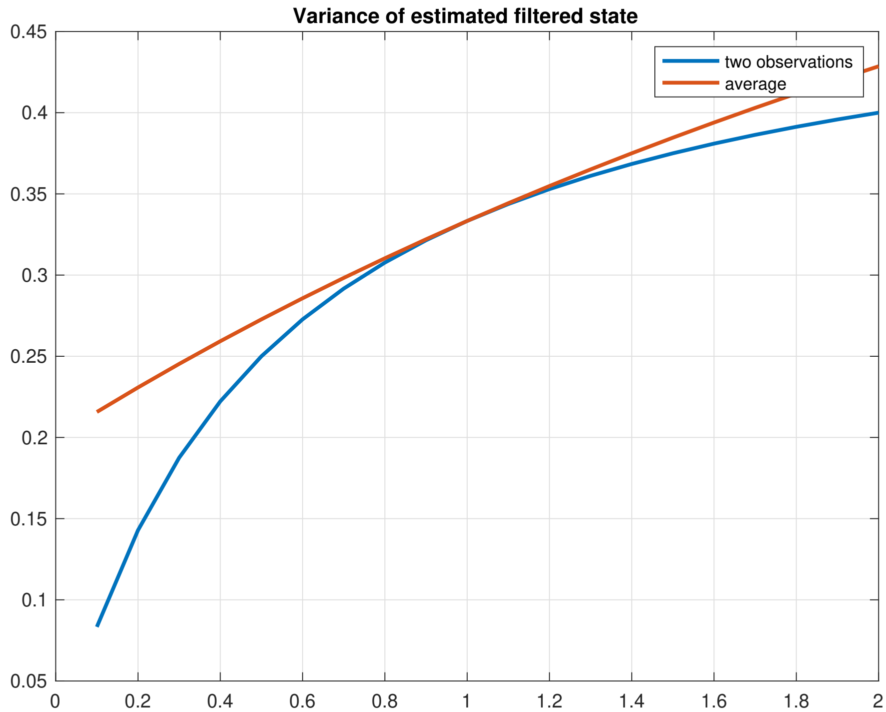

Figure A-8: Variance of estimation error.

the main point with a numerical example. We fix the variance of w_t^1 at 1 and vary the variance of w_t^2 from 0.1 to 2. We then compute the estimated variance for both environments (one with two observables, one with the average observable). Figure A-8 shows our main result: it is always preferable to use more disaggregated data. The only point of indifference occurs when the variances of the w shocks are exactly equal. Turning to the analytical solutions, $var(\varepsilon_t|I_t)$ in the case when we observe both sectors separately is given by

$$var^{\text{two sectors}}(\varepsilon_t|I_t^{\text{two sectors}}) = 1 - (1\ 1)\left(\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} \Sigma^1 & 0 \\ 0 & \Sigma^2 \end{pmatrix}\right)^{-1} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{A-29}$$

The corresponding formula for the case where the average is observed is

$$var^{\text{average}}(\varepsilon_t|I_t^{\text{average}}) = 1 - \frac{1}{1 + \frac{1}{4}(\Sigma^1 + \Sigma^2)} \tag{A-30}$$

Both these equations are standard Kalman filtering formulas. One can then show that the following always holds:

$$var^{\text{two sectors}}(\varepsilon_t|I_t^{\text{two sectors}}) \leq var^{\text{average}}(\varepsilon_t|I_t^{\text{average}}) \tag{A-31}$$

Furtmore, the equality is strict unless $\Sigma^1 = \Sigma^2$. The proof amounts to tedious but straightforward algebra. The result should not be surprising: you can never be worse off by using more information.

Note that one could in our simple example take a weighted average of the sectors to achieve the same variance as in the case with two observables, but in practice this is not feasible because the weights would depend on the variances of the noise terms (the w terms), which are not known before estimation.

K Sectoral Impulse Responses

Sectoral impulse responses, sorted by C/Y and the prior impact to household consumption shock (which is not the same as C/Y, as it also varies with differences in overall volatility of sectoral innovations).

