

# Figuring Out the Fed - Beliefs about Policymakers and Gains from Transparency

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## Abstract

In this paper, I use a Markov Chain Monte Carlo algorithm to estimate a model of private-sector behavior that does not feature private-sector knowledge of the monetary policymaking process and, instead, leaves firms and households uncertain about how monetary policy is set. The private sector entertains two competing views of monetary policymaking, which I estimate. Firms and households use Bayes' law on a rolling data sample to distinguish between those two models.

I use this setup to study the evolution of beliefs about the Federal Reserve and the possible gains from transparency.

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# 1 Introduction

From 1960 to 2005 there have been five Federal Reserve chairmen, each with possibly different views about monetary policy and faced with differing degrees of political pressure and different economic environments, from the oil price shocks of the 1970s to the prosperity of the 1990s. The Federal Funds rate has varied greatly within this period, from 2 % to 18 % annualized, showing a great degree of variation in the Federal Reserve's main policy instrument. Figure (1) displays those large movements in economic conditions and, in particular, the Federal Funds rate. The alternating grey and white bars represent the terms in office of the five chairmen during this sample.

FIGURE 1 HERE

Given these large changes over time it seems natural to ask how the private sector's view of Federal Reserve policy has evolved. This paper is concerned with calculating objects that can be interpreted exactly as representing the private sector's view of monetary policy since 1960, and inferring the effects of changes in these beliefs on macroeconomic outcomes, namely output and inflation. In particular, I will ask what the *gains from transparency* are: how would these outcomes have changed if the private sector's beliefs about monetary policy coincided with the actual policy conduct of the Federal Reserve?

A standard New Keynesian model (or any rational expectations model featuring monetary policy for that matter) posits a policy rule that is both the rule the central bank in that model follows and the policy rule that firms and households use to form beliefs about the path of future interest rates. I remove the assumption of rational expectations of firms and households from such a New Keynesian model. Instead, the private sector is uncertain about how monetary policy is set and uses Bayes' law on a rolling data sample to update the model probabilities on two models of

monetary policymaking. The models (i.e. monetary policy rules) that the private sector is endowed with are solutions to optimal policy problems, one under discretion and one under commitment. The preferences of the hypothetical central banks in the two models are allowed to be different from each other. I estimate these parameters jointly with the other parameters governing private-sector behavior using a Metropolis-Hastings algorithm to provide both a well-fitting model and estimates of statistical uncertainty for the policy experiments in later sections. While I will sometimes refer to one view of monetary policymaking as that associated with commitment (and the other view with discretion), it is important to keep in mind that the two views differ not only in that dimension. The learning problem that private agents face is about distinguishing between two central bank types that could possibly differ along many dimensions. One of the goals of the paper is to find out what different central bank types considered by the public help explain the data<sup>1</sup>.

Firms and households do know the preferences of the hypothetical central banks within each model, so the statistical problem that private agents face each period is to discriminate between two models with known parameter values. Disregarding possibly large shifts in beliefs induced by learning could lead to substantial misspecification of the model, which would invalidate the counterfactuals I present in this paper. It is important to note that in this paper I only model the *beliefs* about monetary policy held by private agents. I do not model the *actual* behavior of the Federal Reserve during the sample<sup>2</sup>. It is exactly the assumption of learning by private agents that lets me disentangle beliefs about policy rules from actual policy rules. My modeling approach is thus consistent with a number of different assumptions about how the Federal Reserve

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<sup>1</sup>The online appendix describes the fit of alternative specifications, one of which features two policymakers that only differ with respect to discretion and commitment, and shows that the fit is worse.

<sup>2</sup>This will become clearer once I discuss my estimation approach

actually set policy during the sample I consider (e.g. the Fed learning about the economy as in Primiceri (2006) and Sargent, Williams & Zha (2006) or the Fed being aware of the private agents' learning as in Cogley, Matthes & Sbordone (2011) and Molnar & Santoro (2010)). Other papers in the literature on learning (e.g. Sargent et al. (2006)) have assumed that the agents who learn use a misspecified model of the economy. Here, instead, I show how to infer the parameters governing the behavior of the agents who learn without having to take a stand on whether or not their views are misspecified.

Using data on inflation, output, and interest rates, I find that the private sector in the 1960s firmly believed that the Federal Reserve was a discretionary policymaker, a finding in line with anecdotal evidence. Only in 1980 were policy actions of the Volcker Federal Reserve able to significantly move the private sector's beliefs toward a central bank that acts under commitment and prefers lower inflation. However, this shift is less pronounced than what is often believed.

I calculate a series of counterfactuals, varying both the path of nominal interest rates and the private sector's beliefs about the Federal Reserve. These counterfactuals lead to the main finding of this paper: transparency matters - a central bank can have trouble reaching its goals even if it follows a 'good' policy (in a sense that will become clear later) as long as agents have doubts about the central bank's behavior.

I also compute the private sector's expectations of inflation and the output gap, and find that these expectations are reasonable when compared to both actual outcomes and survey measures of expectations. This paper thus provides evidence that models of optimal policymaking under commitment and discretion can be helpful to understand the evolution of

output and inflation in the US since 1960<sup>3</sup>. In order to fit the data well, we need to allow for differences in preferences across these hypothetical policymakers though.

This study is closely related to work by Bianchi (2013) and Liu, Waggoner & Zha (2009), where the private sector also considers two possible models of policymaking. However, in those models the private sector does not face a learning problem. Rather, the private sector in these models knows the type of central bank in power today, but considers the possibility of a future change in the policy rule, a feature absent in this paper<sup>4</sup>. Hence, the approach taken by these papers and the approach taken here are complementary. The approach I take appears to be more convenient for addressing the possible gains from transparency. Another key difference between these papers and the work presented here is the set of monetary policy models that the private sector is endowed with. I endow the private sector with models derived from optimal policy problems under different assumptions about the level of commitment while the papers mentioned before follow most of the literature on New Keynesian models and use Taylor-type rules<sup>5</sup>. Bianchi & Melosi (2013) do model learning in an environment in which the policy rule follows a discrete state Markov chain, but they only endow agents with a very specific kind of uncertainty: agents in their model are aware of the policy rule coefficients in place today (in contrast to the agents in this paper), but they are uncertain how long this policy rule will be in place.

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<sup>3</sup>Other papers that have looked at the empirical implications of optimal policymaking include Ireland (1999) and Ruge-Murcia (2003), who find that models in the spirit of Barro & Gordon (1983) are helpful for understanding US economic outcomes. Those papers focus on discretionary policymaking and do not feature private-sector learning.

<sup>4</sup>The learning algorithm employed by the agents will be set up in such a way that they can detect changes in the model generating the interest rate rather quickly, thus offering some 'implicit insurance' against changes in the model governing monetary policy.

<sup>5</sup>For an in-depth discussion of the class of models used by Bianchi (2013) and Liu et al. (2009), see Farmer, Zha & Waggoner (2009).

Roberds (1987), Schaumburg & Tambalotti (2007), Debortoli & Nunes (2007) and Debortoli & Nunes (2008) use a framework where a policy-maker can reoptimize (and thus renege on prior commitments) at random points in time. Hence, their approach could give a possible explanation for changes in private sector beliefs about committed vs discretionary policymakers if that approach was embedded in a learning framework.<sup>6</sup> However, similarly to the papers mentioned in the previous paragraph, firms and households in this strand of the literature do not face a learning problem because they can observe when a reoptimization takes place. This paper is also connected to a line of research in which models with optimizing central banks are estimated (instead of central banks following an ad-hoc, but well fitting policy rule). Papers that follow this approach include Ozlale (2003), Surico (2007) and Adolfson, Laseen, Linde & Svensson (2010). To the best of my knowledge, these papers do not consider private sector learning or estimation of preferences of both discretionary and committed central banks.

Technically this paper contributes to the growing literature on the estimation of learning models in macroeconomics by presenting a likelihood-based approach that allows the econometrician to leave unspecified the "true" data-generating process for the aspects of the economy about which the economic agents are uncertain. Instead, the econometrician can focus on the perceived law of motion of the agents. This approach is embedded in a Markov Chain Monte Carlo algorithm to calculate posterior distributions of statistics of interest.

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<sup>6</sup>Debortoli & Nunes (2008) also allow for changes in preferences when the policy-maker can renege on its previous promises. I find in my estimated model that the two hypothetical central banks that the private sector considers have substantially different preferences.

## 2 The Model

The private sector in the economy described in this section holds beliefs about monetary policy that evolve over time, as described in the second subsection. These beliefs influence not only the agents' expectations of future short-term nominal interest rates, but also their views about steady state inflation and, as a consequence, the steady state nominal interest rate. The uncertainty about how monetary policy is set is the only uncertainty (besides uncertainty about future exogenous variables) that the private sector faces. In particular, firms and households know all parameter values of both monetary policy models; however they do not know which of the two models generates the nominal interest rate.