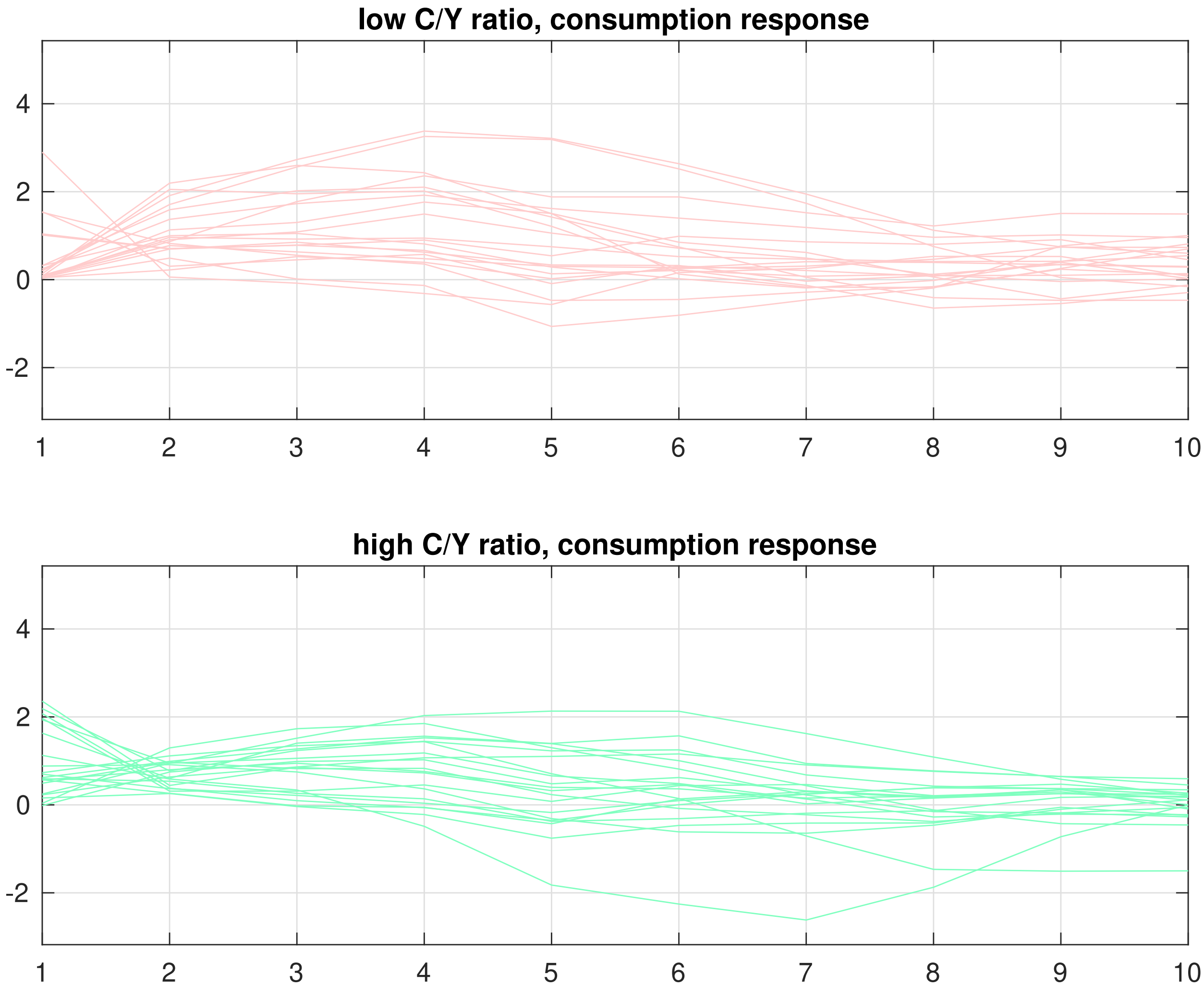

Figure A-9: Sectoral IRFs, high C/Y vs. low C/Y

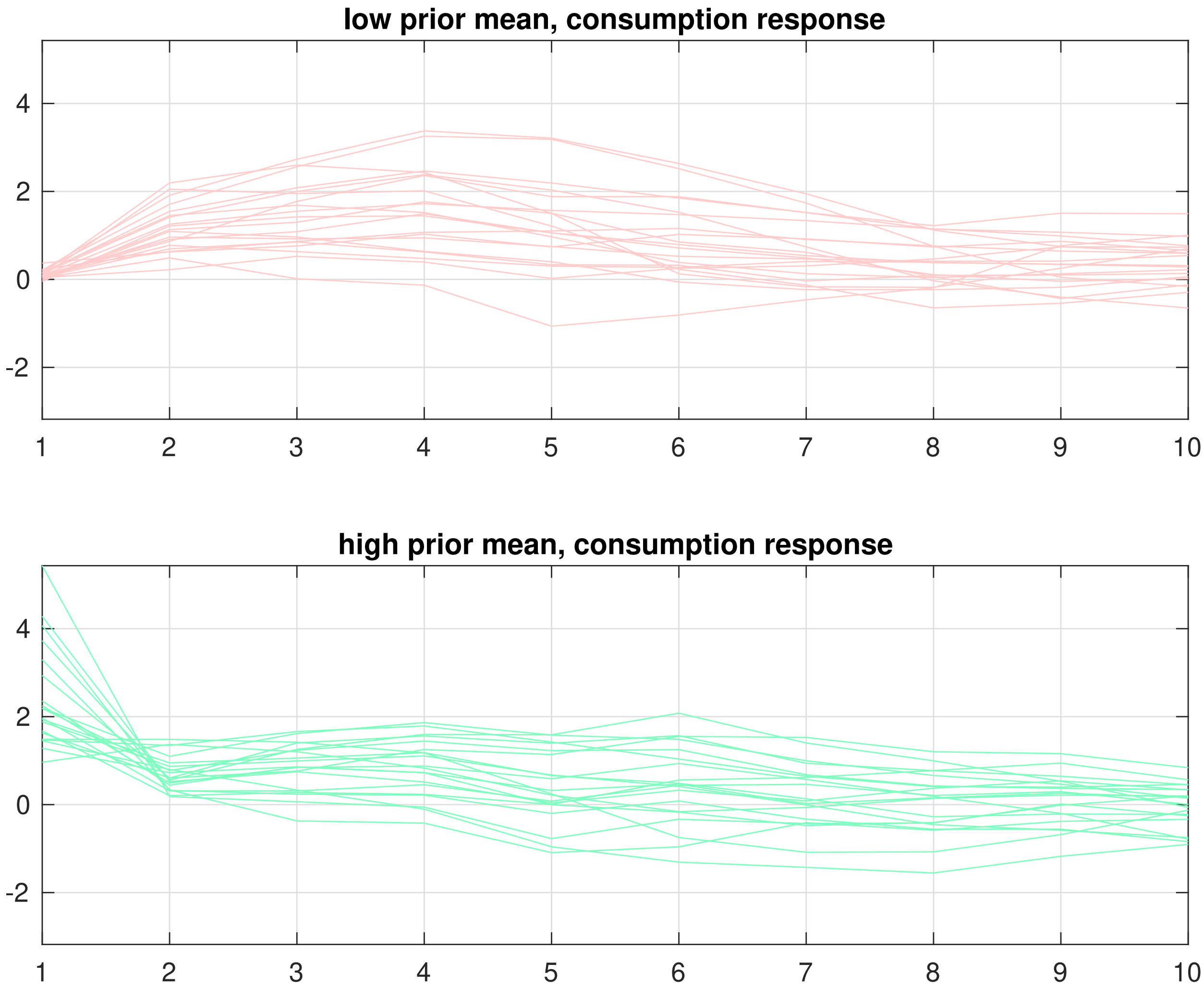

Figure A-10: Sectoral IRFs, high prior mean vs. low prior mean

L Impulse Responses to Other Economic Shocks

Note that the responses to the household consumption shock and the monetary shock are in the main text (Figures 3 and 8).

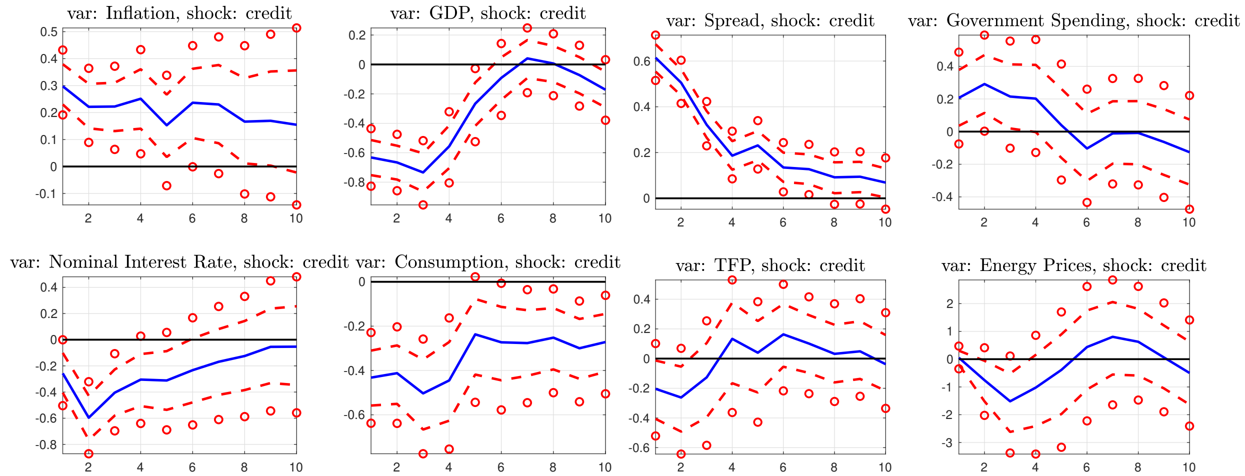

Figure A-12: Responses to Credit Shock. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis Shows Time in Quarters.

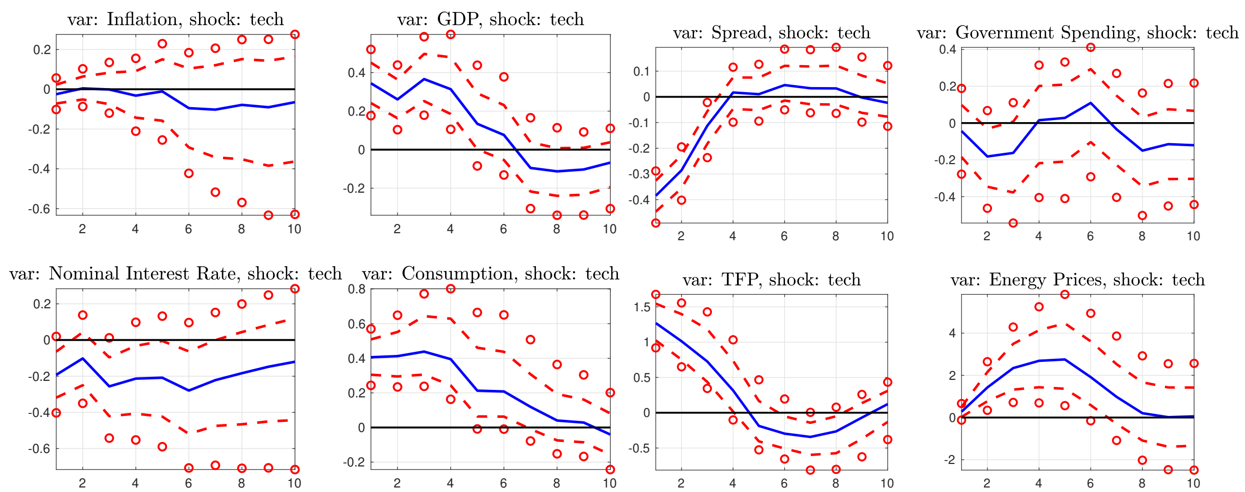

Figure A-11: Responses to Technology Shock. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis Shows Time in Quarters.