### 2.1 Private Sector Behavior Conditional on Beliefs

Conditional on the perceived steady state of inflation and one-step-ahead expectations of inflation  $\pi$  and (log) output deviations from trend  $y^7$ , current period values of those variables are determined by a New Keynesian Phillips Curve with full indexation to past inflation and the representative household's Euler equation:

$$\pi_t - \bar{\pi}_t = \frac{\beta}{1 + \beta} E_t(\pi_{t+1} - \bar{\pi}_t) + \frac{1}{1 + \beta} (\pi_{t-1} - \bar{\pi}_t) + \frac{\kappa}{1 + \beta} y_t - z_t \quad (1)$$

$$y_t = -\sigma^{-1} (i_t - \bar{i}_t - E_t(\pi_{t+1} - \bar{\pi}_t)) + E_t y_{t+1} + g_t \quad (2)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \quad (3)$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \quad (4)$$

Variants of these equations under rational expectations (and thus a constant perceived steady state, which equals the true steady state) can be

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<sup>7</sup>Theoretically  $y$  is defined as the log ratio of output and the efficient level of output. It turns out, conveniently for the empirical application below, that this output gap seems to behave very similar to standard measures of the output gap such as deviations from an HP-filtered trend, as found by Justiniano & Primiceri (2008).

derived as an approximation to the equilibrium conditions of a non-linear representative agent model with monopolistically competitive firms. One model that leads to the equations given here can be found in Del Negro & Schorfheide (2004), with the exception that Del Negro & Schorfheide (2004) do not use an indexation scheme for prices, leading to a New Keynesian Phillips curve that does not include a lagged inflation term. A price setting scheme that leads to the Phillips curve described here can be found in Christiano, Eichenbaum & Evans (2005)<sup>8</sup>.

The exogenous shocks  $z_t$  and  $g_t$  can represent a wide variety of stochastic disturbances hitting the economy, depending on the exact set-up of the underlying non-linear model. For the purposes of this paper the exact interpretation of these shocks is not crucial (a similar approach has been taken by, among others, Bianchi (2009) and Lubik & Schorfheide (2004)). Both exogenous shocks follow AR(1) processes, where the innovations  $\varepsilon_t^z$  and  $\varepsilon_t^g$  follow a bivariate normal distribution with variances  $\sigma_z^2$  and  $\sigma_g^2$  and a contemporaneous correlation coefficient of  $\rho_{gz}$ . This correlation is necessary if we want to allow for the possibility of structural shocks in the underlying non-linear economy that feed into both  $z_t$  and  $g_t$ , such as in Lubik & Schorfheide (2004)<sup>9</sup>.

The parameter  $\kappa$  governs the slope of the New Keynesian Phillips curve<sup>10</sup>. Its value is inversely related to degree of price stickiness in the underlying non-linear economy.  $\bar{\pi}_t$  and  $\bar{i}_t$  denote the perceived steady state variables of inflation and the nominal interest rate, where the perceived steady state value of the latter is given by the sum of perceived

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<sup>8</sup>I choose a small scale New Keynesian model as my point of departure since I want to show that even a small scale model, augmented with learning, can do a good job at capturing broad movements and regularities in US economic time series.

<sup>9</sup>The description of a micro-founded model which leads to correlated shocks in the log-linear approximation of the equilibrium conditions can be found in Bianchi (2009), for example.

<sup>10</sup>Since I assume full indexation to lagged inflation, the Phillips curve coefficients are independent of the perceived level of steady state inflation. See Ascari (2004) for a discussion of this result.



steady state inflation  $\bar{\pi}_t$  and the steady state real interest rate  $r$ . The steady state real interest rate  $r$  is known by the agents and is independent of their views regarding monetary policy. Note that because of the specific form of the New Keynesian Phillips curve assumed here  $\bar{\pi}_t$  actually drops out of (1). Furthermore, because of the relationship between the perceived steady states of inflation and nominal interest rates, one could rewrite (2) as a function of  $r$  instead of  $\bar{\pi}_t$  and  $\bar{i}_t$ . Thus, changes in perceived steady states from period to period only influence output and inflation via  $E_t(\pi_{t+1})$  and  $E_t(y_{t+1})$ .

## 2.2 Belief Formation - Calculating Model Probabilities

The private sector is endowed with two models of monetary policymaking<sup>11</sup>. Each model is a description of how a fictitious central bank in that model would set the nominal interest rate given a set of variables that are observable to the private sector (the members of that set will depend on the model at hand). The policy prescriptions of these two models will be denoted by

$$i_t^c = f^c(X_t^c) \tag{5}$$

$$i_t^d = f^d(X_t^d) \tag{6}$$

where  $c$  and  $d$  denote the two models and  $X^c$  and  $X^d$  denote the observable variables governing policy in each model. How the private sector comes up with these two policy rules is the subject of the next two subsections. It is important to highlight here that these policy rules *only* represent private agents' beliefs about monetary policy, not the actual conduct of

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<sup>11</sup>I will refer to those two models as "submodels" throughout the paper.

monetary policy. I use an estimation approach (described in detail below) that allows me to remain silent on the actual data generating process for the nominal interest rate.

Given these two policy rules, the private sector calculates the likelihoods  $l_t^c$  and  $l_t^d$  of the observed interest rate data at time  $t$  given each model and the sequence of relevant right-hand side variables. Therefore, the private sector only learns about the two models via the observed interest rates. Because I will assume throughout that the state variables for each policy model,  $X_t^c$  and  $X_t^d$ , can be observed by the private sector, the private sector could use (5) and (6) to immediately infer which of the two models, if any, is correct when it observes the interest rate  $i_t$  at period  $t$ . To make the agents' learning problem more interesting, I will assume that the agents know that both policy models are imperfect descriptions of monetary policymaking. The private sector realizes that each period there will be a difference between the two models' policy prescriptions and the observed nominal interest rate. The private sector assumes that the differences between each models' prescription and the observed interest rate follows the same distribution, as can be seen below.

$$i_t = i_t^j + \nu_t^j, j \in \{c, d\} \quad (7)$$

$$\nu_t^j \sim N(0, \sigma_\nu^2) \forall j, t$$

Equation (7) is then used to form the likelihoods  $l_t^c$  and  $l_t^d$  each period. Deviations of prescribed interest rates are penalized equally across models, as both error terms are assumed to follow the same distribution when the private sector calculates the likelihoods. The error terms can be interpreted as a manifestation of the private sector's realization that the models of monetary policymaking it considers are imperfect descriptions of the data. However, this setup is open to other interpretations as well.

The error terms could also be interpreted as the private sector realizing that the nominal interest rate cannot be perfectly controlled by the central bank, for example. To economize on notation, I define  $p_t^j$  and  $E_t^j(\cdot)$  as follows:

$$p_t^j \equiv p(\text{model } j | \mathcal{I}_t) \quad (8)$$

$$E_t^j(\cdot) \equiv E(\cdot | \mathcal{I}_t, \text{model } j), j \in \{c, d\} \quad (9)$$

where  $\mathcal{I}_t$  is the information set at time  $t$ .

Given prior model probabilities  $p^c$  and  $p^d$ , the private sector then uses a Quasi-Bayesian approach to attach probabilities to each model each period by calculating model probabilities the following way:

$$p_t^c = \frac{(\prod_{i=t-t^*}^t l_i^c) p^c}{\sum_{j=c,d} (\prod_{i=t-t^*}^t l_i^j) p^j} \quad (10)$$

For a fixed  $t^*$ , this approach is Quasi-Bayesian because it only applies Bayes' Law to a rolling sample of observations instead of the entire available sample<sup>12</sup>. The private agents in this model thus ask the question: "In the past  $t^*$  periods, which of the models of policymaking is more likely to have generated the data?"

If, instead, we set  $t - t^*$  equal to period 1 of the entire data sample for each  $t$ , the equation given above is equal to the more common recursive

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<sup>12</sup>Two comments about this setup are in order:

1. If the available sample includes less than  $t^* + 1$  datapoints the private sector will use all available data until the sample becomes large enough so that the restriction to start the model probability calculation at time  $t - t^*$  becomes meaningful.
2. This restriction is similar in spirit to constant gain least squares learning, which is used in the majority of the literature on learning in macroeconomics. That approach puts greater weight on more recent observations versus observations that are further in the past. For more details on this approach see, for example, Evans & Honkapohja (2001).

representation of Bayes' law:

$$p_t^c = \frac{p_{t-1}^c l_t^c}{\sum_{j=c,d} p_{t-1}^j l_t^j} \quad (11)$$

Allowing the private sector not to use all past data leads to a situation where model probabilities adjust more quickly to new evidence in favor of one of the submodels. Choosing a fixed  $t^*$  (and thus having agents put no weight on observations more than  $t^*$  periods in the past) can be interpreted as agents having a view of the world where the true policy rule changes over time in an infrequent manner. To be more specific, the approach taken in this paper can be viewed as an approximation to a situation where agents think that the central bank policy rule follows a two-state Markov chain and the Markov state is very persistent (most likely not a bad approximation for the application in this paper).

The online appendix discusses the calculation of one-period ahead expectations. Why is it enough to focus on one-period ahead expectations? For a given set of beliefs  $p_t^j$ , agents can form expectations of any variable of interest at any horizon as a probability weighted average of outcomes under the two different models of monetary policymaking. Here I assume that agents think that probabilities will not change in the future, which is akin to the anticipated utility assumption often invoked in learning models (see for example Primiceri (2006)). In the context of this paper we have very good reason to make this assumption: Bayesian model probabilities follow martingales under the subjective probability measure used by agents<sup>13</sup>. Thus agents do not expect the probabilities to change in the future.