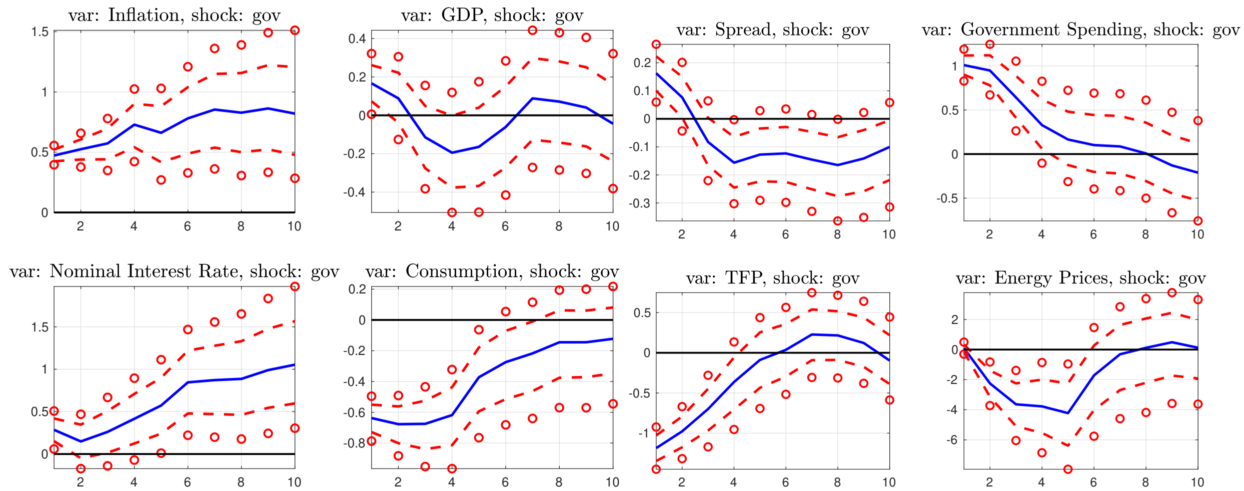

Figure A-13: Responses to Government Spending Shock. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis Shows Time in Quarters.

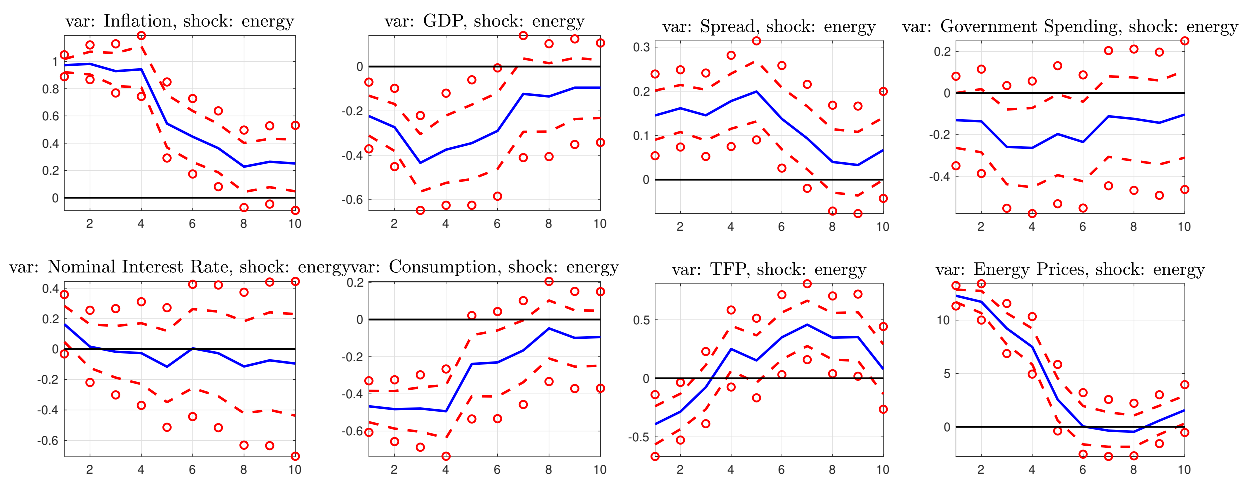

Figure A-14: Responses to Energy Price Shock. Dashed lines are 16th and 84th Posterior Percentile Bands, Dots are 5th and 95th Posterior Percentiles. The x-axis Shows Time in Quarters.

L.1 Sentiment Shock

We can examine whether the sentiment series is a good IV for the consumption shock, by estimating impulse responses to a consumer "sentiment" shock using the series for consumer sentiment as an IV (figure A-15). In particular, to estimate the IRFs, sentiment is ordered first in the VAR(4) and identification of the sentiment shock is achieved via Cholesky decomposition. We use the Canova and Ferroni (2021) toolbox to implement Minnesota priors with estimated hyperparameters (Giannone et al., 2015) and otherwise use standard prior settings as implemented by Canova and Ferroni (2021)

We find that they look similar to the IRFs for the consumption shock in some but not all instances. In particular, it is also associated with increased TFP and stable inflation, indicating that consumer sentiment also captures the response of household expectations to productivity news.