Given this perceived (or subjective) probability measure, we can take standard first order conditions for any infinite horizon decision problem and arrive at standard optimality conditions (such as the consumption

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<sup>13</sup>See the online appendix for a proof.

Euler equation) that link state variables, date  $t$  decision variables and expectations of variables dated  $t + 1$ <sup>14</sup>.

## 2.3 Optimal Policy Under Commitment

The policy rule  $f^c(X_t^c)$  is derived as the solution to an optimal policy problem<sup>15</sup> where the hypothetical central bank solves the optimal policy problem described below once, and then commits to that policy rule forever. The hypothetical policymaker in this submodel minimizes the objective function (12) subject to constraints (1)-(4) under the additional assumption that the expectations in the objective function and the constraints are formed conditional on this model being true (in other words, rational expectations conditional on this being the true model of monetary policy), i.e. not taking into account the private sector's algorithm for forming expectations over two models of monetary policymaking<sup>16</sup>. This simplifying assumption allows me to use a standard algorithm (see Soderlind (1999) or Backus & Driffill (1986)) to solve this optimal policy problem, which will make estimation of the model feasible.

$$E_0^c \sum_{t=0}^{\infty} \beta^t [(\pi_t - \pi^c)^2 + \lambda^c (y_t)^2 + \lambda_i^c (i_t - i^c)^2] \quad (12)$$

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<sup>14</sup>Only one-period ahead expectations appear in the equilibrium conditions because the agent's view of the world as encoded in their perceived law of motion allows them to use the law of iterated expectations. Private agents here are fully aware of the market structure and the knowledge that other private agents possess, which allows them to use the law of iterated expectations, in contrast to the agents in Preston (2005).

<sup>15</sup>Note that the policy here is optimal for an ad-hoc loss function whose parameters are estimated. The same is true for the corresponding policy problem under discretion. Having possibly different values for loss function parameters for the two specifications is crucial for the fit of the model (see the online appendix, which is available on my website).

<sup>16</sup>The hypothetical central banks studied in this paper ignore the impact that their actions might have on the probabilities attached to each type of policymaker. For a study of a central bank that does take into account that its actions influence the learning process of private agents (and does so optimally) see Cogley et al. (2011).

As is standard in the solution of optimal policy problems under commitment, lagged Lagrange multipliers on the constraints that feature forward-looking expectations ( $\lambda_{NKPC,t-1}$  on (1) and  $\lambda_{IS,t-1}$  on (2)) become state variables that influence the optimal choice of the nominal interest rate in this model each period. These lagged Lagrange multipliers represent the influence of past commitments on current policy actions. Thus  $X_t^c$  is given by

$$X_t^c = \begin{pmatrix} \pi_{t-1} \\ z_t \\ g_t \\ \lambda_{NKPC,t-1} \\ \lambda_{IS,t-1} \end{pmatrix} \quad (13)$$

The preference parameters  $\pi^c$ ,  $\lambda^c$  and  $\lambda_i^c$  will be estimated later. On the other hand, the interest rate target  $i^c$  is not a free parameter, but is instead set equal to  $r + \pi^c$ .

### 2.3.1 How the Private Sector Calculates the Lagrange Multipliers

To calculate the interest rate prescribed by the optimal policy problem under commitment, households and firms need to know the values for the multipliers  $\lambda_{NKPC,t}$  and  $\lambda_{IS,t}$ . The solution to the optimal policy problem under commitment delivers a VAR representation for  $X_t^c$ :

$$X_t^c = AX_{t-1}^c + B\varepsilon_t \quad (14)$$

where  $\varepsilon_t$  is a vector of consisting of  $\varepsilon_t^z$  and  $\varepsilon_t^g$ . Given initial values for the Lagrange multipliers and other state variables, the private sector uses the observed shocks  $\varepsilon_t$  every period to update the Lagrange multipliers and calculate the optimal interest rate under commitment.

## 2.4 Optimal Policy Under Discretion

$f^d(X_t^d)$  is derived as the solution to an optimal policy problem under discretion, i.e., it is part of a Markov Perfect equilibrium. Each period the hypothetical central bank in this submodel takes as given future policy and private-sector behavior when choosing the nominal interest rate to minimize an expected discounted quadratic loss function. Hence, the solution to this optimal policy problem describes the behavior of an opportunistic central bank that reoptimizes every period instead of honoring past commitments.  $f^d(X_t^d)$  is the policy rule that arises in a situation where the policy that the hypothetical central bank expects to be followed in the future coincides with its own response to the state variables given that policy. The state variables for this problem are given by

$$X_t^d = \begin{pmatrix} \pi_{t-1} \\ z_t \\ g_t \end{pmatrix} \quad (15)$$

Consequently this hypothetical central bank solves the Bellman equation (16) subject to constraints (1)-(4).

$$V(X_t^d) = \min_{i_t} (\pi_t - \pi^d)^2 + \lambda^d (y_t)^2 + \lambda_i^d (i_t - i^d)^2 + \beta E_t^d V(X_{t+1}^d) \quad (16)$$

As in the previous section, I assume that the expectations that are calculated when solving this policy problem do not take into account the private sector's learning problem, again allowing me to use a standard algorithm (linear quadratic value function iteration in this case) to solve for the optimal policy. The solution algorithm is described in great detail in Soderlind (1999) and Backus & Driffill (1986). Similarly to the commitment case, the preference parameters  $\pi^d$ ,  $\lambda^d$  and  $\lambda_i^d$  will be estimated, while the interest rate target  $i^d$  is equal to  $r + \pi^d$ . Given the fact that both central banks do not have a non-zero target for the output gap the tradi-

tional *average inflation bias* of Kydland & Prescott (1977) is not at work here. However, a *stabilization bias* of discretionary policy is present: the committed central bank can smooth the effect of shocks more efficiently over multiple periods, as it can credibly commit to such a response. The only endogenous state variable that allows for history dependence in the case of discretion is the lagged inflation rate<sup>17</sup>.

### 3 Estimation

I estimate the parameters of the model (except for the discount factor, which I calibrate) using a Bayesian approach. To do so, I need to combine the likelihood of the data with a prior distribution of the model's parameters. Because the model is one of private-sector decision making, it does not have anything to say about how actual monetary policy is set and, instead, focuses on how the private sector thinks monetary policy will evolve. Accordingly, the model implies a likelihood function for the output gap and inflations that conditions on the observed path of nominal interest rates:

$$p(y^T, \pi^T | \Theta, I, i^T) \tag{17}$$

$\Theta$  denotes the vector of parameters,  $x^T$  the sample of size  $T$  for variable  $x$  and  $I$  the vector of initial conditions needed to initiate the learning algorithm (both initial values of observables as well as shocks and Lagrange multipliers). Using Bayes' law we can see how such a likelihood function can be used to calculate the posterior distribution of the parameters of

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<sup>17</sup>As the two hypothetical central banks are allowed to have different inflation targets, it would be hard to distinguish empirically the effect of different inflation targets vs. an average inflation bias (i.e., the result of a non-zero target for the output gap). Furthermore, it is convenient to be able to interpret directly the inflation targets as the long-run average inflation levels within each of the submodels.



interest:

$$\underbrace{p(\Theta|y^T, \pi^T, I, i^T)}_{\text{posterior}} \propto \underbrace{p(\Theta|I, i^T)}_{\text{prior}} \underbrace{p(y^T, \pi^T|\Theta, I, i^T)}_{\text{likelihood}} \quad (18)$$

The prior in the previous equation is not standard in that it is conditional on the path of nominal interest rates. Equation 18 shows that using such a conditional prior is valid when combined with a likelihood function that conditions on the same variables. I will discuss the approximation to  $p(\Theta|I, i^T)$  that I use below. The online appendix describes the results of a different approximation to  $p(\Theta|I, i^T)$ . Conditioning the prior and the likelihood on the path of observed nominal interest rates allows me to avoid one source of misspecification, the actual monetary policy rule. This does not come for free: if that misspecification was not severe, then modeling the actual policy rule could help inference. The online appendix shows where these gains could come from, but also highlights that the gains in efficiency are likely to be modest. The following sections describe the different elements entering (18). I initialize the Lagrange multipliers for the commitment problem on the forward-looking equations (1) and (2) at 0, implying that this hypothetical central bank does not have to honor any prior commitments at the start of the sample. The prior model probabilities  $p^c$  and  $p^d$  are each set to 0.5. Those initial values could be estimated, but, to economize on the number of estimated parameters, I calibrate them for now. I let the private sector use a sample of 10 years to form their model probabilities using (10). Using 10 years for  $t^*$  makes it easier to compare my results with the previous literature on learning, which has used learning algorithms that discount past data. However, using a different sample length leads to results that are similar to those reported below (see the online appendix for the case in which agents use the entire sample to form beliefs). The data used in this paper is described in the online appendix <sup>18</sup>. The online appendix also discusses models in

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<sup>18</sup>The estimation approach used here could be extended to include inflation expectations as observables. I choose not to do that and, instead, check in section 6 if the

which private agents consider other combinations of policymakers: two discretionary or committed policymakers with different preferences or a committed and a discretionary policymaker who share the same preferences and have a non-zero output target so that an average inflation bias is present. The appendix shows that the main model presented here fits the data better than those alternatives and also that the higher inflation target for the discretionary policy maker can indeed be interpreted as a stand-in for an average inflation bias arising from non-zero output target.

### 3.1 The Likelihood Function

The likelihood function can be rewritten as follows:

$$p(y^T, \pi^T | \Theta, I, i^T) = p(y_1, \pi_1 | \Theta, I, i_1) \prod_{t=2}^T p(y_t, \pi_t | \Theta, I, i^t, y^{t-1}, \pi^{t-1}) \quad (19)$$

Each of the densities in (19) can be evaluated by using the distributional assumptions on  $\varepsilon_t$ , equations (1)-(4) and the learning algorithm described earlier<sup>19</sup>. A more detailed description of the calculation of the likelihood can be found in the online appendix.

This likelihood function will be used as an input for a Metropolis-Hastings algorithm, which will generate draws from the posterior distribution of the parameters. For a discussion of this algorithm see, for example, An & Schorfheide (2007).

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estimated inflation expectations from my model line up with observed survey expectations.

<sup>19</sup>Note that the timing assumption used for the formation of expectations is very useful when writing down (19), as the expectations in (1) and (2) are functions of variables in the conditioning set of each of the densities in (19) at each point in time.

## 3.2 Priors

This section describes the priors that are used in combination with the likelihood described above to calculate the posterior distribution of the parameters. It is unfortunately not feasible to directly calculate the conditional prior  $p(\Theta|i^T, I)$ <sup>20</sup>. Thus, I approximate the conditional prior  $p(\Theta|i^T, I)$  by the product of two prior distributions: one that I will call 'standard' in that it resembles priors used in previous studies on New Keynesian models, and one that puts more prior probability on regions of the parameter space where the learning models yield reasonable policy prescriptions<sup>21</sup>:

$$\begin{aligned} \underbrace{p(\Theta|y^T, \pi^T, I, i^T)}_{\text{posterior}} &\propto \underbrace{p(\Theta|I, i^T)}_{\text{prior}} \underbrace{p(y^T, \pi^T|\Theta, I, i^T)}_{\text{likelihood}} & (20) \\ \propto \underbrace{p_1(\Theta)}_{\text{"standard" prior}} &\underbrace{p_2(\Theta|i^T, I)}_{\text{prior on reasonable submodels}} &\underbrace{p(y^T, \pi^T|\Theta, I, i^T)}_{\text{likelihood}} \end{aligned}$$

It turns out that when only the prior  $p_1(\Theta)$  is used, the learning models of the agents will not fit particularly well, as that lack of fit is not directly punished by either the likelihood (which conditions on interest rates) or the prior. As long as the expectations in (1) and (2) lead to relatively small errors in these equations, the likelihood function will not be close to zero even if the interest rate prescriptions are very different from the observed interest rate. The fitted interest rates coming out of the two submodels are still highly correlated with the actual interest rate, but their variance is much higher, leading to highly improbable policy recommendations from each model.

I find it highly implausible that the private sector would use models

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<sup>20</sup>One could, in theory, approximate  $p(\Theta|i^T, I)$  using a Metropolis-Hastings algorithm. This would increase the computational burden substantially, which is why I choose to instead approximate this density. This paper is not the first one to approximate a prior to facilitate inference. Another example (using a substantially different approximation) is Del Negro & Schorfheide (2008).

<sup>21</sup>The product of these two distributions forms a valid kernel for a probability density function, which is all that is needed for inference using the Metropolis-Hastings algorithm.

that give unreasonable policy recommendations for over 40 years to learn about monetary policy. Thus, I want to put more prior mass on regions of the parameter space where the two submodels yield reasonable policy prescriptions. A shortcut to doing so is described below.

### 3.2.1 "Standard" Priors

I use a set of independent probability distributions to characterize "standard" prior beliefs about the parameters of the model. Priors for those parameters that are standard in New Keynesian models are in line with those used by Lubik & Schorfheide (2004) and Bianchi (2009) and can be found in table 1, along with the priors for the loss functions of the hypothetical central banks. The discount factor  $\beta$  is calibrated and set equal to 0.99.

The priors for the loss function parameters of the two hypothetical central banks are set to capture the prior belief that the commitment central bank has a lower inflation target and a lower weight on output in the loss function. The variances on these priors are relatively large to ensure that the data have the final word on the values of these parameters<sup>22</sup>. To make the loss function parameters more easily interpretable, the reported priors for the weights on the output gap are scaled by 16, as it seems more natural to think about a loss function where output deviations are compared to deviations of annualized inflation and interest rate from their respective targets, while the variables  $\pi_t$  and  $i_t$  in the model are defined as quarterly inflation and the quarterly nominal interest rate. This kind of rescaling is standard in the analysis of optimal

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<sup>22</sup>The goal of this paper is to endow agents with well-fitting models of monetary policy. A priori, it seems reasonable to assume that one possible view of monetary policymaking is a policymaker that cares more about inflation being low and stable than about output. I associate this view with the commitment type since, as mentioned before, the difference in inflation targets is motivated as a stand-in for an average inflation bias. The means for the inflation targets are chosen so that the average of the two is roughly the sample average of inflation. The associated standard deviation is chosen to be quite large.

policy in New Keynesian models<sup>23</sup>.

### 3.2.2 Prior on Reasonable Policy Models

To approximate the dependence of  $p(\Theta|i^T, I)$  on  $i^T$ , I multiply the prior described in the previous section with the following probability distribution<sup>24</sup>:

$$\prod_{t=1}^T (0.5l_t^c + 0.5l_t^d) \quad (21)$$

This is a weighted average of the likelihoods of the two policy models. It is important to remember that those are the likelihoods that the private sector uses to form model probabilities. Using this probability distribution as additional prior information ensures that regions of the parameter space that yield badly fitting policy models receive low prior probability.

An important issue with this construction of a prior is whether it too profoundly influences model probabilities. By looking at (21) one might have the impression that it will push the model probabilities towards .5, as it penalizes the likelihoods of both submodels symmetrically. While this is a priori a fair concern, it turns out that this concern is unwarranted; we will see below that while both submodels fit reasonably well there are still large swings in the estimated model probabilities.

One advantage of using (21) is that it penalizes a model with unreasonable policy prescriptions, even in periods when that model has a low

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<sup>23</sup>The output gap targets are chosen to be roughly in line with the literature - for example, the associated weight on the output gap in Adolfson et al. (2010) is around 1. I pick a prior mean for the commitment type slightly lower (0.8) and a higher one for the discretionary type (2), but with a standard deviation of 0.5. The prior mean weight on the interest rate term is set (admittedly in an arbitrary fashion) to 10 percent of the weight on inflation. The prior standard deviation is 0.1. This term in the loss function is introduced to minimize the violations of the zero lower bound, as described in Woodford (2003). As such, a relatively small weight seems reasonable (even with such a small prior weight, expected interest rates rarely fall below 0).

<sup>24</sup>Another interpretation of what I do here is that I take a penalized likelihood approach, with the penalty being data-dependent. For an introduction to penalized likelihood estimation see Green (1999).

model probability attached to it, and thus strengthens the identification of the loss function parameters. To see this, assume instead that I would use the following construction to penalize unreasonable learning models:

$$\prod_{t=1}^T (p_{t-1}^c l_t^c + p_{t-1}^d l_t^d) \quad (22)$$

While this might seem more appealing on theoretical grounds (after all, it is the perceived likelihood of the private sector), it only contains information about the loss function parameters of the hypothetical central bank in one of the submodels when the probability of that model is significantly different from 0. A prior of this sort would use a much smaller effective sample to estimate the loss function parameters. I found this not only to be a theoretical concern, but also to dramatically hinder the numerical estimation.

A shortcoming of this approach is that the object in equation 21 also depends on inflation and output gap data. A viable alternative is to approximate  $p(\Theta|i^T, I)$  by putting a prior directly on the moments (say, mean and variance) of the policy recommendations, where the moments of the prior density are determined by the observed path of nominal interest rates. This approach would not depend on the specific realizations of inflation and the output gap. Unfortunately the theoretical moments of the policy recommendations (which are needed to evaluate the prior) are in general not available in closed form, necessitating simulations of the model to evaluate the prior. This approach has been advocated by Gallant & McCulloch (2009). The online appendix uses this approach to check for the robustness of the estimation results described in the following sections and finds broadly similar results.

### 3.3 Estimation Results

Table 1 contains the parameter values at the posterior mode, while the online appendix also displays the posterior mean and 95 % posterior density intervals.

In general, the mean and mode are very similar, with the only main difference between them being the weight on the output gap in the commitment loss function, which is almost five times larger at the posterior mean. Nonetheless, at both the posterior mean and the posterior mode, the same picture across the two sets of central bank preference parameters arises; the weight on the output gap, the weight on the interest rate deviations from the target and the inflation target are substantially larger for the hypothetical discretionary central bank.

TABLE 1 HERE

To interpret the weight on the interest rate deviation from target and the inflation target, table 1 reports the estimated annualized inflation targets in percent at the posterior mode and the rescaled weight on the output gap, which I multiply by 16 to have a weight on the output gap that is comparable to weights on the inflation and interest targets, which are reported in annualized (instead of quarterly) units.

The inflation target of the hypothetical commitment central bank is in the range of 2 %, a commonly assumed target value for the Federal Reserve today. The inflation target for the discretionary policymaker on the other hand seems very high at 5.48 %. Remember, however, that for reasons of identifiability I have abstracted from an average inflation bias, which could explain why this estimate is so high.