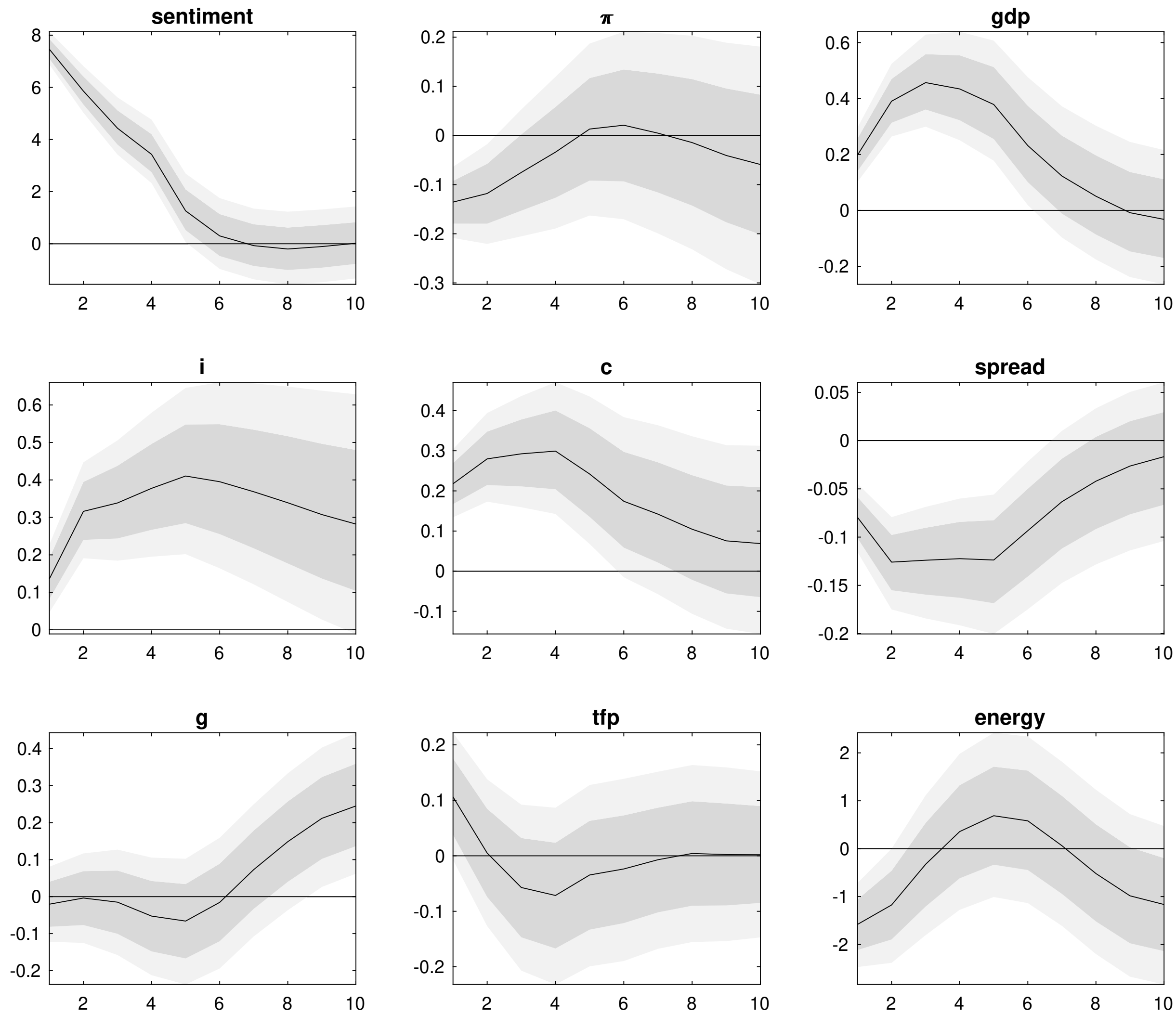

Figure A-15: Impulse response to a one-standard deviation sentiment shock. Black line is the posterior median, error bands represent 68% (darker area) and 90% posterior probability.

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M Further Robustness checks

To economize on space, we focus in our robustness checks on the importance/variance decomposition (for business cycle frequencies) of the consumption shock for aggregate variables. Relative to the main text we also show the 5th and 95th percentiles of this variance decomposition. Therefore, we start by showing the results for our benchmark case. Throughout all these specifications the household consumption shock remains a key driver of economic activity.

M.1 Benchmark

	5th Percentile	Mean	95th Percentile
Inflation	8.4	13.9	18.7
GDP	24.8	33.9	39.7
Nominal Interest Rate	21.2	22.9	26.0
Consumption	37.6	42.6	48.7
Spread	6.2	9.8	12.6
Government Spending	6.1	26.1	35.5
TFP	4.6	10.9	14.6
Energy Prices	5.8	8.3	13.3

Table A-4: Variance decomposition across business cycle frequencies, consumption shock. Benchmark specification.

M.2 Aggregates only identification

To show the marginal gain from using sectoral data for identification of shocks, we show the variance decomposition when only informative priors on the effect of aggregate shocks are used.

	5th Percentile	Mean	95th Percentile
Inflation	4.7	14.9	21.7
GDP	13.4	19.7	44.2
Nominal Interest Rate	11.5	18.5	26.1
Consumption	42.2	45.5	48.4
Spread	13.3	17.1	25.1
Government Spending	12.4	29.8	47.0
TFP	2.9	9.2	15.5
Energy Prices	8.1	12.9	15.9

Table A-5: Variance decomposition across business cycle frequencies, consumption shock, only aggregate identification restrictions.

M.3 Larger Prior Variance on Impact of Consumption Shock

Next, we increase the prior standard deviation for the impact of the consumption shock on aggregate consumption equal to $1/2 \times abs(E[D_c])$, where D_c is the prior mean of the impact of the household shock on aggregate consumption.[9]

	5th Percentile	Mean	95th Percentile
Inflation	18.2	22.5	29.1
GDP	27.1	34.7	40.9
Nominal Interest Rate	25.0	33.0	40.2
Consumption	38.9	45.3	49.7
Spread	14.7	17.7	22.8
Government Spending	8.6	25.2	32.1
TFP	6.0	11.0	20.2
Energy Prices	3.1	4.4	9.0

Table A-6: Variance decomposition across business cycle frequencies, consumption shock. Larger prior variance.