The rescaled values for the weights on the output gap show a big difference in the weight being placed on the output gap, with the discretionary policymaker caring over seven times more about the output gap than the hypothetical commitment central bank. Both values, though,

are smaller than one, indicating that both hypothetical central banks care more about inflation than about the output gap.

Turning to the 95 % posterior density intervals, it is evident that all parameters are estimated tightly. This is due, at least in part, to the use of the second set of priors. Using these priors leads to a more peaked posterior, as regions of the parameter space where one of the two sub-models does not fit well are given low posterior probability, even if the other model fits well and has much higher model probability attached to it in that region of the parameter space.

The estimates for the AR coefficients  $\rho_g$  and  $\rho_z$  are quite a bit lower than what is usually found in the literature (see, for example, Lubik & Schorfheide (2004)). It seems that the learning algorithm presented in this paper creates enough endogenous persistence to allow these parameters to be substantially lower than their prior mean (which is centered around usual estimates in the literature).

The estimate of the correlation between the innovations in (3) and (4),  $\rho_{gz}$ , is relatively high. Again, this is not surprising, as many non-linear models imply that the error terms in the linearized Euler equation (2) and New Keynesian Phillips curve 1 are correlated because an exogenous error term enters both of these equations. If shocks are redefined, as they are here, allowing for contemporaneous correlation seems important to do the underlying non-linear model justice.

Besides needing less exogenous persistence than other models in the literature, the estimate of  $\kappa$  is higher than what is found in most other studies (a notable exception being Lubik & Schorfheide (2004)<sup>25</sup>). This high value implies a much higher probability of a firm being able to adjust its price each period in a Calvo framework. Note that the slope of

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<sup>25</sup>Like this paper Lubik & Schorfheide (2004) uses Hodrick-Prescott detrended output as an observable (even though the exact detrending method differs, see the online appendix, while a lot of other studies use output growth and make actual output an unobservable state variable in a state-space representation of the linear(ized) model. This is a possible explanation for the similarity of some of the parameter estimates.



the Phillips curve is  $\kappa/(1 + \beta)$ , not  $\kappa$  itself, so if one compares the slope estimates of New Keynesian Phillips curves across studies, the estimate found here, while being on the high end, is not unheard of.

## 4 Evolution of Beliefs

This section analyzes the private sector's beliefs by focusing mainly on the posterior mode estimates. Since the parameters are all tightly estimated, the error bands around most of the statistics of interest are narrow.

Figure 2 plots the sequence of estimated posterior model probabilities of the commitment model. The private sector quickly learned in the 1960s that the Federal Reserve was a discretionary policymaker. It took the Volcker disinflation at the beginning of the 1980s to slowly induce a change in these beliefs. While the policy actions in 1980 caused the posterior probability of the commitment model to increase substantially, the following 15 years are associated with large swings in the estimated model probabilities. There is an upward trend in  $p_t^c$  during that period, but fluctuations around that trend are volatile. Also, it is worth noting that towards the end of the sample  $p_t^c$  decreases again.

FIGURE 2 HERE

In general, these model probabilities are estimated precisely, which is why I focus on posterior mode estimates<sup>26</sup>.

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<sup>26</sup>The online appendix of this paper shows the mean model probabilities and 90 % probability bands for the estimated model probabilities. The mean probabilities are very similar to the model probabilities at the posterior mode estimates. Interestingly, there turns out to be substantial model uncertainty during the last year of the sample, making it hard to distinguish between the two submodels on the ground of the data used here for that period. For the other calculations in this paper (and the probability calculations for all other parts of the sample), posterior quantile bands do not convey substantial additional information beyond calculations carried out using the posterior mode estimates.

The initial lack of credibility of the Volcker Federal Reserve has also been documented prominently by Goodfriend & King (2005). The approach there is different in that it is more narrative (even though an equilibrium model is used to illustrate the points made in that paper), and, in addition, uses long-term interest rates as a measure of inflation expectations, which the authors in turn interpret as a measure of credibility attributed to the Federal Reserve by the bond market. While it is beyond the scope of this paper, it would be very interesting to incorporate term structure information into the private sector's learning algorithm.

Comparing the results on this paper with those in Bianchi (2013) is not straightforward since the models that govern (in my model perceived) monetary policy are different across the two papers. Nonetheless, if the reader is willing to roughly equate Bianchi's "Hawk" regime with the committed policymaker in this paper, we see that the estimated model probability in this paper and the smoothed (i.e. based on the full sample) model probability in Bianchi's paper share a similar pattern once Volcker comes into office.

## **5 Interest Rate Prescriptions and Perceptions**

This section analyzes the prescribed interest rates for the commitment and discretion submodels and the path for expected interest rates that those prescriptions and the calculated model probabilities imply.

At the end of each period, after inflation and the output gap for that period are realized, agents in this economy know the state variables for both submodels,  $X_t^c$  and  $X_t^d$  and they can determine the model probabilities taking into account that period's realization of exogenous shocks  $z_t$

and  $g_t$ . Firms and households can then calculate what they think interest rates should have been that period given the data<sup>27</sup>:

$$p_t^c i_t^c + p_t^d i_t^d \tag{23}$$

Panel 1 of figure 3 plots the historical path of the interest rate and the perceived path calculated using (23). While the perceived interest rate is more volatile than the actual Federal Funds rate, it does track medium- and low-frequency movements of the actual policy instrument reasonably well. Because the zero lower bound on nominal interest rates is not modeled here, perceived interest rates can go below zero and actually do so for a small number of periods. Taking into account the zero lower bound would require using techniques that would make the estimation of this model infeasible<sup>28</sup>.

## 6 Private Sector Expectations

Because one-step-ahead forecasts of inflation and the output gap are important factors in determining date  $t$  values for those variables, as is evident from the New Keynesian Phillips curve (1) and the representative household’s consumption Euler equation (2), it is important to ask what kind of expectations agents hold when they use the learning algorithm described in this paper. Figure 3 plots  $E_t \pi_{t+1}$  and  $E_t y_{t+1}$  versus the actual outcomes those expectations try to predict.

FIGURE 3 HERE

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<sup>27</sup>I use the private sector’s information set at the end of each period so that I do not have to take a stand on how the agents forecast the Lagrange multipliers that are state variables for the commitment problem.

<sup>28</sup>For a treatment of optimal policy under discretion and commitment that explicitly takes into account the zero lower bound on nominal interest rates see Adam & Billi (2006), Adam & Billi (2007) and Eggertsson & Woodford (2003).

The private sector's expectations track the actual inflation rate very well and do reasonably well for the output gap. The learning algorithm presented in this paper thus endows the private sector with very reasonable expectations of future economic outcomes.

A related question is how well the expectations calculated using the learning algorithm presented here track survey measures of inflation expectations. Figure 4 contrasts the median one-year-ahead inflation expectation of the University of Michigan's Survey of Consumers <sup>29</sup> with one-year-ahead inflation expectations coming from the model. Note that the model consistent inflation expectations plotted in figure 4 are not directly comparable to those model consistent expectations plotted in figure 3 because the former are one-year-ahead expectations, while the latter are one-quarter-ahead expectations. Also, the dates on the x-axis in figure 4 refer to the dates at which expectations are formed, in contrast to the dates on the x-axis of the relevant panel of figure 3, which refers to the date inflation is actually realized.

#### FIGURE 4 HERE

The model consistent expectations track the lower frequency movements of survey expectations well. The decrease in expected inflation at the beginning of the 1980s is evident in both series. Because there are a number of issues commonly raised when it comes to survey measures of inflation, I will not compare the two series in greater detail. It is, however, worth remembering that the learning algorithm presented here endows the agents with expectations broadly similar to those measured in surveys.

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<sup>29</sup>Inflation expectations are surveyed every month. I use a three month average to make the survey comparable to the output of the model, which uses quarterly variables. The source for the survey data is the FRED database of the Federal Reserve Bank of St. Louis. This series is only available from the second half of the 1970s.

## 7 The Perceived Inflation Target and Estimated Shocks

As the private sector updates model probabilities, it also changes its perceptions of the long-run level of inflation. The model presented here can thus be reinterpreted as a theory of perceived changes in the inflation target. It is important to emphasize that this model of private-sector behavior has nothing to say about whether or not the actual inflation target of the central bank (which is not modeled here) changed. Instead, by focusing on the private sector's perceptions, this model complements studies such as Erceg & Levin (2003), Ireland (2005) and Liu, Waggoner & Zha (2007) that explicitly model changes in the inflation target of the central bank in their models. It is important to emphasize that a setup in which the two hypothetical policymakers share the same preferences, there can still be movements in the perceived steady state level of inflation because of an average inflation bias under discretion. A model of this kind is estimated in the online appendix and does indeed find an average inflation bias that leads the perceived steady state of inflation to have a similar shape as in the case discussed here (it does fit the data worse than the benchmark specification, though). As mentioned before, I abstract from an average inflation bias in the main model for identification purposes, but we can interpret differences in estimated inflation targets as a stand-in for the average inflation bias under discretion as long as the discretionary central bank has a higher inflation target (which it does here).

Figure 3 plots the perceived steady state value of inflation. The perceived steady state moves to the inflation target of the discretionary central bank at the beginning of the sample, even though inflation is still relatively low. The reason for this is that the private sector learns from interest rates and not from inflation directly and as a consequence  $p_t^c$  can

move to 0, even though inflation is still far away from the inflation target of the discretionary central bank. Looking at figure 3, it is evident that a substantial part of the low-frequency movement of inflation expectations is governed by changes in the perceived steady state of inflation, but that there are considerable medium- and high-frequency dynamics in inflation expectations beyond those dynamics in perceived steady states. Changes in these medium- and high-frequency dynamics are a result of substantially different dynamics and volatilities of endogenous variables under the two policy rules considered by private agents. The fact that the volatilities of endogenous variables are different across the two submodels means that the model of private sector behavior displays stochastic volatility as the agents are learning even though the volatilities of exogenous disturbances remains constant. The link between learning models and stochastic volatility has been emphasized by Cogley et al. (2011). These results can also be viewed through the lens of the "good luck" vs. "good policy" debate as sources of the great moderation. VAR-based studies such as Primiceri (2005) and Sims & Zha (2006) tend to favor the "good luck" hypothesis, whereas investigations based on DSGE models favor the "good policy" (see for example Lubik & Schorfheide (2004)). Agents here only entertain the possibility of changes on policy, but since the different models of policy will lead to different volatilities of endogenous variables, a VAR-based estimation will likely attribute some of the changes in the volatilities of endogenous variables to changes in exogenous variables. A similar point has been made in the context on full information rational expectations models by Benati & Surico (2009), who consider a one time switch from one rational expectations equilibrium to another.

That it is in fact changes in beliefs that govern the dynamics of the model, and not unusually large shocks, can be seen by inspecting the bottom two panels of figure 3, which plot the estimated series of exogenous

disturbances at the posterior mode estimates.

## 8 Gains from Transparency

To address the issue of gains from transparency, I will calculate several counterfactual scenarios. In what follows, I will mainly focus on the gains from transparency of a commitment central bank (with the estimated preferences) since those gains translate into substantially lower inflation.<sup>30</sup>

At this point it is important to remember that even though I call one hypothetical central bank 'committed' and the other 'discretionary', the policy protocol is not the only difference between these two hypothetical policymakers - they also differ because they have different preference parameters<sup>31</sup>. The counterfactual scenarios differ from the actually estimated model along two dimensions: the interest rate set in the economy and the beliefs of the private sector<sup>32</sup>. The first counterfactual asks what inflation and the output gap would have been if the Federal Reserve had actually set the interest at its historical levels, but the private sector believed that the Federal Reserve was acting under commitment ( $p_t^c = 1 \forall t$ ). As we will see below, this change in beliefs will have substantial effects on inflation and the output gap. It is thus reasonable to question the

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<sup>30</sup>The results also suggest that there are cases (i.e. parameter values) for which the discretionary central bank could improve economic outcomes by convincing the public that it is a central bank acting under commitment. This type of scenario is analyzed by, among others, Barro (1986), King, Lu & Pastén (2008), Levine, McAdam & Pearlman (2008) and Sleet & Yeltekin (2007). A closely related question is that of optimal policy when the central bank has an informational advantage over the private sector. For a recent treatment of this question, see Mertens (2008), who solves for the Markov perfect equilibrium of an economy similar to the one presented in this paper when the private sector does not directly observe the central bank's time varying output gap target.

<sup>31</sup>The online appendix also contains one robustness check where the only difference between policymakers is the policy protocol. That specification fits the data substantially worse than the benchmark case.

<sup>32</sup>Counterfactuals are calculated using a subsample of 50000 draws from the original Metropolis-Hastings sample. The reported results are averages across draws. Because most parameters are tightly estimated, these averages adequately characterize the posterior distribution of outcomes under the counterfactual scenarios.

assumption that the Federal Reserve would have set the Federal Funds rate to the historically observed levels. To remedy this shortcoming and to analyze a situation where beliefs about policy are correct, I turn to the second counterfactual. This counterfactual asks what output and inflation would have been if the Federal Reserve had not only convinced the public that it was a central bank acting under commitment, but had also actually followed that policy ( $i_t = i_t^c$ )<sup>33</sup>. I then ask how outcomes differ if certainty about the Federal Reserve's conduct of monetary policy is removed from the private sector. The third counterfactual sets  $p_t^c = 0.5 \forall t$  while retaining the assumption that interest rates are actually set according to the commitment policy rule. The fourth counterfactual endows the private sector with the belief that Federal Reserve is acting under discretion and assumes that this belief is indeed correct (so that  $i_t = i_t^d$ ). The fifth and final counterfactual examines a situation in which the central bank is acting under discretion, but the private attaches a probability of .5 to each model of monetary policymaking, similarly to counterfactual 3. Comparing outcomes under the second and third counterfactuals gives an estimate of the effects of transparency on inflation and the output gap for a committed central bank. Table 2 shows the mean and variance of inflation and the output gap multiplied by 100 for the data and the five counterfactuals as well as the relative gains from transparency and counterfactual sacrifice ratios discussed below.

#### TABLE 2 HERE

A few patterns emerge when comparing these outcomes across counterfactuals:

Comparing the data to counterfactuals 1 and 2, we see that endowing the private sector with the belief that the Federal Reserve is a central bank

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<sup>33</sup>In all counterfactuals, I set the policy shocks  $\nu$  to zero.



acting under commitment ( $p_t^c = 1$ ) leads to inflation that is both lower on average and less volatile than what we have observed in the data.

This belief alone can lead to a substantially lower and more volatile output gap compared to the data, if the policy the Federal Reserve actually follows does not agree with the beliefs of the private sector. This can be seen by comparing the data to counterfactual 1.

Comparing counterfactuals 1 and 2, we see that if the Federal Reserve had actually followed the commitment policy and convinced the public of that, it could have achieved both low and stable inflation without a deterioration in the output gap, both in average terms and in terms of volatility.

Removing the private sector's uncertainty is crucial in achieving lower and less volatile inflation even when the Federal Reserve follows the commitment policy rule, as can be seen by comparing counterfactuals 2 and 3.

A discretionary Federal Reserve, in combination with correct beliefs of the private sector about monetary policy, would have led to very high and volatile inflation. In particular, the average inflation level would have been substantially higher after 1980.

A comparison of counterfactuals 2 and 3 highlights the *gains from transparency* for a committed central bank, while looking at counterfactual 1 shows that convincing the public of the central bank's policy rule even without necessarily following through would have lead to lower and less volatile inflation, albeit at a cost in terms of output. Figure 5 plots the counterfactual inflation series <sup>34</sup>.

Contrasting the last two counterfactuals shows how a discretionary central bank would have influenced economic outcomes if the private sector remained uncertain about the true nature of the policymaker throughout

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<sup>34</sup>I focus on counterfactual inflation since the output gap is not very different across counterfactuals. The counterfactual inflation series for the final counterfactual is very similar to the series for the third counterfactual, so I omit it here.

the sample. We see that the once the private sector attaches substantial probability throughout the sample that it faces the committed central bank, average inflation is quite low and less volatile than it would be if the private sector was aware of the nature of monetary policymaking in this scenario. This again emphasizes the crucial role of beliefs about policymakers rather than the actual policy rule being used.

Table 2 also shows the relative loss for a discretionary and a committed central bank (relative to the loss associated with the actually observed outcomes). We see that the discretionary central bank would have gained less than the committed central bank by setting the interest rate in an environment in which the public is perfectly informed about the policymaker.

The counterfactual calculations so far use the entire sample. Next, I turn to an analysis of the run-up in US inflation and the eventual disinflation, the period from 1970 to 1984. Looking at figure 5, we see that the first three counterfactuals lead to substantially lower inflation in that period. Following Bianchi (2009), I calculate *counterfactual sacrifice ratios*, which are defined as follows:

$$\sum_{t=1970:Q1}^{1984:Q1} (y_t - y_t^{CF}) / \sum_{t=1970:Q1}^{1984:Q1} (\pi_t - \pi_t^{CF}) \quad (24)$$

where variables with a  $CF$  superscript denote counterfactual outcomes. These counterfactual sacrifice ratios can be interpreted as the cost of lowering inflation in terms of the output gap. Table 2 also shows the counterfactual sacrifice ratios for the first three counterfactuals <sup>35</sup>.

While the sacrifice ratio is highest in the first counterfactual, owing to the large deterioration in output gaps, there is also a substantial dif-

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<sup>35</sup>As there is no disinflation in the fourth counterfactual, the counterfactual sacrifice ratio would have been harder to interpret for that counterfactual.

ference between the second and the third counterfactual. This highlights again the *gains from transparency*.

## 9 Conclusion

This paper presents a theory of private-sector decision making with a particular focus on belief formation about monetary policy<sup>36</sup>. The theory confirms anecdotal evidence that the Federal Reserve was seen as a policymaker acting under discretion with a high inflation target in the 1970s and that it took the drastic policy measures of the Volcker Federal Reserve to change those beliefs. However, the policy actions of the 1980s did not leave firms and households certain that the Federal Reserve acted under commitment to reduce average inflation.

The gains from transparency for a central bank acting under commitment with the estimated preferences would have been large for the sample considered here. Such a transparent central bank could have avoided the large increase in inflation in the 1970s.

The estimation algorithm used here highlights the fact that the estimation of learning models is possible without taking a stand on the true data-generating process of the aspects of the economy about which economic agents are uncertain.