M.4 Shorter Sample

To assess whether or not our results are driven by the Great Recession, we re-estimate the model ending our sample in 2004:Q3.

	5th Percentile	Mean	95th Percentile
Inflation	22.3	24.5	27.9
GDP	16.5	21.6	25.4
Nominal Interest Rate	23.2	28.2	39.2
Consumption	28.7	30.9	32.7
Spread	6.6	13.3	17.2
Government Spending	10.4	19.4	24.0
TFP	16.4	18.5	22.8
Energy Prices	13.5	17.7	20.4

Table A-7: Variance decomposition across business cycle frequencies, consumption shock. Shorter sample.

[9] For our benchmark, we use $0.1 \times abs(E[D_c])$. The prior standard deviation for the aggregate impact of the other aggregate shocks is set in the same fashion.

M.5 Fewer Lags

We now reduce the number of lags L and L^X to 4 from our benchmark specification of 6.

	5th Percentile	Mean	95th Percentile
Inflation	8.8	17.8	39.4
GDP	27.3	34.9	38.1
Nominal Interest Rate	11.4	16.8	26.2
Consumption	34.5	37.4	38.4
Spread	11.5	12.1	12.8
Government Spending	5.0	8.6	12.3
TFP	6.1	9.4	21.0
Energy Prices	3.2	8.0	15.6

Table A-8: Variance decomposition across business cycle frequencies, consumption shock. Fewer lags.

M.6 Investment specific technology shock

In this robustness check we modify our benchmark specification in two ways:

1. We add year-over-year growth in investment to our set of aggregate observables. As a measure of investment we use Real Gross Private Domestic Investment (FRED mnemonic GPDIC1).
2. We also identify an investment shock. This shock moves aggregate investment positively on impact (the prior is set in the same fashion as for our consumption shock, for example). At the sectoral level, it decreases inflation while increasing quantities. These effects are stronger the higher the investment intensity for a sector is, which we measure as the ratio between the value of goods produced in the sector that go towards gross capital formation and its total gross output.

As displayed in Table A-9, our consumption shock still remains the main driver of business cycle fluctutations.

	5th Percentile	Mean	95th Percentile
Inflation	17.8	27.3	45.0
GDP	16.5	24.4	29.7
Nominal Interest Rate	22.3	34.0	50.0
Consumption	34.4	40.4	45.9
Spread	13.3	19.4	30.7
Government Spending	7.6	20.1	27.7
TFP	6.3	11.3	16.8
Energy Prices	7.6	11.1	20.4
Investment	9.0	15.1	23.4

Table A-9: Variance decomposition across business cycle frequencies, consumption shock. Specification with investment-specific technology shocks .

M.7 Sample starting in 1985

To assess whether or not our results are driven by the Great Inflation, we re-estimate the model starting our sample in 1985:Q1.

	5th Percentile	Mean	95th Percentile
Inflation	7.5	11.4	20.0
GDP	22.6	24.8	30.7
Nominal Interest Rate	10.1	13.7	16.3
Consumption	27.3	30.4	37.4
Spread	7.2	10.0	11.3
Government Spending	10.3	12.6	16.6
TFP	6.8	11.2	14.0
Energy Prices	6.9	9.7	11.0

Table A-10: Variance decomposition across business cycle frequencies, consumption shock. Sample starting in 1985.

M.8 Comparison of Main Business Cycle Shock in Angeletos et al. (2020)

Our results suggest that consumption shocks are one of several important shocks, rather than a single main business cycle shock. We tested this by regressing the main business cycle shock from Angeletos et al. (2020) on the various shocks we identify. We found that this main shock has a small correlation with the consumption shock and can be better understood as a combination of various shocks, with the coefficients shown in Table M.8. This supports the view that multiple shocks play significant roles in business cycles, and the consumption shock plays a prominent but not dominant role.

	tech	credit	demand	gov	energy	monetary	investment
β_i	0.2	-0.1	0.0	-0.0	-0.1	-0.2	0.3