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<sup>36</sup>The learning algorithm presented here endows the private sector with a large amount of information and only leaves firms and households uncertain about a particular feature of the economy. In contrast, a considerable amount of previous work on learning in macroeconomics, such as Milani (2007), endows the private sector with considerably less information. Another approach that endows economic agents with more information about the structure of the economy is presented in Preston (2005).

## References

- Adam, K. & Billi, R. M. (2006), 'Optimal monetary policy under commitment with a zero bound on nominal interest rates', *Journal of Money, Credit and Banking* **38**(7), 1877–1905.
- Adam, K. & Billi, R. M. (2007), 'Discretionary monetary policy and the zero lower bound on nominal interest rates', *Journal of Monetary Economics* **54**(3), 728–752.
- Adolfson, M., Laseen, S., Linde, J. & Svensson, L. E. O. (2010), Optimal monetary policy in an operational medium-sized DSGE model. forthcoming, *Journal of Money, Credit and Banking*.
- An, S. & Schorfheide, F. (2007), 'Bayesian analysis of DSGE models', *Econometric Reviews* **26**, 113–172.
- Ascari, G. (2004), 'Staggered prices and trend inflation: Some nuisances', *Review of Economic Dynamics* **7**(3), 642–667.
- Backus, D. & Driffill, J. (1986), The consistency of optimal policy in stochastic rational expectations models, CEPR Discussion Papers 124, C.E.P.R. Discussion Papers.
- Barro, R. J. (1986), 'Reputation in a model of monetary policy with incomplete information', *Journal of Monetary Economics* **17**(1), 3–20.
- Barro, R. J. & Gordon, D. B. (1983), 'A positive theory of monetary policy in a natural rate model', *Journal of Political Economy* **91**(4), 589–610.
- Benati, L. & Surico, P. (2009), 'VAR analysis and the Great Moderation', *American Economic Review* **99**(4), 1636–52.
- Bianchi, F. (2009), Three Essays in Macroeconometrics, PhD thesis, Princeton University.

- Bianchi, F. (2013), 'Regime switches, agents' beliefs, and post-world war II U.S. macroeconomic dynamics', *Review of Economic Studies* **80**(2), 463–490.
- Bianchi, F. & Melosi, L. (2013), Constrained discretion and central bank transparency, Technical report.
- Christiano, L., Eichenbaum, M. & Evans, C. (2005), 'Nominal rigidities and the dynamic effects of a shock to monetary policy', *Journal of Political Economy* **113**(1), 1–45.
- Cogley, T., Matthes, C. & Sbordone, A. M. (2011), Optimal disinflation under learning, Technical report.
- Debortoli, D. & Nunes, R. (2007), Loose commitment, International Finance Discussion Papers 916, Board of Governors of the Federal Reserve System (U.S.).
- Debortoli, D. & Nunes, R. (2008), The macroeconomic effect of external pressures on monetary policy, International Finance Discussion Papers 944, Board of Governors of the Federal Reserve System (U.S.).
- Del Negro, M. & Schorfheide, F. (2004), 'Priors from general equilibrium models for VARs', *International Economic Review* **45**, 643–673.
- Del Negro, M. & Schorfheide, F. (2008), 'Forming priors for DSGE models (and how it affects the assessment of nominal rigidities)', *Journal of Monetary Economics* **55**(7), 1191–1208.
- Eggertsson, G. B. & Woodford, M. (2003), 'The zero bound on interest rates and optimal monetary policy', *Brookings Papers on Economic Activity* **34**(2003-1), 139–235.
- Erceg, C. J. & Levin, A. T. (2003), 'Imperfect credibility and inflation persistence', *Journal of Monetary Economics* **50**(4), 915–944.

- Evans, G. W. & Honkapohja, S. (2001), *Learning and Expectations in Macroeconomics*, Princeton University Press.
- Farmer, R. E., Zha, T. & Waggoner, D. F. (2009), Understanding Markov-switching rational expectations models, NBER Working Papers 14710, National Bureau of Economic Research, Inc.
- Gallant, A. R. & McCulloch, R. E. (2009), 'On the determination of general scientific models with application to asset pricing', *Journal of the American Statistical Association* **104**(485), 117–131.
- Goodfriend, M. & King, R. G. (2005), 'The incredible Volcker disinflation', *Journal of Monetary Economics* **52**(5), 981–1015.
- Green, P. J. (1999), 'Penalized likelihood', *Encyclopaedia of Statistical Sciences* **3**, 578–586.
- Ireland, P. N. (1999), 'Does the time-consistency problem explain the behavior of inflation in the United States?', *Journal of Monetary Economics* **44**(2), 279–291.
- Ireland, P. N. (2005), Changes in the Federal Reserve's inflation target: Causes and consequences, Boston College Working Papers in Economics 607, Boston College Department of Economics.
- Justiniano, A. & Primiceri, G. (2008), Potential and natural output. Working Paper, Northwestern University.
- King, R. G., Lu, Y. K. & Pastén, E. S. (2008), 'Managing expectations', *Journal of Money, Credit and Banking* **40**(8), 1625–1666.
- Kydland, F. E. & Prescott, E. C. (1977), 'Rules rather than discretion: The inconsistency of optimal plans', *Journal of Political Economy* **85**(3), 473–91.

- Levine, P., McAdam, P. & Pearlman, J. (2008), 'Quantifying and sustaining welfare gains from monetary commitment', *Journal of Monetary Economics* **55**(7), 1253–1276.
- Liu, Z., Waggoner, D. & Zha, T. (2007), Has the Federal Reserve's inflation target changed?, Working Paper, Federal Reserve Bank of Atlanta.
- Liu, Z., Waggoner, D. & Zha, T. (2009), 'Asymmetric expectation effects of regime shifts in monetary policy', *Review of Economic Dynamics* **12**(2), 284–303.
- Lubik, T. A. & Schorfheide, F. (2004), 'Testing for indeterminacy: An application to U.S. monetary policy', *American Economic Review* **94**(1), 190–217.
- Mertens, E. (2008), Managing beliefs about monetary policy under discretion, Working Papers 08.02, Swiss National Bank, Study Center Gerzensee.
- Milani, F. (2007), 'Expectations, learning and macroeconomic persistence', *Journal of Monetary Economics* **54**(7), 2065–2082.
- Molnar, K. & Santoro, S. (2010), Optimal monetary policy when agents are learning, Technical report.
- Ozlale, U. (2003), 'Price stability vs. output stability: tales of federal reserve administrations', *Journal of Economic Dynamics and Control* **27**(9), 1595–1610.
- Preston, B. (2005), 'Learning about monetary policy rules when long-horizon expectations matter', *International Journal of Central Banking* **1**(2).

- Primiceri, G. (2005), 'Time varying structural vector autoregressions and monetary policy', *Review of Economic Studies* **72**(3), 821–852.
- Primiceri, G. (2006), 'Why inflation rose and fell: Policymakers' beliefs and US postwar stabilization policy', *The Quarterly Journal of Economics* **121**, 867–901.
- Roberds, W. (1987), 'Models of policy under stochastic replanning', *International Economic Review* **28**(3), 731–55.
- Ruge-Murcia, F. J. (2003), 'Does the Barro-Gordon model explain the behavior of US inflation? A reexamination of the empirical evidence', *Journal of Monetary Economics* **50**(6), 1375–1390.
- Sargent, T., Williams, N. & Zha, T. (2006), 'Shocks and government beliefs: The rise and fall of American inflation', *American Economic Review* **96**(4), 1193–1224.
- Schaumburg, E. & Tambalotti, A. (2007), 'An investigation of the gains from commitment in monetary policy', *Journal of Monetary Economics* **54**(2), 302–324.
- Sims, C. A. & Zha, T. (2006), 'Were there regime switches in u.s. monetary policy?', *American Economic Review* **96**(1), 54–81.
- Sleet, C. & Yeltekin, S. (2007), 'Recursive monetary policy games with incomplete information', *Journal of Economic Dynamics and Control* **31**(5), 1557–1583.
- Soderlind, P. (1999), 'Solution and estimation of RE macromodels with optimal policy', *European Economic Review* **43**(4-6), 813–823.
- Surico, P. (2007), 'The Fed's monetary policy rule and U.S. inflation: The case of asymmetric preferences', *Journal of Economic Dynamics and Control* **31**(1), 305–324.



Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

## A Tables for Main Body of the Paper

Table 1: Description of "standard" priors and the posterior mode. For parameters with restricted range (relative to how the usual range for the relevant distribution) the moments refer to the prior distribution *without* the restriction.

Variable	Prior Distribution	Range	Prior Mean	Prior St. Dev.	Posterior Mode
$\kappa$	Gamma	$[0, \infty]$	0.3	0.1	0.70
$\sigma$	Gamma	$[0, \infty]$	2	0.2	1.61
$\rho_g$	Inverse Gamma	$[0, \infty]$	0.8	0.1	0.40
$\rho_z$	Inverse Gamma	$[0, \infty]$	0.8	0.1	0.57
$\sigma_\nu$	Inverse Gamma	$[0, \infty]$	0.01	0.005	0.006
$\sigma_z$	Inverse Gamma	$[0, \infty]$	0.01	0.005	0.008
$\sigma_g$	Inverse Gamma	$[0, \infty]$	0.004	0.003	0.016
$\rho_{gz}$	Normal	$[-1, 1]$	0	0.3	0.61
$400 * \pi^c$	Normal	$[-\infty, \infty]$	2	1	1.76
$400 * \pi^d$	Normal	$[-\infty, \infty]$	6	1	5.48
$16 * \lambda^c$	Normal	$[0, \infty]$	0.8	0.5	0.07
$16 * \lambda^d$	Normal	$[0, \infty]$	2	0.5	0.49
$\lambda_i^c$	Normal	$[0, \infty]$	0.1	0.1	0.11
$\lambda_i^d$	Normal	$[0, \infty]$	0.1	0.1	0.31
$r$	Normal	$[-\infty, \infty]$	0.005	0.0025	0.0076

Table 2: Counterfactual outcomes. First row gives the imposed policy rule and beliefs for each counterfactual.

	data	(actual $i_t, p_t^c = 1$ )	( $i_t^c, p_t^c = 1$ )	( $i_t^c, p_t^c = 0.5$ )	( $i_t^d, p_t^d = 1$ )	( $i_t^d, p_t^d = .5$ )
mean, inflation	3.72	1.13	1.91	3.40	5.17	2.97
variance, inflation	6.45	1.57	1.38	2.28	5.14	2.42
mean, 100*output gap	0.01	-0.27	0.02	0.05	0.06	0.01
variance, 100*output gap	2.64	2.86	2.40	2.35	2.34	2.62
relative loss for committed CB	1	-	0.30	-	-	-
relative loss for discretionary CB	1	-	-	-	0.95	-
counterfactual sacrifice ratio	-	0.42	0.17	0.27	-	0.27

## B Figures for Main Body of the Paper

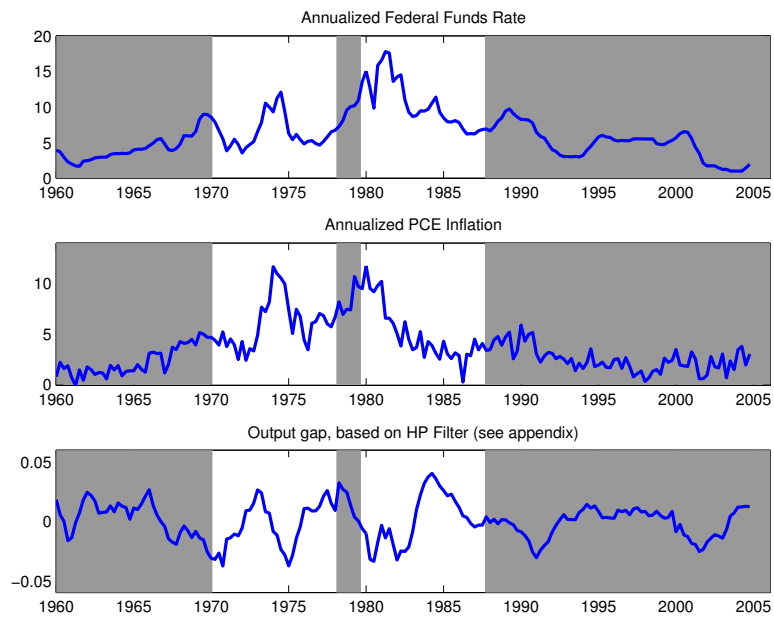


Figure 1: Data used in this paper

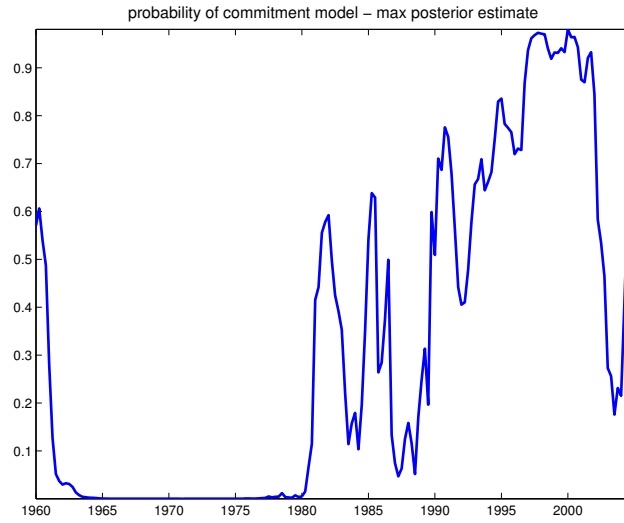


Figure 2:  $p_t^c$  at the posterior mode estimates

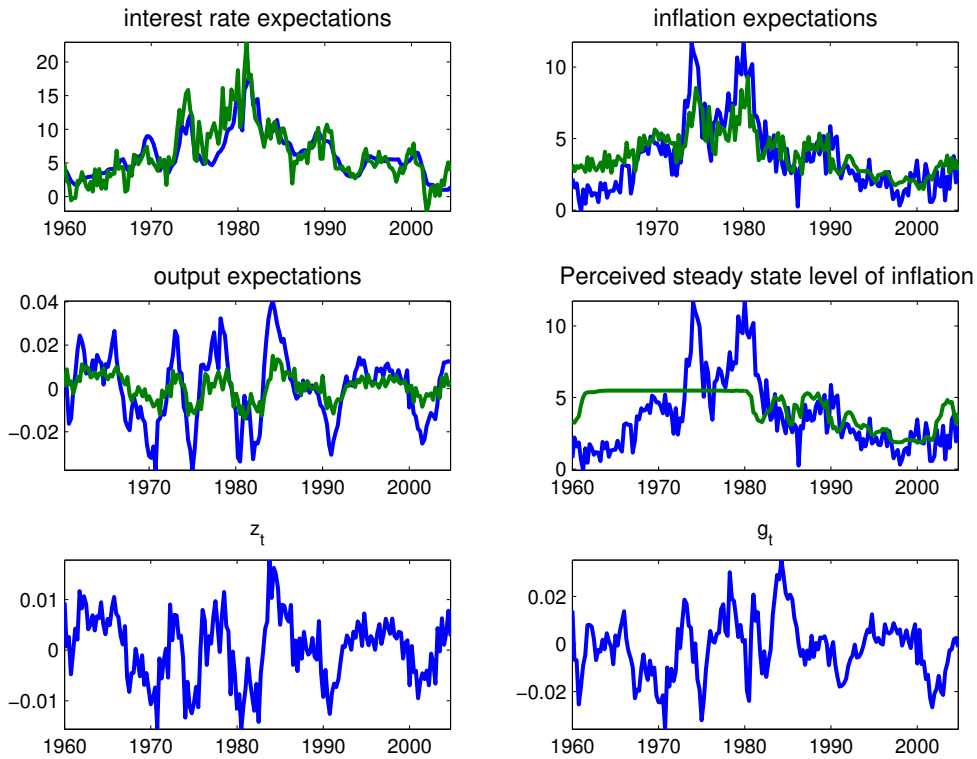


Figure 3: Expectations, perceptions and estimated shocks. All based on the posterior mode. Expectations/perceptions are in green, outcomes/data in blue

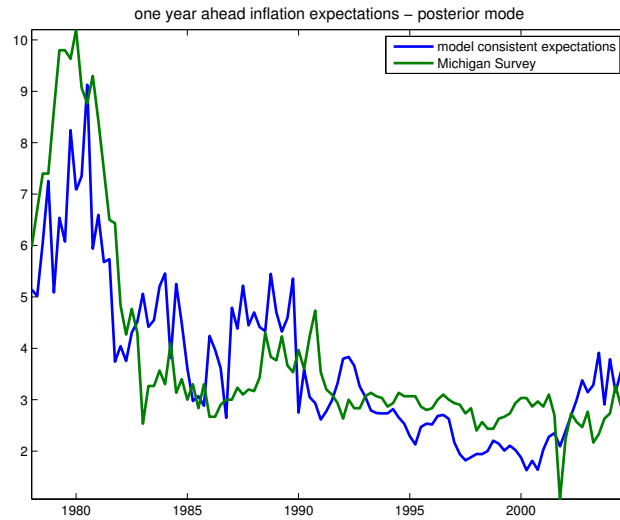


Figure 4: Private sector expectations from the model and a survey measure of inflation expectations

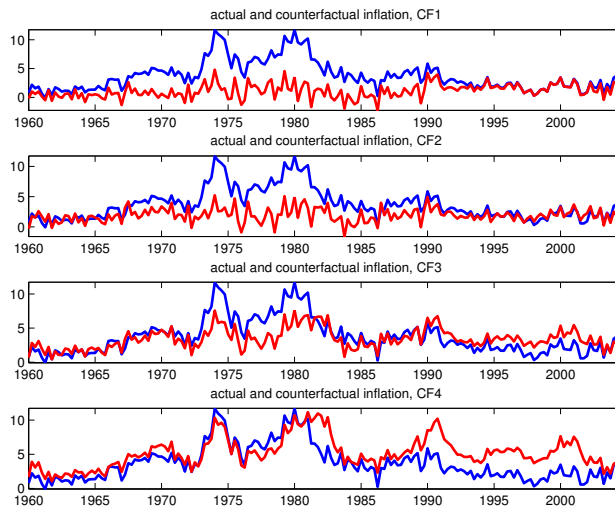


Figure 5: Counterfactual inflation series